229/2012

Raport Badawczy

Research Report

RB/55/2012

Generalization of the Nash Solution Concept to a Multicriteria Bargaining Problem

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Warszawa 2012

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1 Introduction

The paper deals with decision situations in which two agents acting on a market consider a possibility of cooperation. The cooperation is possible if it is beneficial for both of them. Decision makers representing the agents negotiate - bargain conditions of the cooperation. It is assumed that each of them has his individual set of objectives which he would like to achieve. Achievements of the objectives are measured by given vectors of criteria, which are in general different for each agent. The criteria are conflicting in the case of an individual agent as well as between them. Each agent has also his individual preferences defined in his space of criteria.

The bargaining process will succeed if the final cooperation conditions satisfy desirable benefits of each agent measured by the criteria and valuated according to the individual preferences. Information about possibilities and preferences of each agent is confidential. In many situation, at beginning of the bargaining process, agents can not be conscious of their preferences if they have not enough information about possible results of the cooperations. The preferences he can define knowing and comparing attainable variants of the cooperation. Let us consider the simplest buying - selling bargaining problem. A buyer and a seller propose prices of a good trying to find a consensus. The consensus is possible if there exists an interval of prices beneficial for both sides, called as an agreement set. In the case of positional negotiation an impasse is frequently observed, and negotiations can not succeed even if the agreement set is not empty. This can be resolved by applying a respective mediation procedure. The problem is much more complicated if each agents valuates variants of cooperations with use of his own vector of criteria. A variant should be found which will be accepted by both sides despite the fact that the criteria are conflicting in the case of each agents as well between them.

In this paper a multicriteria bargaining problem is formulated describing the mentioned decision situation. A bargaining process is proposed with use of a mediation procedure. In the procedure agents look for preferred variants of cooperation using reference point method supporting multicriteria analysis, while mediation proposals are generated with use of ideas of the Nash cooperative solution concept, satisfying respective fair play rules. The original Nash solution concept has been formulated for bargaining problem in which benefits of bargaining sides are scalar. A special construction is proposed generalizing the Nash solution concept on the case of multicriteria payoffs of bargainers. Using the idea an algorithm is proposed assuring that the mediation proposals reflect preferences of bargainers. It is assumed that the algorithm can be implemented in a computer based system supporting bargainers in negotiations.

Formulation and analysis of different solution concepts to the bargaining problem with scalar payoffs of players have been presented in many papers including (Nash 1950, 1953, Kalai and Smorodinsky 1975, Roth 1979, Thomson 1980, Peters 1986, Moulin 1988) and others. Papers dealing with multicroteria payoffs of players in bargaining are relatively rare. This paper continues the line of research presented in papers (Kruś and Bronisz 1993, Kruś 1996, 2001, 2011).

2 Problem formulation

Let us consider two decision makers negotiating conditions of possible cooperation. Each decision maker has defined decision variables, denoted by vector
$x_i = (x_{i1}, x_{i2}, \dots x_{ik^i})$, $x_i \in I\!R^{k^i}$, where
k^i is a number of decision variables of decision maker $i = 1, 2$, and $I\!R^{k^i}$ is a space of his decisions. Decision variables of all decision makers are denoted by vector $x = (x_1, x_2) \in I\!R^K$, $K = k^1 + k^2$, where $I\!R^K$ is cartesian product of the decision spaces of decision makers 1 and 2.

It is assumed that results of the cooperation are measured by a vector of criteria which is in general different for each decision maker. Criteria of decision maker i, $i = 1, 2$ presenting his payoff are denoted by vector
$y_i = (y_{i1}, y_{i2}, \dots y_{im^i}) \in I\!R^{m^i}$,
where m^i is a number of criteria of decision maker i, and $I\!R^{m^i}$ is a space of his criteria. Criteria of all decision makers are denoted by $y = (y_1, y_2) \in I\!R^M$, where $M = m^1 + m^2$. Space $I\!R^M$ is cartesian product of the citeria spaces of all decision makers.

We assume that a mathematical model is given describing payoffs of decision makers being result of decision variables undertaken by them. The model implemented in a computer based system will be used to derive payoffs of decision makers for given variants of decision variables. Formally we assume that the model is defined by a set of admissible decisions $X_0 \subset I\!R^K$, and by a mapping $W : I\!R^K \rightarrow I\!R^M$ from the decision space to the space of criteria. A set of attainable payoffs, denoted by $S_0 = W(X_0)$ is defined in the space of criteria of all decision makers. However ech decision maker has access to information in his criteria space only. In the space of criteria of ith decision maker a set of his attainable payoffs S_{0i}, can be defined, being subset of the set S_0. The set of attainable payoffs of every decision maker depends on his set of admissible decisions and on the set of admissible decisions of other decision maker.

