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Abstract

The paper deals with multicriteria decision support in conflict situations described as a multicriteria noncooperative game. Generalization of the noncooperative game theory for the multicriteria case is required for construction of such decision support. New theoretical results in this case are presented. They include parametric characterization of the multicriteria gains representing preferences of the players as well as relations of equilibria in the multicriteria games and respective classical games. An algorithm supporting analysis of payoffs in the multicriteria game and derivation of the best response strategies satisfying preferences of the players is proposed. It is shown that proposed parametrization of the multicriteria game allows to derive equilibria representing preferences of the players.

Key words: Multicriteria noncooperative games, noncooperative equilibrium, multicriteria decision making, decision support systems.

1 Introduction

Decision support problems in the case of conflict situations that can be described as multicriteria noncooperative games are discussed in the paper. The general theory of noncooperative games has already been intensively developed starting from fundamental papers by Nash [12], [13], and after him by Arrow, Debreu, Hurwicz, to mention only the precursors of the theory. In the references only selected papers are mentioned: Nash [12], [13], Arrow and Debreu [1], Arrow and Hurwicz [2], Aubin [3]. The last of the references includes a broad bibliography on the subject. The theory has been developed hitherto as a background of analysis of conflict situations under the assumption that each player has an explicitly given one-dimensional utility function measuring his outcome.

In practical problems, it is typical that a player deals with not one but with several criteria which he would like to satisfy. The player has rather inmind preferences on the criteria. The utility function aggregating the criteria is in general not given explicitly. What more, in practice, the decision maker - player can modify his preferences when obtaining new information about possible gains. The decision support system is considered as a computer-based tool that allows the players to make an analysis of the conflict situation, taking into account their preferences. The analysis can be done using an interactive, learning procedure utilizing methods of multicriteria optimization. To construct such procedures, a development of the theory of noncooperative games an its generalisation for the multicriteria case is required, that is, on the case where different objectives of the players are considered explicitly without the use of a given utility function.

Formulation of multicriteria noncooperative games has already been introduced in Tzafestas [15] and Szidarovsky et al. [15]. The existence of equilibria in the games has been analyzed by Wang [16], [17], Kruś and Bronisz [10]. Wierzbicki [21] developed concepts and a theory of multicriteria decision analysis in such games.

In this paper some theoretical results on n-person noncooperative multicriteria games described in strategic form (normal form) are presented. They relate to the definitions of the noncooperative equilibria and the theorems on the relations of the multicriteria game equilibria to the Nash equilibrium in the respective classical (unicriteria) game. On the basis of the theorems we can simplify the derivation of the multicriteria game equilibria taking into account players preferences. The discussion of decision support problems in the case of the multicriteria noncooperative games is presented. In the papers (Kruś and Bronisz [9], Kruś [8], [7], [6], [5]) ideas of computer-based decision support in the case of the multicriteria bargaining problems is developed. These ideas are proposed to be applied in the case of the noncooperative games considered here.

2 Problem formulation

We assume a given, finite set of the players $N = \{1, \ldots, n\}$.

Let each player i has a strategy space X^i. The space of multistrategies is the Cartesian product of the players' strategies and is denoted by

$$X^N = \prod_{i=1}^{n} X^i.$$

We assume also that there exists a set $X(N) \subset X^N$ of feasible multistrategies x. Each player i has a gain function $g_i : X(N) \to \mathbb{R}^k$ associating with any multistrategy x a vector of real numbers representing values of criteria $g_i(x) = (g_{i1}(x), g_{i2}(x), \ldots, g_{ik}(x))$ measuring his gains. The multistrategy set $X(N)$ can be discrete or continuous (linear or nonlinear). For simplicity of notation, without loss of generality, we assume that each player has the same number of criteria. Let $\mathbb{R}^{NK}$ denote the multigain space $\prod_{i=1}^{n} \mathbb{R}^k$. The multigain operator is defined by

$$G : X(N) \to \mathbb{R}^{NK},$$

where $G(x) = (g_1(x), g_2(x), \ldots, g_n(x)) \in \mathbb{R}^{NK}$.

The operator defines multicriteria gains of all the players for the strategies undertaken by all of them. The gains are elements of the multi-gain space which is the Cartesian product of the multicriteria spaces of the gains of particular players.

