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Computational Analysis of Cooperative Solutions in an Education System

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COMPUTATIONAL ANALYSIS OF COOPERATIVE SOLUTIONS IN AN EDUCATION SYSTEM

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Abstract

The paper deals with a model of an education system. The model describes cooperation of university with students in the presence of risk. A bargaining problem is formulated. Three cooperative solution concepts (Nash, Raiffa and Egalitarian) to the problem are analyzed. The analysis is made from axiomatic point of view, taking into account properties of the solutions concepts. Next a computational analysis is made. An experimental system based on the model has been constructed. A number of computational experiments have been made. Results of the experiments are presented and discussed.

1. Introduction

The paper deals with economic analysis of education. The education is considered as a process in which several actors, for example a university and students take part and can achieve some benefits. The analysis is made using tools of cooperative game theory and numerical methods. The analysis is based on a model originally proposed by Kulikowski (2002d).

A literature on the economics of education includes among others Blaug (1970), Cohn (1979), Schultz (1976), Knauff (2002). The Kulikowski's model describes expected benefits of a university organizing studies and benefits of students obtaining knowledge and higher qualifications, including the risk attached to both the sides. The risk is taken into account according to the URS (Utility-Return-Safety) methodology developed in papers (Kulikowski, 2000, 2002a,b,c) and applied also in (Krus, 2002) for analysis of Nash cooperative strategies in the case of innovation activity.

In this paper the Kulikowski's model is a base for formulation of a cooperative game, namely a bargaining problem in which a university and a student are considered as bargaining sides. The problem consists in a proper allocation of benefits resulting from the cooperation. The proper allocation means that it will be unanimously accepted by booth the sides. In other case, i.e. when there is no agreement (the cooperation will not occur), the payoffs of the sides are defined by a status quo point. There is a wide literature devoted to the bargaining problem starting from Nash, (1950), Kalai, Smorodinsky (1975), Roth (1979), Thomson (1997) and many others. A number of different solution concepts have been proposed and discussed. In this paper three frequently used solution concepts, namely Egalitarian solution, Raiffa-Kalai-Smorodinsky, and Nash solutions are applied to the formulated bargaining problem describing the university – student cooperation. Outcomes of the sides resulting from the solutions are compared and discussed using axiomatic approach, i.e. analyzing properties of the solutions. After that a computational analysis is made. Optimization problems have been formulated. An experimental computer-based system has been designed and implemented. The system includes model description and a computational part, which enables solving the optimization problems and calculating payoffs of the sides. Some number of calculation experiments has been made with use of the system. Obtained results are presented in the paper and discussed. Payoffs of the sides due to the three solution concepts are compared. It is shown for example how the payoffs depend on the preferences of the sides regarding the risk, considered and taken into account in the model.

2. Model

The model describes cooperation of university with students. Quantities and relations of the model are presented in the following for the university and the student respectively.

2.1. University

The university, organizing studies, has to cover some cost. A part of the cost is constant. Other part of the cost depends on the number of students in a given year. Therefore overall cost of university per annum, per one student can be defined by:

$$C_1^b = C_0/N + C_1,$$

where

C_0 denotes constant cost per annum,

N - number of students,

C_1 - variable cost per one student.

The present value of receipts:

$$P = P_0\,\rho,$$

where

P_0 is average tuition of one student per annum,

$\rho = 1 + \frac{1}{1+r} + \ldots + \frac{1}{(1+r)^{T_0}}$,

r - discount rate, T_0 - period of studies in years.

The ratio of the tuition to the overall cost is considered as a decision variable

$$x = P_0/C_1^b.$$

Therefore the receipts of the university can be described by

$$P = C_1^b\,\rho\,x.$$

The rate of return:

$$R_1(x) = (P - C)/C, \text{ where } C = C_1^b\,\rho.$$

In the above description, the present value of the receipts is compared to the present value of the overall cost. It depends on the decision variable x.

2.2. Student

In the case of student we compare cost of studies he has to pay and value of education he obtains. Cost of studies is considered as the present value of tuition paid during the period of studies [0, T_0]:

$$P=P_0\,\rho.$$

The value of education is calculated as the excess of salary that the student will obtain in the future period of time [T_0, T] for qualifications he will have being graduated.

$$V = V_0 \sum_{t=T_0}^{T} (1+k)^{-t}, \qquad \text{(V)}$$

where V_0 is the expected excess of salary per annum for the qualifications, and k is a discount rate.

Rate of return

$$R_2(x) = (V - P)/P = V/(x\,C) - 1.$$

2.3 Utilities

Both the sides, university and student act in a presence of risk. Their risks are included in description of utilities with use of safety index, the notion introduced by Kulikowski (2000, 2002) within URS methodology.

The utility of university

$$U_1(x) = C_1^b \rho R_1(x) S_1^{\beta_1}.$$

The student's utility

$$U_2(x) = P R_2(x) S_2^{\beta_2}.$$

According to the URS methodology the safety index of random variable R' representing the rate of return, with expected value R and standard deviation σ, is described by $S=1-\kappa\sigma/R$, where κ is quantile of probability distribution function, defining probability p^w of the worst case, $p^w = Pr\{R^t \leq R - \kappa\sigma\}$. The safety index takes values from the interval [0, 1].

In our case both the rates of return are random variables with expected values $R_1(x)$ and $R_2(x)$. The safety indices S_1 and S_2 express risk attached to the university and the student respectively. Let us see that grater risk results in lower value of the safety index.

During the time the student stays at university, the risk relates to the fact that the student may not pass exams and can resign. In such a case the student will waste the tuition already paid. The university will not obtain the full tuition and will have not covered part of the constant cost C_0/N already spent. After the studies the risk relates solely to the student. It is the risk that the student will not obtain the salary as high as he has assumed, and in the worst case that he can be unemployed. The future excess of salary V is a random variable. The risk is expressed by dispersion of the variable. It can be taken into account with use of the discount rate k in the formula (V).

3. Cooperative solutions

The model presented above describes utilities of the university U_1, and the student's utility U_2 as functions of the expected return and the safety index. The university and the student can be considered as sides (players) in a bargaining problem. Let us see that each side tries to select the decision variable, namely the variable x to maximize its own utility. Set of attainable utilities denoted further by $A \in \boldsymbol{R}^2$ is given by admissible values of the decision variable and the model relations. Minimum values of utilities accepted by each of the sides are defined as a status quo point. The values can be calculated in general case using BATNA concept formulated by Fisher, Ury (1981). According to the concept each of the sides, preparing to negotiation in bargaining process, should analyze its position and evaluate his BATNA (Best Alternative To Negotiation Agreement).