A partial ordering is introduces in criteria spaces. Let $I\!R^m$ denote a space of criteria. Each of m criterions can be maximized or minimized. However, to simplify the notation and without loss of generality we assume that decision makers maximize all their criteria.

Let $z, y \in I\!R^m$, we say, that
a vector z **weakly dominates** y and denote $z \geq y$, when $z_i \geq y_i$ for $i = 1, 2, \ldots, m$,
a vector z **dominates** y and denote $z > y$, when $z_i \geq y_i, z \neq y$ for $i = 1, 2 \ldots, m$,
a vector z **strongly dominates** y and denote $z \gg y$, when $z_i > y_i$ for $i = 1, 2 \ldots, m$.

A vector $z \in I\!R^m$ is **weakly Pareto optimal** (weakly nondominated) in set $Y_0 \subset I\!R^m$ if $z \in Y_0$ and does not exist $y \in Y_0$ such, that $y \gg z$.
A vector $z \in I\!R^m$ jest **Pareto optimal** (nondominated) in set $Y_0 \subset I\!R^m$ if $z \in Y_0$ and does not exist $y \in Y_0$ such, that $y \geq z$.

A bargaining problem with multicriteria payoffs of decision makers (multicriteria bargaining problem) can be formulated by a pair (S, d), where element $d = (d_1, d_2) \in S \subset I\!R^M$ is called a disagreement point, and set S is agreement set. The agreement set $S \subset S_0 \subset I\!R^M$ is the subset of the set of attainable payoffs dominating the disagreement point d. The agreement set defines payoffs attainable by all decision makers but under their unanimous agreement. If such an agreement is not achieved, the payoffs of all decision makers are defined by the disagreement point d.

The multicriteria bargaining problem is analyzed under the following general conditions:

C1 agreement set S is compact and convex,

C2 agreement set S jest nonempty and includes at least one point $y \in S$ such, that $y \gg d$,

C3 disagreement point $d \in S_0$, additionally for any $y \in S$, we have $y > d$.

We assume, that each decision maker i, $i = 1, 2$, defines $d_i \in I\!R^{m^i}$ as his reservation point in his space of payoffs. Every decision maker, negotiating possible cooperation, will not agree for payoffs decreasing any component of the point. A decision maker can assume the point as the status quo point or analyzing alternative options, he can defined it on the basis of BATNA concept presented in (Fisher, Ury 1981). The BATNA (abbreviation of Best Alternative to Negotiated Agreement) concept, is frequently applied in processes of international negotiations, in a prenegotiation step. According to the concept, each side of negotiations should analyze possible alternatives to the negotiated agreement and select the best one according to its preferences. The best one is called as BATNA. It is the alternative for decision maker if negotiation will not succeed.

A question arises, how each decision maker can be supported in processes of decision analysis and in finding the agreeable solution. The analysis should include valuation of payoffs for different assumptions on their own decisions and decisions of the second decision maker, aiding in derivation of nondominated solution defining payoffs of decision makers in the agrement set. The solution should fulfil fair play rules such that it could be accepted by both decision makers as a cooperative solution. In this paper an interactive procedure is proposed including multicriteria decision support of each decision maker using reference point method developed by A.P. Wierzbicki (Wierzbicki 1986), (Wierzbicki, Makowski, Wessels 2000) and applies an idea of the Nash cooperative solution for derivation of mediation proposals. The solution has been originally formulated under axioms describing fair play distribution of cooperation benefits, that can be accepted by rational players in bargaining problem. The Nash solution (Nash 1950, 1953) has been originally proposed to the bargaining problem under assumptions of scalar payoffs of players. It can not be applied directly in the multicriteria bargaining problem considered here. This paper presents a construction enabling application of this idea in the case of multicriteria payoffs of decision makers.

3 A procedure - general view

The procedure is realized in some number of rounds $t = 1, 2, ..., T$. In each round t:

- each decision maker makes independently interactive analysis of nondominated payoffs in his multicriteria space of payoffs (the analysis is called further as unilateral) and indicates a direction improving his payoff in comparison to the disagreement point. The direction is selected by him according to his preferences as an effect of the multicriteria analysis.