Definition 2.1 *An n-person noncooperative multicriteria game* $\{X(N), G\}$ *is described in the strategic form (normal form) by a multigain operator* G *mapping a multistrategy set* $X(N) \in X^N$ *into the multigain space* $\mathbb{R}^{NK}$.

In the classical case of the noncooperative game in strategic form, the gain of each player is described by a scalar function. In the paper, we assume that the gain of each player is described by a vector function defining values of the criteria for given decision strategies of all the players.

For simplicity of notation, let $\bar{i} = N \backslash \{i\}$. From the point of view of player i, the set of all strategies X^N can be split into the set of strategies of the player i and the strategies of other players: $X^N = X^i \times X^{\bar{i}}$, where $X^{\bar{i}} = \prod_{j \neq i} X^j$. If p^i and $p^{\bar{i}}$ denote the projections from X^N onto X^i and $X^{\bar{i}}$, we set $x = p^i x$ and $x^{\bar{i}} = p^{\bar{i}} x$.

We introduce the domination relation in vector spaces.

Definition 2.2 *For any space* $\mathbb{R}^m$ *and for any* $x, y \in \mathbb{R}^m$ *we say that a vector* y ***dominates*** *a vector* x *and write* $y > x$ *if* $y_i \geq x_i, y \neq x$ *for* $i = 1, 2, \ldots, m$. *We say that a vector* y ***strictly dominates*** *a vector* x *and write* $y >> x$ *if* $y_i \geq x_i$ *for* $i = 1, 2, \ldots, m$.

Definition 2.3 *We say that a multistrategy* $x \in X(N)$ *is a* ***weak noncooperative equilibrium*** *in the n-person multicriteria game* $\{X(N), G\}$ *if for each player* $i \in N$, *there does not exist a multistrategy* $x' \in X(N), p^{\bar{i}} x' = x^{\bar{i}}$ *satisfying* $g_i(x') >> g_i(x)$.

A multistrategy $x \in X(N)$ *is a* ***noncooperative equilibrium*** *in the n-person multicriteria game* $\{X(N), G\}$ *if for each player* $i \in N$, *there does not exist a multistrategy* $x' \in X(N), p^{\bar{i}} x' = x^{\bar{i}}$ *satisfying* $g_i(x') > g_i(x)$.

Remarks

A multistrategy is a weak equilibrium if no player i can obtain a higher gain for all his criteria (i.e. a gain better according to the strict domination relation), by making an alternative choice under the assumption that the remaining players make no change in their strategies.

A multistrategy is an equilibrium if no player can obtain a higher gain for some of his criteria, not decreasing his other criteria (i.e. a gain better according to the domination relation), by making an alternative choice under the assumption that the remaining players make no change in their strategies.

It is easy to show that if a multistrategy $x \in X(N)$ is a noncooperative equilibrium then it is also a weak noncooperative equilibrium.

In the unicriteria case, i.e. when $k = 1$, these definitions are equivalent and define the Nash equilibrium.

The measure of gain has only ordinal meaning (not cardinal), i.e. for a given criterion, a gain function compares two elements for ordering purposes. We do not also introduce explicitly any "weights of importance"' or "priorities" of criteria aggregating them.

Theorem 2.1 *Suppose that the multistrategy set $X(N)$ is a convex, compact subset, and that for each player i, the gain function g_i is continuous and concave with regard to each coordinate, for all $x \in X(N)$. Then there exists a noncooperative equilibrium.*

The proof is given in (Kruś and Bronisz [10]). It is similar to that proposed by Aubin [3]. It is based on the Ky Fan theorem.

Remarks

The theorem does not say anything about the uniqueness of equilibria. In many cases, there is a set of equilibria.

If we compare our game and a game formulated as a game of $n \times k$ players, i.e. in which each criterion of every player is treated as a "player" in a classical noncooperative game, then the sets of equilibria will be different.

3 Parameter characterization of efficient outcomes of the multicriteria game

In multicriteria optimization problems, characterization of the set of in some sense efficient outcomes serves as a mathematical background for the construction of decision support systems enabling the decision maker to scan and analyze the efficient outcomes. Most of the characterizations utilize some substitute scalarizing function. The function typically depends on the objective function but also on additional parameters, for example weighting coefficients (Chankong and Haimes [4]), or levels of objective functions interpreted as reference aspiration levels (Wierzbicki [22], [24]).