In the case of university the minimum accepted utility could be calculated from the following acceptation condition:

$$U_1(x) \geq U_{1min} = C_1^{\,b} R_F,$$

where R_F denotes risk free rate of return calculated on the base of governmental bonds. The condition means that the university utility should not be less than the utility obtained from investment in risk free governmental bonds. According to the condition the minimum value of decision variable

$$x_{\min} = 1 + \frac{R_F}{S_1^{1-\beta_1}},$$

accepted by university can be calculated.

In a similar way the minimum utility accepted by student can be derived:

$$U_2(x)\,) \geq U_{2min} = P\, R_F.$$

Using the model relation the maximum value of the decision variable

$$x_{\max} = \frac{V}{C_1^b + C_1^b / S_2^{1-\beta_2}}$$

accepted by student can be derived. We assume that the student does not accept utility less than the utility he could obtain investing in risk free governmental bonds the sum of money equal to his discounted expenditures (spent on the studies).

The status quo point denoted by $d \in \boldsymbol{R}^2$ can be then defined by $d = (U_{1min}, U_{2min})$.

Analyzing possible cooperation of the sides we look for payoffs - pairs $(U_1, U_2) \in A$ which are attainable and would be unanimously accepted by both the sides as fair. More precisely we try to find a function $f(.)$ defining a unique point $f(A,d) = U^C = (U_1^C, U_2^C) \in A$. The function is called cooperative solution to the bargaining problem. In the theory of cooperative games some number of solution concept have been formulated for different assumption (axioms) about feeling of the sides what the fairness mean. In the following several axioms, frequently assumed will be discussed as a base for analysis of three types of cooperative solutions: Egalitarian, Nash and Raiffa-Kalai-Smorodinsky solution concepts.

Egalitarian solution $f^E(A, d)=(U_1^E, U_2^E)$ is the maximal point of A of equal coordinates in comparison to status quo, i.e.

$(U_1^E - U_{1min}) = (U_2^E - U_{2min}), (U_1^E, U_2^E) \in A.$

In our case it can be derived solving the following optimization problem:

$\lambda(A, d) = max_x \{t: [(t+U_{1min}), (t+U_{2min})] \in A\}.$

The quantity $\lambda(A, d)$ defines the maximum increase of utility, which can be obtained by each of the players (it is the same for each of them). Solving the above problem we can find the value of decision variable $x=x^E$ and utilities $U_1(x^E)$, $U_2(x^E)$ according to the egalitarian solution concept.

Property 1. Strong monotonicity

Let $f(A,d)$ denote a solution concept to the bargaining problem. We say that the solution concept is strongly monotone if the following condition holds. For any two bargaining problems with agreement sets A and A', if $A' \supseteq A$, then $f(A', d) \geq f(A, d)$.

The property means that if opportunities expand, than all players should weakly gain.

Property 2. Weak Pareto optimality

The outcome $y=f(A, d)$ generated by the solution is weakly Pareto optimal in the set A, i.e. there is no element $z \in A$ such that $z>y$, (*for* $z=(z_1, z_2)$, $y=(y_1, y_2)$, $z>y$ *means* $z_i>y_i$ *for* $i=1,2$).

Property 3. Symmetry.

Let the problem (A, d) be symmetric (i.e. $d_1=d_2$, and if a point $(y_1, y_2) \in A$, then $(y_2, y_1) \in A$). Then $f_1(A, d) = f_2(A, d)$.

The property requires that the solution should not distinguish between the parties if the model does not. It means that if the parties have the same bargaining positions, they should have obtained the same utilities.

Theorem (Kalai, 1977)

The egalitarian solution is the only solution satisfying strong monotonicity, weak Pareto optimality and symmetry properties.

Nash solution $f^N(A,d)=(U_1^N, U_2^N)$ is the point of A at which the product of utility gains is maximized, i.e. $(U_1^N, U_2^N)=arg\ max\ (U_1-U_{1min})\ (U_2-U_{2min})$ for $(U_1, U_2) \in A$.

In the case of considered model the solution can be derived solving the problem:

$max_x\{(U_1(x)-U_{1min})(U_2(x)-U_{2min})$, subject to the constraint $[U_1(x), U_2(x)] \in A$.

The obtained value of decision variable $x=x^N$ defines values of utilities proposed to the sides according to the solution.

Property 4. Independence of Equivalent Utility Representations. (Scale invariance)

Let a_k, b_k be real numbers, $a_k>0$, $k=1,2$, where the subscript $k=1$ relates to the university and $k=2$ to the student. Let for the problem (A,d) we define the problem

(A',d'): $A'=\{y \in \mathbf{R}^2$: *there exists* $z \in A$, *such that* $y_k=a_k z_k+b_k$, $k=1,2\}$, $d_k'= a_k d_k+b_k$, $k=1, 2$.

Then $f_k(A', d')=a_k f_k(A, d) + b_k$, $k=1, 2$.

The property says that the solution is invariant to affine transformations of utilities. Any party can not benefit changing for example the scale of his utility.

Property 5. Collective rationality (Pareto optimality)

The outcome $y=f(A, d)$ generated by the solution is Pareto optimal in the set A, i.e. there is no element $z \in A$ such that $z \gtrsim y$, , (*for* $z=(z_1, z_2)$, $y=(y_1, y_2)$, $z \gtrsim y$ *means* $z_i \gtrsim y_i$ *for* $i=1,2$, *and* $z \neq y$).

According to the property the solution will select an outcome such that no other feasible outcome is preferred by both the parties.

Property 6. Independence of Irrelevant Alternatives.

Let us consider two problems: (A,d) and (B,d) such that $B \subset A$. If $f(A,d) \in B$ then $f(A, d)=f(B,d)$.

It means that if an outcome generated by the solution $f(A,d)$ belongs to a reduced agreement set B, then it has to be also equal to the solution of the problem (B, d).

Theorem (Nash, 1950)

The Nash solution is the only solution satisfying properties of Pareto optimality, symmetry, independence of irrelevant alternatives and independence of equivalent utility representations.

Raiffa-Kalai-Smorodinsky solution is the maximal point of A on the segment connecting the status quo to the ideal point of A. The ideal point $U^I(A)=(U_1^I(A), U_2^I(A))$ is defined by:

$$U_i^I(A)=max\{a_i : a=(a_1,a_2) \in A\}.$$

The solution can be derived solving the problem:

$\lambda(A, d)= max_x \{t: [(t(U_1^I(A)-U_{1min})+U_{1min}), (t (U_2^I(A)-U_{2min})+U_{2min})] \in A\}$.

The optimum value of decision variable $x=x^R$ defines values of utilities proposed to the sides according to the Raiffa-Kalai-Smorodinsky solution concept:

$U_1(x^R)= \lambda(A, d)(U_1^I(A)-U_{1min})$.