- computer-based system collects improvement direction indicated by both decision makers and generates on this basis a mediation proposal d^t,
- decision makers analyze the mediation proposal and correct the preferred improvement directions, afterwards system derives next mediation proposal.

All mediation proposals d^t are generated on basis of the improvement directions indicated by the decision makers and with application of an assumed solution concept of multicriteria bargaining problem:

$d^t = d^{t-1} + \alpha^t[G^t - d^{t-1}]$, dla $t = 1, 2, ...T$,

where

$d^0 = d$,

α^t is so called confidence coefficient assumed by decision makers in round t,

G^t is a solution of multicriteria bargaining problem derived in round t, satisfying required properties. In this case a multicriteria solution concept is proposed which is a generalization of the Nash solution concept to the case of multicriteria payoffs of decision makers in the bargaining problem.

Each decision makers can in each round reduce improvement of payoffs (his own payoffs and at the same time payoffs of other decision maker) assuming respectively small value of the confidence coefficient.

4 Unilateral analysis

Unilateral analysis should lead given decision maker i, $i = 1, 2$ to derivation and selection of the Pareto optimal element in set S and the resulting direction improving his payoff according to his in mind preferences. Within the analysis the given decision maker generates and compares points representing Pareto frontier of set S in his s[ace of criteria. Unilateral analysis is made with use of the reference point method (Wierzbicki 1986, 1993) and with application of the respective achievement function.

Each nondominated point $\overline{y}_i$ of set S in the criteria space of decision maker i, is derived as the solution of the following optimization problem:

$$\max_{x \in X_0} s(y_i, r_i), \tag{1}$$

where
$r_i \in I\!R^{m_i}$ is a reference point of decision maker i in his space of criteria,
x is a vector of decision variables,
$y_i = v_i(x)$ defines vector of criteria of decision maker i, as dependent on decision variables x due to mapping W, under additional constraints assumption that criteria of the second decision maker are on the level of his reservation point $y_{3-i} = d_{3-i}$,
$s(y_i, r_i)$ is an achievement function approximating order in space $I\!R^{m_i}$.

A representation of Pareto frontier of set S can be obtained by solving the optimization problem for different reference points r_i assumed by decision maker i.

A general achievement function (Wierzbicki, Makowski, Wessels 2000) has the form:

$$\overline{s}(y_i, y_i^a, y_i^r) = \min_{1 \le k \le m_i} \sigma_{i,k}(y_{i,k}, y_{i,k}^a, y_{i,k}^r) + \rho \sum_{k=1}^{m_i} \sigma_i(y_{i,k}, y_{i,k}^a, y_{i,k}^r) \tag{2}$$

where $y_i = v_i(x)$, and $y_{i,k}^a, y_{i,k}^r$ denote respectively aspiration and reservation levels defined by decision maker i. Functions $\sigma_{i,k}(.)$ are of the form

$$\sigma_{i,k}(y_{i,k}, y_{i,k}^a, y_{i,k}^r) = \begin{cases} \beta(y_{i,k} - y_{i,k}^r)/(y_{i,k}^r - y_{i,k}^{lo}), & \text{jeśli} \quad y_{i,k}^{lo} \le y_{i,k} \le y_{i,k}^r \\ (y_{i,k} - y_{i,k}^r)/(y_{i,k}^a - y_{i,k}^r), & \text{jeśli} \quad y_{i,k}^r \le y_{i,k} \le y_{i,k}^a \\ 1 + \gamma(y_{i,k} - y_{i,k}^a)/(y_{i,k}^{up} - y_{i,k}^a), & \text{jeśli} \quad y_{i,k}^a \le y_{i,k} \le y_{i,k}^{up} \end{cases} \tag{3}$$

In the considered case $s(y_i, r_i) = \overline{s}(y_i, y_i^a, y_i^r)$, when reference points $y_i^a = r_i$ but the reservation point is assumed on the level of the disagreement point $y_i^r = d_i$. Parameters ρ, β, γ are assumed coefficients of the reference point method , ρ - is relatively small number, $0 < \beta < 1 < \gamma$, points y_i^{up} i y_i^{lo} denote relatively a point dominating the ideal point, and point dominated by the reservation point in the space $\mathbb{R}^{m_i}$. The points y_i^{up} and y_i^{lo} are assumed to normalize the optimization problem.