Using the decision support system, the decision maker can generate some number of efficient outcomes assuming values for the parameters and look for the outcome closest to his preferences. In an analogical way, the scalarizing function could be used in the case of a multicriteria noncooperative game. However, in the last case the problem is much more complicated. Each player has a different vector of objectives. The outcomes are dependent on the strategies of all the players. A question arises: can the scalarizing functions be used for a characterization of efficient outcomes of the game, but also for a characterization of the set of equilibria, or, more precisely, of the set of nondominated equilibria.

Ideas of a selection of game equilibria using the scalarizing function were proposed by Wierzbicki [19]. The scalarizing function can also be considered as a tool aggregating for each player his multicriteria to unicriteria gain, depending on the selected parameter, and therefore to each multicriteria game we can assign a classical game in which each player has his gain defined in a unicriteria way.

In the following we assume that each player has in general his own parameter (vector of weighting coefficients or vector of reference aspiration levels), i.e. we assume that each player i can use a different vector $w^i = (w_1^i, \ldots, w_i^k)$. Let us consider a set W of such a parameters, $W \subseteq I\!R^k$. The simplest scalarization form is made using weight coefficients with each player i who is assumed to have his own vector of weights $w^i = \lambda^i$.

Let A denote the simplex:

$$A = \{\lambda \in I\!R^k : \lambda_i \geq 0, \Sigma_{i=1}^k \lambda_i = 1 fori = 1, 2, \ldots, k\}$$

For a given multicriteria game $\{X(N), G\}$, we consider a class of classical games $\{X(N), G^\lambda\}$ defined for

$$\lambda = (\lambda^1, \lambda^2, \ldots, \lambda^n) \in A^N$$

by

$$G^\lambda : X(N) \to I\!R^N,$$

where

$$G^\lambda(x) = (g_1^\lambda(x), g_2^\lambda(x), \ldots, g_n^\lambda(x)),$$
$$g_i^\lambda(x) = \lambda_1^i g_{i1}(x) + \lambda_2^i g_{i2}(x) + \ldots + \lambda_k^i g_{ik}(x).$$

In the game $\{X(N), G^\lambda\}$, the gain of a player is defined as a linear combination of all criteria. We might interpret λ as a vector of criteria weights of importance for all players. In general each player can have different vector of weights.

Theorem 3.1 *Suppose that x is a Nash equilibrium of a classical game $\{X(N), G^\lambda\}$. Then x is also a noncooperative equilibrium of the multicriteria game $\{X(N), G\}$.*

Proof

Let x be a Nash equilibrium of a game $\{X(N), G^\lambda\}$ for any $\lambda \in A^N$, i.e. for each $i \in N$ and for each $y \in X(N)$ satisfying $p^{\bar{i}}y = x^{\bar{i}}$,

$$g_i^\lambda(x) \geq g_i^\lambda(y).$$

Suppose that x is not an equilibrium of the game $X(N), G$. Then there exist $i \in N$ and $y \in X(N)$ satisfying $p^{\bar{i}}y = x^{\bar{i}}$ such that $g_i(y) > g_i(x)$. Then

$$g_i^\lambda(y, x^{\bar{i}}) = g_i^\lambda(y) = \lambda_1^i g_{i1}(y) + \lambda_2^i g_{i2}(y) + \ldots + \lambda_k^i g_{ik}(y) >$$

$$\lambda_1^i g_{i1}(x) + \lambda_2^i g_{i2}(x) + \ldots + \lambda_k^i g_{ik}(x) = g_i^\lambda(x).$$

Contradiction, because we assume that x is a Nash equilibrium. $\square$

In the following we will consider generalization of the result for a broader class of scalarizing functions having monotonicity properties (so called strictly and strongly monotone - compare Wierzbicki [16]).