$U_2(x^R)=\lambda(A, d)(U_2^I(A)-U_{2min})$.

Let us see that the maximum increases of utilities proposed to the sides $i=1,2$ are in proportion to $U_i^I(A)-U_{imin}$.

Property 7. Individual monotonicity.

For any bargaining problems *(A, d)* and *(B, d)*, if $B \supseteq A$ and components of the ideal points $U_i^I(A) = U_i^I(B)$ for all $i \neq j$ then $f_j(B) = f_j(A)$.

The property means that if opportunities expand in direction of favorable to one of the parties then the party weakly gains.

Theorem (Kalai-Smorodinsky, 1975)

The Raiffa- Kalai-Smorodinsky solution is the only solution satisfying weak Pareto optimality, symmetry, scale invariance and individual monotonicity.

4. Numerical results

An experimental computer-based system has been constructed on the base of the model presented above. The system enables simple simulations, calculating required expenditures and benefits, which each of the sides can obtain. The system enables also derivation of different solutions, calculation of resulting payoffs of the sides and other quantities described in the model. For this reason, the optimization problems formulated in the previous point are implemented in the system and solved. The system enables computational analysis of the problem. It can be checked for example, how the solutions – payoffs of the sides depend on some input variables. Some results obtained with use of the system are presented in the following figures.

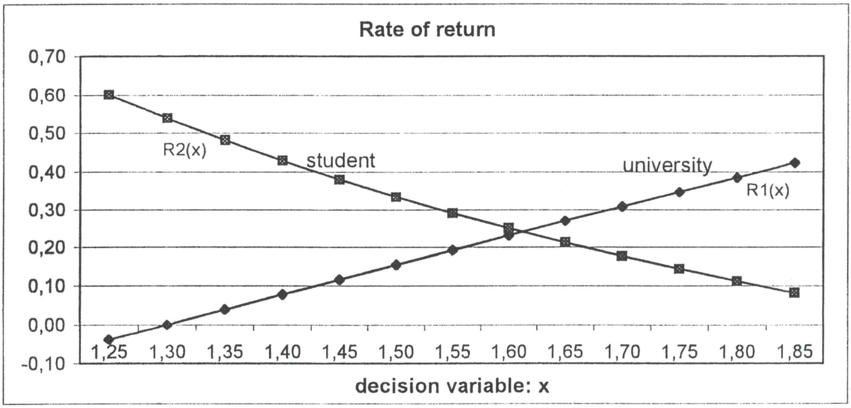
Assumed initial values of the model parameters and quantities are as follows:

- the overall cost of university per annum per one student C_1^b=9000 PLN,
- the discounted value of education calculated as the excess of salary that will be obtained for qualifications: V=18000 PLN,
- the risk free rate of return R_F= 0.08,
- the university's safety index S_1=0.9, and the parameter β_1=0.5,
- the student's safety index S_2=0.9, and the parameter β_2=0.5.

Figure1 presents results of model simulations. It is shown how the main model output variables: rate of return R1(x) obtained by university, R2(x) - obtained by student and respective utilities U1(x, S1), U2(x, S2) depend on the decision variable *x*. The derived utilities are compared to the utilities U1_Rf, U2_Rf that could be achieved by the university and the student respectively, in the case of investing in risk free governmental bonds. The cooperative solution outcomes (Nash, Raiffa and Egalitarian) have been calculated and are presented. Let us see that the university does not agree for *x* lower than 1.41 (when U1(x, S1) is lower than U1_Rf) and the student does not agree for *x* grater than 1,85. The Egalitarian solution gives equal increases of utility for both the sides. In the case of Raiffa solution the increases are in proportion to the maximum increases possible to achieve by each of the sides. The Nash solution gives maximum product of the increases.

Figures 2, 3 and 4 illustrate how the model outputs for the considered solution concepts depend on risk, which incurs the university. The risk is described in the model by safety index S_1. The figures present curves of optimum values of the decision variable x, rates of return R1 and R2, utilities U1 and U2 calculated for university and student respectively (*the notation used in this section is the same as in computer outprints of the system*). Each point presented on the figures has been calculated solving respective optimization problem mentioned in the previous section. The optimum values of decision variable are compared to the minimum and maximum values resulting from the acceptation conditions. The calculated utilities are compared to the utilities achieved in the case of risk free investments. According to the definition of the safety index, if greater risk incurs the university then the safety index is lower. In such a case, for all the solution concepts, the university is then awarded by grater rate of return, but its utility U1 decreases.

Figures 5, 6, 7 present results of similar research in the case of student for the three considered solution concepts. If the risk incurring the student is greater, what results in lower