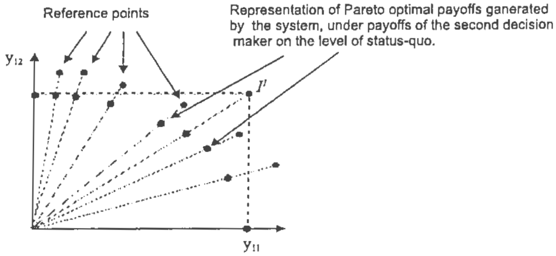

Figure 1: Generation of nondominated payoffs of decision maker 1 for assumed reference points

Fig. 1 illustrates how a decision maker can generate and review his attainable nondominated payoffs. He assumes different reference points and then the system derives respective Pareto optimal solutions. Reference points assumed by the decision maker and Pareto optimal payoffs $\overline{y}_i$ derived by the system are stored in a data base, so the decision maker can obtain a representation of Pareto optimal frontier of set S and can analyze it.

It is assumed that each decision maker i, $i = 1, 2$, finishing multicriteria analysis, indicates his preferred nondominated payoff $\widehat{y}_i$ in his space of criteria. The payoff corresponds to element $y^1 = (\widehat{y}_1, d_2) \in S$ in the case od decision maker $i = 1$ and respectively element $y^2 = (d_1, \widehat{y}_2) \in S$ in the case of decision maker $i = 2$. The last elements are defined in the space of criteria of both decision makers. The stage of unilateral analysis is finished when both decision makers have indicated their preferred payoffs.

Unilateral analysis can be realized in different ways with respect to access to information available for decision makers. In the presented way it is assumed that each decision maker makes unilateral analysis not knowing criteria nor reservation point of the second decision maker. The mediator only has access to the full information. This information is obviously used in calculation of the computer based system. In general any decision maker has not permission to data introduced and generated by the other one.

5 Derivation of mediation proposal

A mediation proposal is derived by the system when both decision makers have indicated their preferred payoffs $\widehat{y}_1, \widehat{y}_2$ in their spaces of criteria and when respective points $y^1, y^2 \in S$ have been calculated by the system.

Let us construct a hyperplane H^2 defined by points d, y^1, y^2. Each point $y \in H^2$ may be defined as

$$y = d + a_1(y^1 - d) + a_2(y^2 - d).$$

Let A denote mapping from H^2 to $\mathbb{R}^2$ defined by $A(y) = A[d + a_1(y^1 - d) + a_2(y^2 - d) + \cdots + a_n(y^n - d)] = (a_1, a_2, \ldots, a_n)$. A two person bargaining problem $(A(S^H), A(d))$ can be considered on hyperplane H^2. Set $S^H = S \cap H^2$ in the problem and payoffs of decision makers are scalar on the hyperplane.

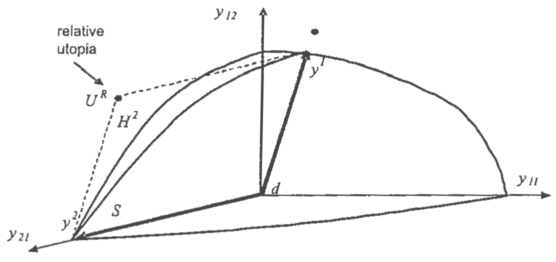

Figure 2: Construction of hyperplane H^2.

A generalization of the Nash cooperative solution concept can be constructed using hyperplane H^2. Fig. 2 presents a construction of plane H^2 for a multicriteria bargaining problem of two decision makers. In this example decision maker 1 has two criteria $y_{1,1}$ and $y_{1,2}$ respectively, decision maker 2 has only one criterion $y_{2,1}$. Let point y^1 be defined according to preferences of the first decision maker. The preferred point y^2 of the second one is defined by the maximal attainable value of his payoff. Hyperplane H^2 is defined by points d, y^1 and y^2.

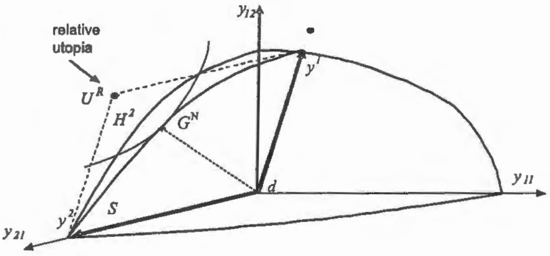

Figure 3: Construction of generalized Nash solution to the multicriteria bargaining problem

A construction of the solution to the multicriteria bargaining problem, which is based on the Nash idea is presented in Fig. 3. Arrows drown on the figure present improvement directions leading to the nondominated payoffs y_{11}, y_{12} selected by decision maker 1 and to the nondominated payoff y_{21} selected by decision maker 2 respectively.