Definition 3.1 *For any parameter $w \in W$, a scalarizing function $s : \mathbb{R}^k \times W \to \mathbb{R}$ is* ***strictly monotone*** *with respect to y if, for any y', $y" \in \mathbb{R}^k$,*

$$y' \textit{ strictly dominating } y" \ (y' \gg y") \textit{ implies } s(y', w) > s(y", w).$$

The function s is ***strongly monotone*** *with respect to y if, for any y', $y" \in \mathbb{R}^k$,*

$$y' \textit{ dominating } y" \ y' > y" \textit{ implies } s(y', w) > s(y", w).$$

Let us consider multicriteria game $\{X(N), G\}$ and a class of associated classical games $\{X(N), G^w\}$ defined for a given player's parameters $w^i \in W$, $i = 1, 2, \ldots, n$, and for a scalarizing function $s(y, w)$ as follows:

$$G^w(x) = (g_1^w(x), \ldots, g_n^w(x)),$$

where

$$g_i^w(x) = s(g_i(x), w^i),$$

with

$$g_i(x) = (g_{i1}(x), \ldots, g_{ik}(x)).$$

In the associated classical game, the gain g_1^w of player i is defined as an aggregation of his multicriteria gains using a scalarizing function.

The scalarizing function depends on a parameter $w_i = (w_1^i, \ldots, w_k^i)$, where i is the number of the player, $i = 1, \ldots, n$. Using the parameter, the player i can express his preferences among his criteria.

Theorem 3.2 *Let $x \in X(N)$ be a Nash equilibrium of a classical game $\{X(N), G^w\}$, i.e. for each player $i \in N$,*

$$g^w(x) = max\{g_i^w(x') : x' \in X(N), p^{\bar{i}}x' = x^{\bar{i}}\}.$$

If the scalarizing function $s : \mathbb{R}^k \times W \to \mathbb{R}$ is ***strongly monotone*** *with respect to y for any $w \in W$, then x is also a* ***noncooperative equilibrium of the multicriteria game*** *$\{X(N), G\}$.*

Proof

If $x \in X(N)$ is a Nash equilibrium of the classical game $\{X(N), G^w\}$ for any $w \in W$, then from the definition of the equilibrium, for each $i \in N$ and for each $x' \in X(N)$ satisfying $p^{\bar{i}}x' = x^{\bar{i}}, g_i^w(x) \geq g_i^w(x')$. Let us assume that x is not an equilibrium of the mult)criteria game $\{X(N), G\}$. In that case, there exists $i \in N$ and $x' \in X(N)$ satisfying $p^{\bar{i}}x' = x^{\bar{i}}$ such that $g_i(x') > g_i(x)$. Then

$$g_i^w(x') = s(g_i(x'), w^i) > s(g_i(x), w^i),$$

according to the strong monotonicity of s, where $s(g_i(x), w^i) = g_i^w(x)$. So we have a contradiction to $g_i^w(x') > g_i^w(x)$, i.e. to the assumption that x is a Nash equilibrium. □

Theorem 3.3 *Let $x \in X(N)$ be a Nash equilibrium of a classical game $\{X(N), G^w\}$.*

If the scalarizing function $s : \mathbb{R}^k \times W \to \mathbb{R}$ is ***strictly monotone*** *with respect to y for any $w \in W$, then x is also* ***a weak noncooperative equilibrium of the multicriteria game*** *$\{X(N), G\}$.*

Proof

Let, like in the proof of the previous theorem, x be not a weak equilibrium of the multicriteria game $\{X(N), G\}$. In that case, there exists $i \in N$ and $x' \in X(N)$ satisfying $p^{\bar{i}}x' = x^{\bar{i}}$, such that $g_i^w(x) >> g_i^w(x')$. Then

$$g_i^w(x') = s(g_i(x'), w^i) > s(g_i(x), w^i),$$

according to the strictg monotonicity of s, where $s(g_i(x), w^i) = g_i^w(x)$. So we have a contradiction to $g_i^w(x') > g_i^w(x)$, i.e. to the assumption that x is a Nash equilibrium. □

The scalarization of gain has only ordinal meaning (not cardinal), i.e. for a given player, the scalarization function compares two strategies for ordering purposes. If player i specifies his parameter w^i properly, the scalarization function should reflect his preferences, and the noncooperative equilibrium should be satisfying.

4 Decision support in the case of multicriteria noncooperative games

New methodological problems related to the concepts and construction of decision support systems arise in the case of multicriteria noncooperative games.

By the decision support system we mean a tool that should aid the players in a selection of rational strategies. The system should support players, first in learning their situation in the game, showing equilibria outcomes, possible conflict escalation, as well as possible cooperation outcomes. The analysis of the game should be made in a multicriteria context. It should also allow the players to make a multicriteria analysis depending on their preferences among the criteria measuring their outcomes.

A simulation of the game is the simplest prototype aiding the players in making such an analysis. In this case the players assume some strategies and the system calculates their outcomes.