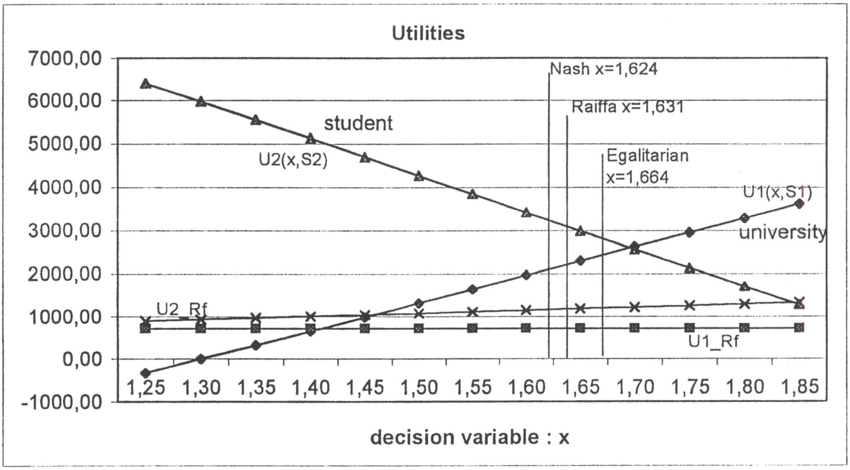


Fig. 1. Selected results of model simulations

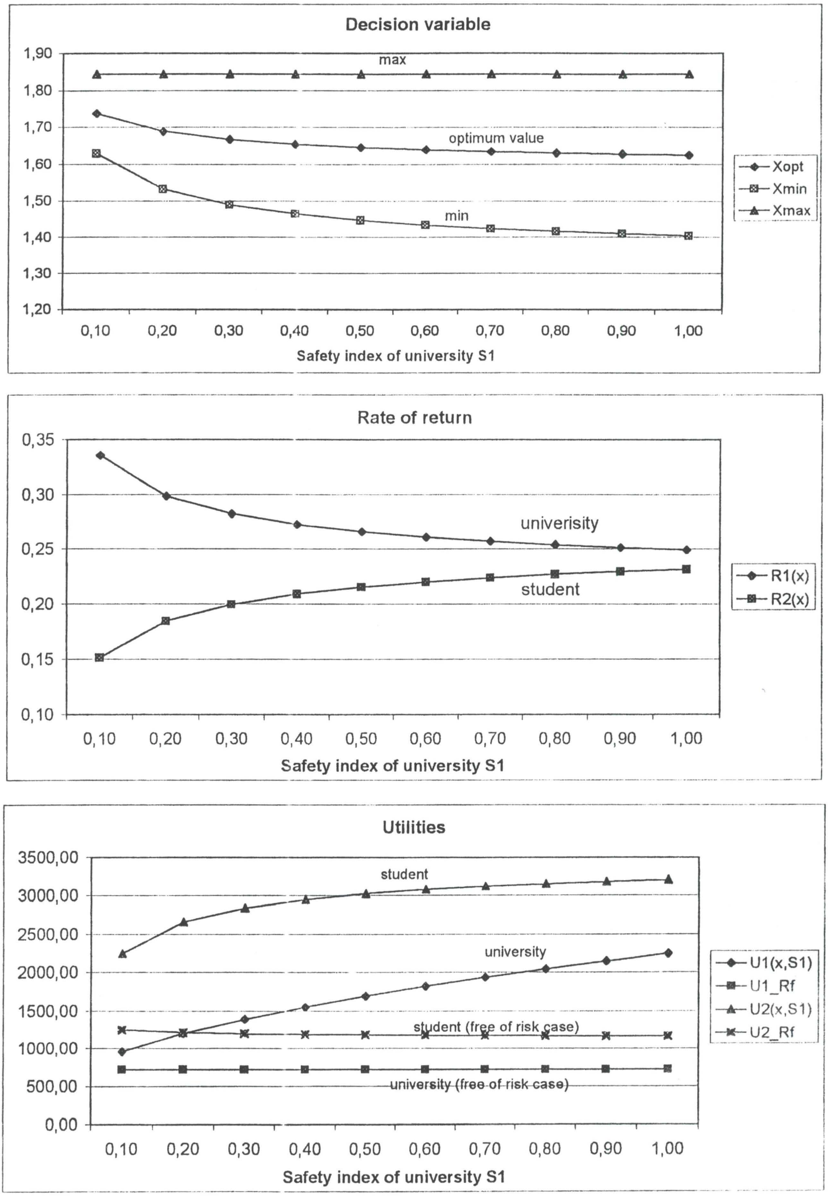


Fig. 2. Nash cooperative solution for different values of university safety index S_1, S_2=0.9, $\beta_1 = \beta_2$=0.5.

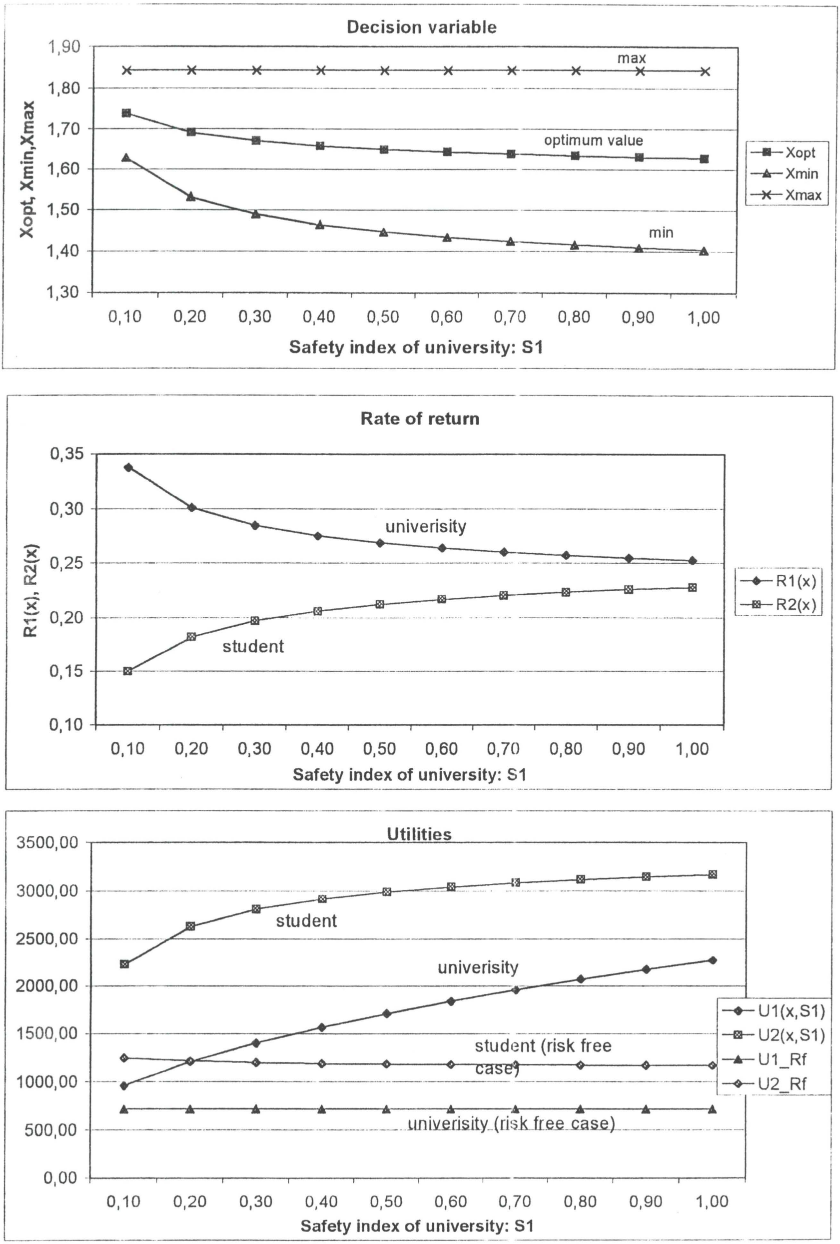


Fig. 3. Raiffa cooperative solution for different values of university safety index S_1, S_2=0.9, $\beta_1 = \beta_2$=0.5.