The Nash cooperative solution $y^N = f^N(S,d)$ to bargaining problem (S,d) is defined as the point of set S maximizing product of the payoffs increases for decision makers 1 and 2 on hyperplane H^2. The point fulfills the following axioms for final payoffs $y \in I\!R^2$ under assumptions that preferences of decision makers are expressed by points y^1 i y^2:

(A1) Pareto-optimality
$y^N = f^N(S,d)$ is Pareto-optimal in set S,

(A2) Individual rationality
For every bargaining game (S,d), $y^N = f^N(S,d) \geq d$.

(A3) Symmetry
We say, that bargaining problem (S,d) is symmetric, if $d_1 = d_2$ and $(x_1,x_2) \in S$, then $(x_2,x_1) \in S$. We say, that a solution fulfills symmetry property, if for symmetric (S,d) problem, $f_1^N(S,d) = f_2^N(S,d)$.

(A4) Independence of equivalent utility representation
Let L be a affine mapping, i.e. such that $Lx = (a_1x_1 + b_1, a_2x_2 + b_2)$

for any $x \in R^2$, where $a_i, b_i \in R, a_i > 0, i = 1, 2$. We say, that a solution is independent of equivalent utility representation, if $Lf^N(S,d) = f^N(LS, Ld)$.

(A5) Independence of irrelevant alternatives
Let (S,d) and (T,d) be bargaining problems such that $S \subset T$ and $f^N(T,d) \in S$.
Then $f^N(S,d) = f^N(T,d)$.

The last axiom means that if decision makers have agreed solution $f^N(T,d)$ in bargaining problem (T,d), then decreasing of agreement set T to set S which includes the solution, i.e. $f^N(T,d) \in S$, should not change the final payoffs of decision makers.

According to the Nash theorem (Nash 1950), for any bargaining problem (S^H, d) satisfying assumptions C1 - C3 there exists one and only one solution $f^N(S^H, d)$ of the form:

$$f_N(S^H, d) = \arg\max_{y \in S^H} ||y_1 - d_1|| \cdot ||y_2 - d_2||,$$

satisfying axioms A1 - A5.

$||.||$ is a distance measured on hyperplane H^2.

Axioms A1 - A5 can be treated as fair play rules satisfied by the mediation proposal constructed according to the Nash solution concept. Axiom A1 assures efficiency of the solution in set S. The solution is individually rational according to axiom A2. Axiom A3 means that both decision makers are treated in the same way. Axiom A4 prevent possible manipulation of decision makers by changing scales measuring their payoffs, i.e. any decision maker will not benefit by changing scales measuring his payoffs. Comparison of different solution concepts to the multicriteria bargaining problem, based on ideas of Raiffa-Kalai-Smorodinsky, Lexicographic, Nasha, Equitable Solutions and their properties can be found in (Kruś 2011). For example Equitable solution does not fulfill Axiom A4. In this case distribution of cooperation benefits defined by this solution is malleable on possible manipulation of decision makers changing the scales.

6 Algorithm

It is assumed that the following algorithm is implemented in a computer-based system. The system supports multicriteria analysis made by decision makers and derives mediation proposals.

Let $d^t \in S$ denote a vector of payoffs in round t for $t = 1, 2, \ldots,$ and $d^0 = d$. Let $S^t = \{y : y \in S, y > d^{t-1}\}$.

Each decision maker i has the following parameters to control proces of multicriteria analysis and derivation of mediation proposal: reference points $r_i^t \in I\!R^{m^i}$, indicated preferred payoff nondominated in set S and confidence coefficient $\alpha_i^t \in (\delta, 1]$, where δ is a relatively small positive number $\delta > 0$.

On the basis of the reference points assumed by given decision maker attainable nondominated payoffs are derived, analyzed further by him. He is asked to indicate the preferred payoff. Each decision maker has access only to information in his own space of criteria. He does not know criteria nor attainable payoffs of the second decision maker.

Each decision maker can reduce improvement of his payoff, and at the same time of payoff of the second decision maker, in given round assuming relatively small value for the confidence coefficient.

Step 1. Set $t = 1$.