The analysis can be made much more effectively when a multicriteria optimization approach, in particular Wierzbicki's aspiration-led approach [22] is used. If we apply the aspiration-led approach in the case of the noncooperative game, each player can analyze the problem assuming his reference points (aspiration levels for his criteria) and assuming the reference points for the counter players. For given reference points assumed by a player i the system generates respective outcome which is Pareto optimal in the set of his attainable outcomes. The system generates the respective Pareto optimal outcome solving an optimization problem with use of so called achievement function.

Let the player i assume a reference point g^{*i} in his space of criteria $I\!R^h$ and assume multictrategies of other players.

The outcome representing the Pareto frontier in the case of the player, $i = 1, 2, \ldots, n$, can be derived solving the optimization problem:

$$\max_{x^i \in X^i} \ [s(g^i(x^i, x^{\bar{i}}), g^{*i}],$$

subject to given strategies of other players $x^{\bar{i}}$,
where:
$g^{*i} = (g_1^{*i}, \ldots, g_k^{*i}$ is a reference point assumed by the decision maker i in the space $I\!R^k$,
$g^i(x)$ defines the vector of criteria of the i-th decision maker, $i = 1, \ldots, k$ which are dependent by the model relations on the vector x of decision variables,
$s(g, g*)$ is an order approximating achievement function.

The following achievement function can be applied:

$$s(g^i(x^i), g^{*i}) = \min_{1 \le j \le k, i=1,2,\ldots,n} [a_j(g_j^i(x^i) - g_j^{*i})] + a_{k+1} \sum_{j=1,\ldots,k} a_j(g_j^i(x^i) - g_j^{*i}),$$

where $g^{*i} \in I\!R^k$ is a reference point, a_j, $1 \le j \le k$, are scaling coefficients, and $a_{k+1} > 0$ is a relatively small number.

The following algorithm is proposed supporting analysis made by a given player $i \in N$. It supports multicriteria analysis of the payoffs and calculation of the best response strategies satisfying preferences of the player.

Step 1. Independent analysis made by a given player.

The system invites the decision maker $i = 1, \ldots, n$ to make independently multicriteria analysis of their nondominated payoffs in the multicriteria noncooperative game.

Set the number of round $t = 1$.

Step 2. The decision maker i assumes multistrategies of other decision makers $x^{\bar{i}t}$.

Step 3. Multicriteria analysis of payoffs. Calculation of the best response strategy satisfying preferences of the decision maker.

Step 3.1 The system presents to the decision maker i information about the ideal point I^{it} in the decision maker criteria space $I\!R^k$. The ideal point is derived as $I^{ti} = (I_1^{ti}, I_2^{ti}, \ldots, I_k^{ti})$, where $I_j^{ti} = \max g_j^i(x)$ calculated with respect to x^i subject to $x \in X^N$.

Step 3.2 The decision maker i writes values of the components of his reference point g_j^{*it}, $j = 1, 2, \ldots, k$.

Step 3.3 The system derives the Pareto optimal solution in the criteria space of the decision maker i, according to the reference point approach and stores the resulting payoff in a data base.

Step 3.4 The decision maker analyzes the generated Pareto optimal payoff. He compares the payoff to other Pareto optimal payoffs stored in the data base, obtained for other different reference points.

Step 3.5 Has the decision maker i finished multicriteria analysis?
If no - go to Step 3.2, to generate next Pareto optimal payoff.
If yes - system writes in the data base the preferred Pareto optimal payoff indicated by the decision maker $\widehat{g}^{it}$ as well as the optimal response strategies $\widehat{x}^{it}$ for the given strategies $x^{\bar{i}t}$ of other decision makers.

Step 4. The system checks whether the decision maker have finished his analysis for all assumed multistrategies of other players.
If no - set the round number $t = t + 1$ and go to the Step 2 to make analysis for another multistrategy of other players.
If yes - there is in the data base a set of the best response strategies of the decision maker and the respective preferred Pareto optimal outcomes.

End of the procedure.

The system can include a part calculating the equilibrium startegies in the multicriteria game. Let each player $i = 1, \ldots, n$ assume his selected Pareto optimal payoff $\widehat{g}^i$. The presented achievement function can be used as the scalarizing function considered in the Section 3 with parameters $w^i = g^i = \widehat{g}^i$. This achievement function is a strongly monotone with respect to the obtained gains. A classical game $\{X(N), G^w\}$ can be constructed for the parameters, and an equilibrium in the game can be derived.