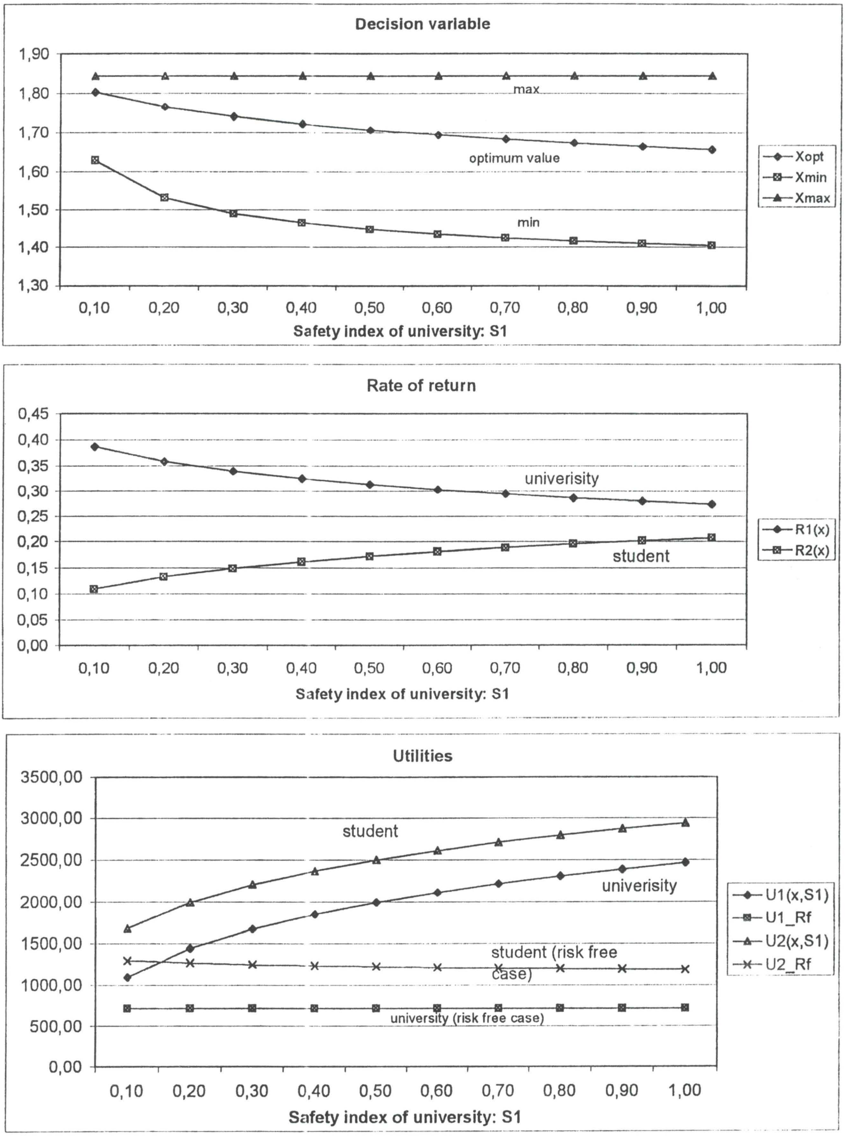


Fig. 4. Egalitarian cooperative solution for different values of university safety index S_1, S_2=0.9, β_1= β_2=0.5.

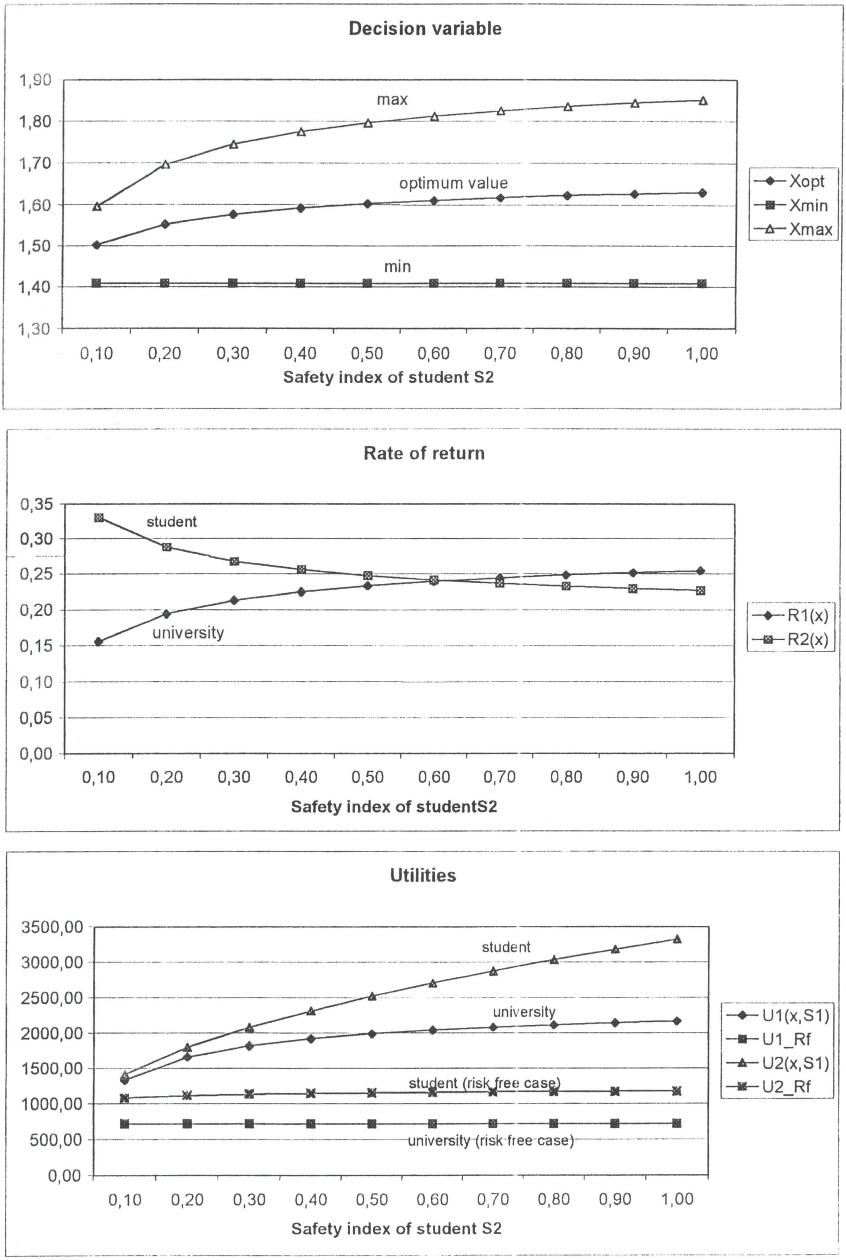


Fig. 5. Nash solution for different student's safety index S_2, S_1=0.9, $\beta_1 = \beta_2$=0.5.