Step 2. System invites decision makers $i = 1, 2$ to make independently analysis of their nondominated payoffs in multicriteria bargaining problem (d^{t-1}, S^t).

Step 2.1 System presents to decision maker i information about the ideal point I_i^t, and the status quo point d_i^{t-1} in the decision maker criteria space. The ideal point is derived as $I_i^t = (I_{i,1}^t, I_{i,2}^t, \ldots, I_{i,m_i}^t)$, where $I_{i,j}^t = \max y_{i,j} : y = (y_1, y_2) \in S^t \wedge y_{3-i} = d_{3-i}$.

Step 2.2 Decision maker i writes values of components of his reference point r_{ij}^t, $j = 1, 2, \ldots, m^i$.

Step 2.3 System derives the nondominated solution in set S, solving optimization problem 1 and stores the solution in a data base.

Step 2.4 The decision maker analyzes generated nondominated payoff (payoffs). If he has enough information to select the preferred payoff, he indicates it as $\widehat{y}_i$ and assumes value for the confidence coefficient α_i^t. He signals finishing of the unilateral analysis phase.

Step 2.5 Has decision maker i finished unilateral analysis?
If no - go to Step 2.2, to generate next nondominated payoff. If yes - system writes the preferred nondominated payoff indicated by the decision maker $\widehat{y}_i$ as well as assumed value of confidence coefficient α_i^t to a data base.

Step 3. System checks whether both decision makers have finished their unilateral analysis, selected their preferred payoffs and defined values of the confidence coefficients. If no - system waits as long as they will finish generation and analysis of payoffs in Steps 2.1-2.5.

Step 4. System derives points $y^1 = (\widehat{y}_1, d_2^{t-1})$ and $y^2 = (d_1^{t-1}, \widehat{y}_2)$. Hyperplane H^2 is defined on this basis.

Step 5. System derives mediation proposal $d^t = (d_1^t, d_2^t)$ at round t,

$$d^t = d^{t-1} + \alpha^t[G^t - d^{t-1}],$$

where $G^t = \arg\max_{y \in S^H} ||y_1 - d_1^{t-1}|| \cdot ||y_2 - d_2^{t-1}||$,
$\alpha^t = min\{\alpha_1^t, \alpha_2^t\}$, $0 < \rho < \alpha_i^t \leq 1$ for $i = 1, 2$.

Step 6. System presents mediation proposal - payoffs d_i^t to decision makers $i = 1, 2$ respectively.

Step 7. System checks the cooperative solution of the round. Is it Pareto optimal in set S?
If yes - end of the procedure.
If no - set number of next round $t = t + 1$ and go to Step 2.

In the algorithm a sequence of bargaining problems (S^t, d^{t-1}) is formulated and analyzed. Decision makers make in each round independent analysis of nondominated payoffs in set S^t using reference points. Then each of them selects his preferred payoff. This is made in Steps 2.1-2.5. The selected payoffs and confidence coefficients assumed by decision makers are used by the system to derive a mediation proposal which is proposed to the decision makers in a given round (Steps 4-7). Proposed construction of the mediation proposal assures that the proposal is consistent with preferences of all decision makers in the given round. Decision makers using confidence coefficients can inflow on the number of following rounds of the procedure. They can again analyze Pareto optimal frontier of set S in these rounds and correct previously indicated preferences. The mediation proposal derived by the system according to ideas of the Nash cooperative solution, defines distribution of the cooperation benefits which fulfills axioms A1 - A5 describing fair play rules.

It can be shown that a sequence of the mediation proposals derived in the procedure converges to the Pareto optimal element in set S, similarly as in the procedure using the generalized Raiffa-Kalai-Smorodinsky presented in (Kruś, 2011). The last paper includes formal prove of the convergency.

7 Conclusions

Construction of a solution concept to the multicriteria bargaining problem is proposed. The bargaining problem describes decision situations in which two decision makers negotiate possible cooperation in realization of a join enterprize and each of them valuates effects of the cooperation by his own different set of criteria. The proposed solution concept generalizes on the multicriteria case the known solution concept proposed by Nash for classical bargaining problem with scalar payoffs of players.

An original algorithm is also proposed to support multicriteria analysis made by the decision makers as well as a mediation process leading the decision makers to a consensus. Multicriteria analysis is made with use of the reference point approach. In the algorithm a sequence of mediation proposals is generated with use of the proposed solution concept, taking into account preferences expressed by the decision makers.

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