Remark

According to the theorem 3.2 the derived equilibrium of the classical game is also the equilibrium of the considered multicriteria game. That means the equilibrium strategies derived for the classical game define also the equilibrium in the multicriteria game. Let us see that the gains $g^i = \widehat{g}^i$ have been selected according to the preferences of the players. The derived equilibrium in the classical as well as in the multicriteria game expresses the preferences of the player.

In the multicriteria games we deal in general with a set of different equilibria outcomes. It can be said that a multicriteria formulation of decision making problems of players typically leads to nonunique equilibria.

In general, the equilibria can be inside the set of attainable outcomes, i.e. the equilibria can be not Pareto optimal. Therefore, there exist cooperative strategies of the players, such that all the players can improve their outcomes in comparison to the case of equilibria strategies. On the other and, the nonunique equilibria can lead to a conflict escalation (see Wierzbicki [19]). The escalation can take place when each of the players tries to apply an equilibrium strategy but related to different equilibria points. With regard to the decision support problems, we follow the argument proposed by Wierzbicki [19], [20], [21]. The calculation of equilibrium strategies can lead to some computational problems. Solving the problems we deal with two-level optimization procedures in witch a nondifferential objective function is maximized on the second level. Further research in this direction is required.

The decision support system should demonstrate to the players the advantage of possible cooperative strategies. Next we face the problem of how to lead the players into a cooperative outcome, being Pareto optimal in the set of attainable outcomes. In this case another decision support mechanism can be applied - an interactive mediation procedure.

In the mediation we consider the following problem: there is given an disagreement point (it can be assumed as an equilibrium point or as a status quo point) and a set of attainable outcomes in the multicriteria space of all the players. The problem consists in aiding the players in finding a mutually beneficial, unanimously accepted outcome in the set of attainable outcomes. The final outcomes should be selected according to the preferences of each of the players. The mediation procedure can be made according to the rules of the multicriteria bargaining support developed in the papers by Kruś and Bronisz [9], Kruś [7], [6], [5]. That approach utilizes the new results of multicriteria bargaining problems and the interactive multicriteria, aspiration-led approach. Initial practical experience of such a support has been obtained when the experimental computer-based system (Kruś et al. [11]) was constructed.

5 Final remarks

In this paper we discuss the problem of decision analysis and support in noncooperative games in a multicriteria context. A theoretical research on the multicriteria games is still required to make a background for construction of decision support systems in the games. In this paper several results in the subject have been presented. These include an analysis of equilibria in n-person, multicriteria noncooperative games. In particular, the new theorems on relations of the equilibria in a multicriteria game to the Nash equilibria in the classical (unicriteria) game are presented and proved, where the classical game is defined by parametrization of the multicriteria game. An algorithm supporting multicriteria analysis of payoffs in the game is proposed. Using the algorithm each player can make the multicriteria analysis and derive the best response strategies satisfying his preferences.

The decision support system in this case is considered as a tool supporting the players: first, in the analysis of the game, and second, in aiding the selection of a mutually acceptable, cooperative, Pareto optimal outcome. Application of the aspiration-led approach of multicriteria optimization, bargaining, interactive mediation procedures seems to be useful in constructing such a system.

Let the reference point approach (called also aspiration-led approach) be applied by each player to make multicriteria analysis of attainable multicriteria gains in the game. According to the approach each player proposes reference points in the space of his criteria in a sequence of steps and compares respective Pareto optimal outcoms derived by a computer-based system. He selects finally the preferred outcome. Let in the reference point approach an achievement function be applied which is strongly monotone with respect to multicriteria payoffs for any reference point assumed by the player during the analysis. Applying the achievement function, a classical - unicriteria game can be constructed. Scalar payoffs in the game are calculated according to the achievement function for given reference points assumed by players due to their preferences. If the reference points in the function are applied as the preferred outcomes selected by players during the multicriteria analysis than the gains in the unicriteria game express preferences of the players. According to the theorem 3.2 the decision variables for which the noncooperative equilibrium is obtained in the classical game, define also the noncooperative equilibrium in the multicriteria game. Therefore the multicriteria equilibrium is defined for outcomes representing preferences of the players.

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