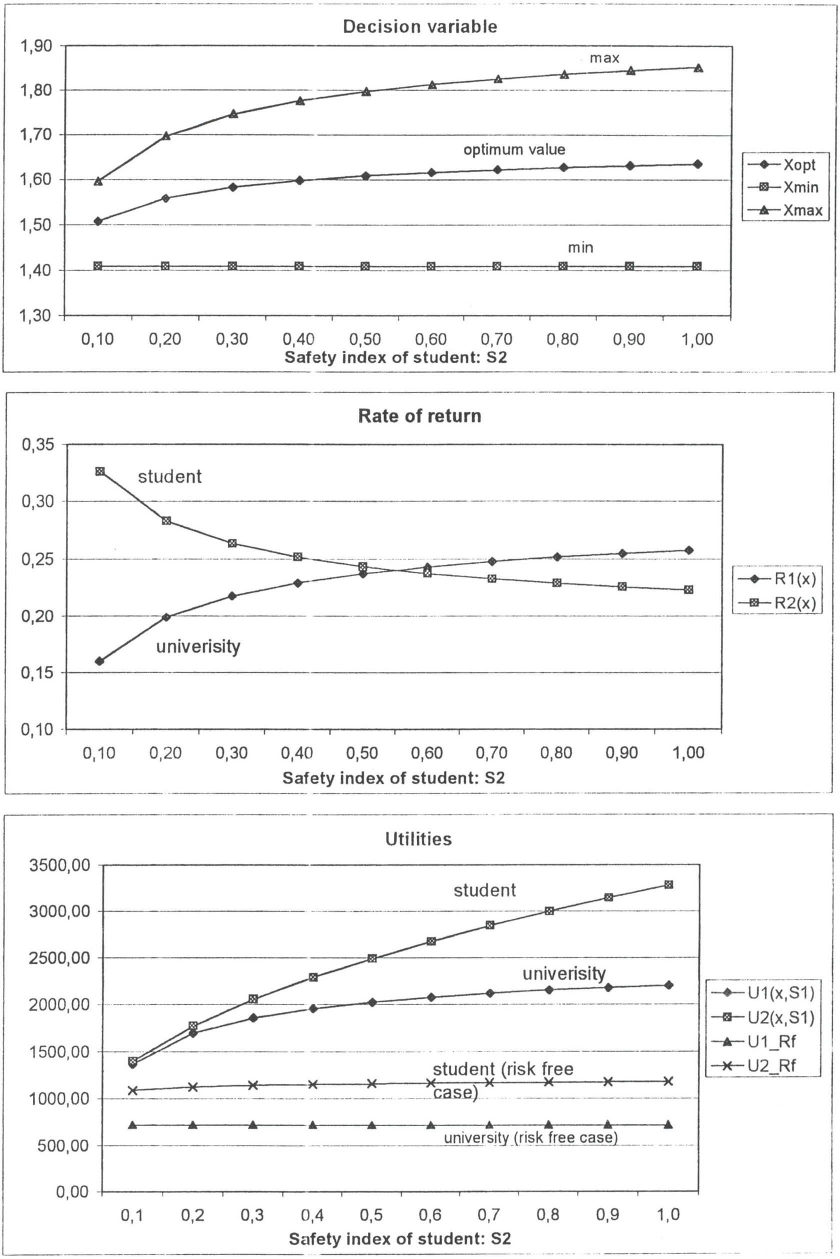


Fig. 6. Raiffa solution for different student's safety index S_2, S_1=0.9, $\beta_1 = \beta_2$=0.5.

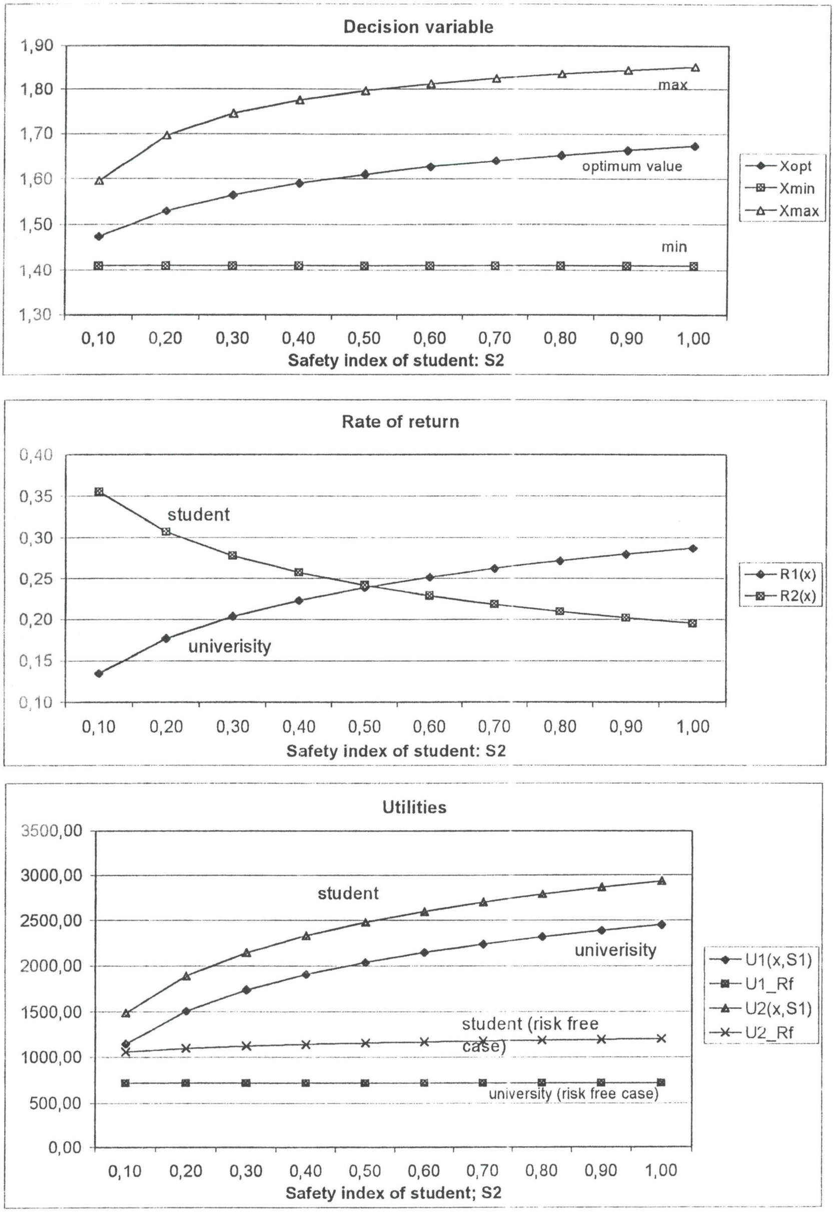
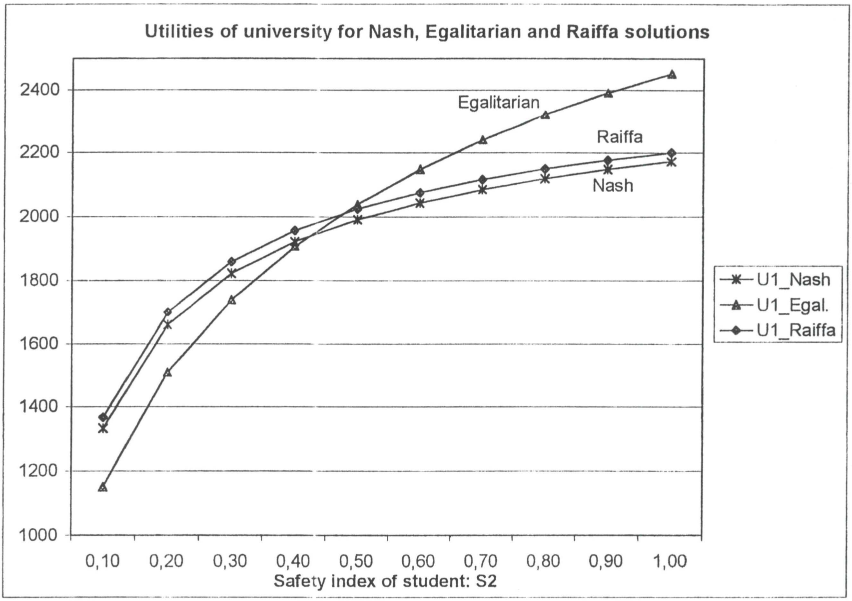


Fig. 7. Egalitarian solution for different student's safety index S_2, $S_1=0.9$, $\beta_1=\beta_2=0.5$.



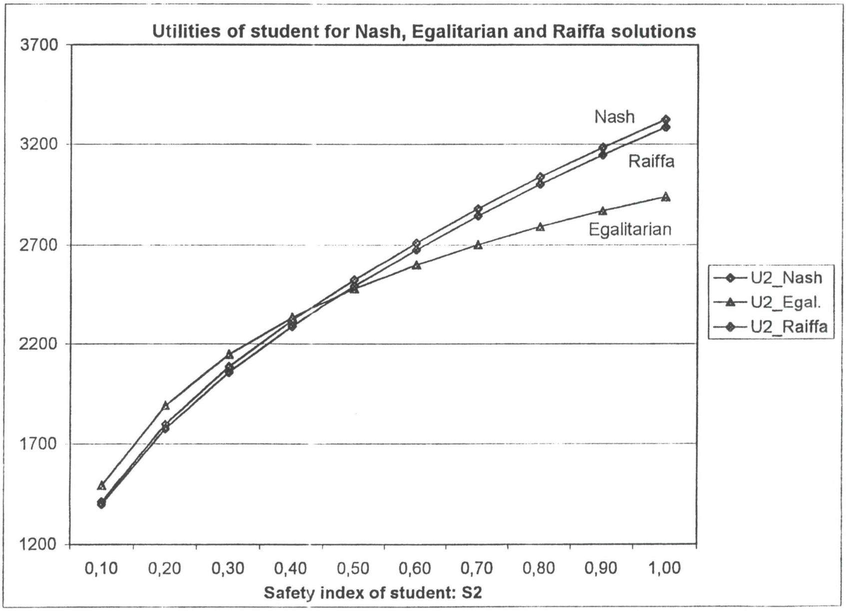
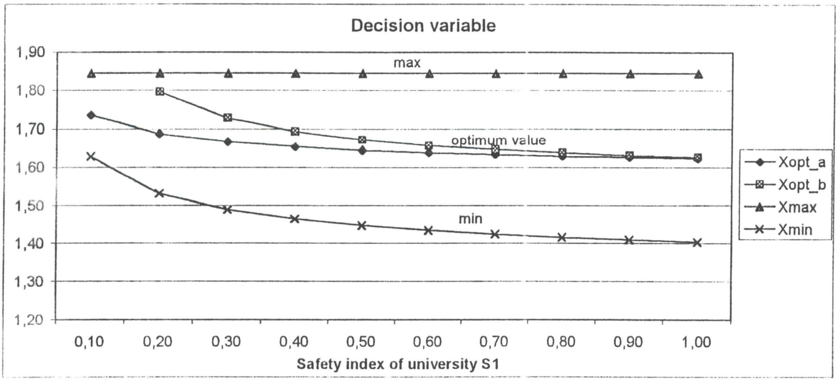
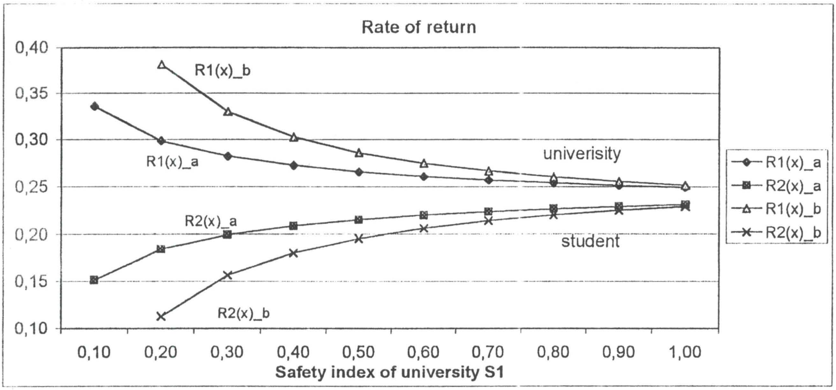


Fig. 8. Comparison of Nash, Raiffa and Egalitarian cooperative solutions.





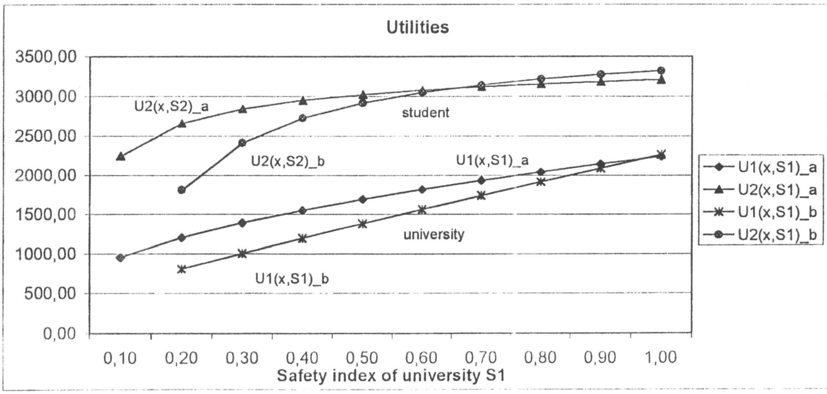
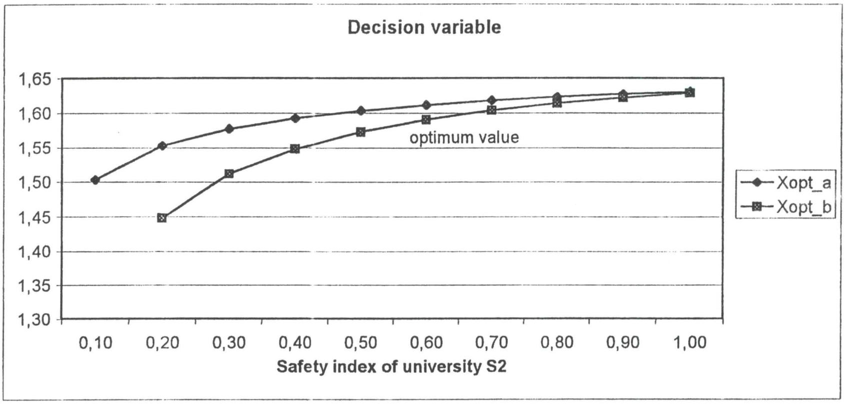
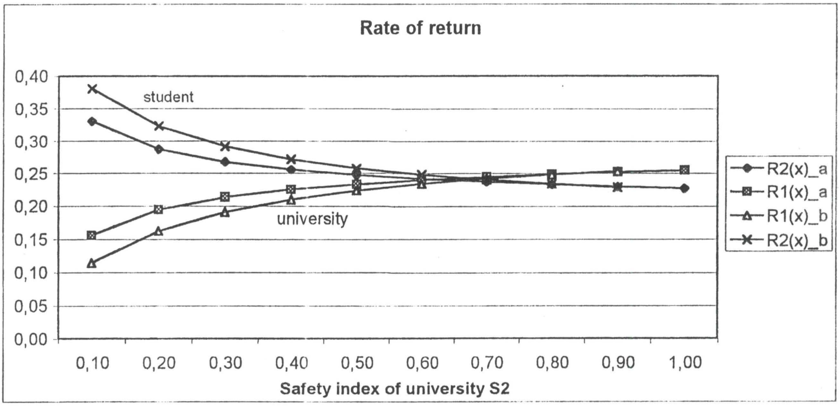


Fig. 9. Nash cooperative solution for different values of university safety index S_1, S_2=0.9, β_1=0,1, β_2=0.9.





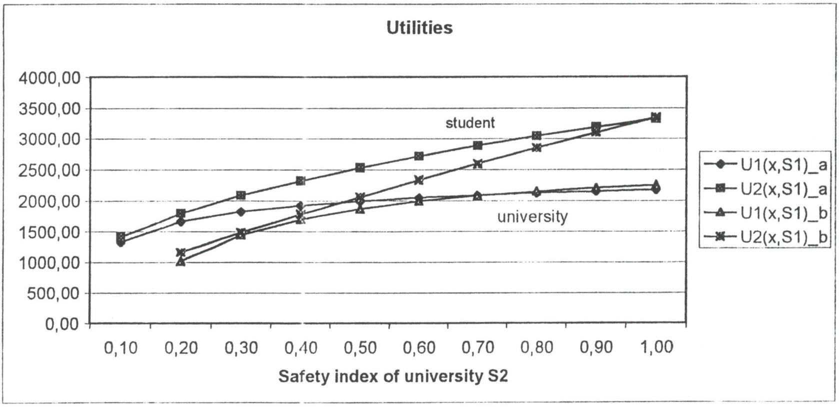


Fig. 10. Nash solution, S2 varying, S1 =0,9, beta1=0,9, beta2=0,1.

safety index S2, then the optimum value of the decision variable is lower (lower tuition) and the rate of return obtained by the student is greater.

Figure 8 includes comparison of utilities depending on safety index for the three considered cooperative solutions. Greater safety of the student gives greater utilities for both the student and the university. It means that both the sides benefit. Let us see that the derived utilities for Nash and Raiffa solutions are close each other. The utility of university in the case of Egalitarian solution increases much more than in the case of Raiffa and Nash solution. In contrary, the student's utility in the case of egalitarian solution increases less than in the case of the other solution concepts.

Figure 9. shows how the Nash cooperative solution results depend on β parameters of utility functions. The results of two variants: **a.** and **b.** are compared. In the variant **a.**: $\beta_1=\beta_2=0,5$. In the variant b.: $\beta_1=0.1$, $\beta_2=0.9$. Values of decision variable, rates of return and utilities have been calculated according to the Nash solution for $S_2=0,9$ and varying safety index S_1 of university. In the variant **b.**, for small values of S_1 we observe grater value of decision variable (greater tuition), grater rate of return R_1 achieved by university, lower student's rate of return R_2, lower utilities of both the sides. The results can be interpreted that lower parameter β of university, for increasing risk, causes greater repayments (grater tuition and greater rate of return) on the cost of other side, but final utility is lower.

Figure 10. shows also results of Nash solution for asymmetric values $\beta_1=0.9$, $\beta_2=0.1$ (variant **b.**) compared to the same variant **a.** as in the previous figure. The safety index of university $S_1=0.9$. The student's safety index S_2 is varying. Lower value of S_2 causes lower decision variable (lower tuition), greater rate of return R_2 and lower utility U_2 in the variant **b.** in comparison to the respective quantities in the variant a. In the variant **b.**, increasing risk of the student is repaid by lower tuition and grater rate of return, however both the sides achieve lower utilities.

5. Final remarks

The bargaining problem describes the benefits the university and the student can achieve when they cooperate. The division of the benefit depends solely on their agreement. Cooperative solution describes payoffs of the sides, in comparison to a given status quo, under assumed set of axioms – properties. The axioms express attitudes of the sides to the cooperation. Nash formulated first the bargaining problem and his well-known solution concept. The solution satisfies axioms of collective rationality, symmetry, independence of equivalent utility representations, independence of irrelevant alternatives. The last axiom was an object of criticism, and other solution concepts were formulated with axiomatic motivation. In this paper, the Nash cooperative solution concept is compared to Egalitarian and Raifa solution concepts, in application to analysis of an education system. The discussed model of the system has been implemented in a form of experimental computer system. The optimization problems enabling derivation of the cooperative solution have been formulated and also implemented in the computer system. The numerical results obtained in experiments illustrate the model and show how the cooperative solutions depend on some input variables.

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