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ITERATIVE SOLUTION TO THE MULTICRITERIA BARGAINING PROBLEM BASED ON THE NASH SOLUTION CONCEPT

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1 Introduction

A multicriteria bargaining problem is a generalization of the classical bargaining problem introduced by Nash (1950). A group of decision makers is engaged in a bargaining process. Each decision is the result of a negotiation. Each decision maker has his own criteria which value any decision. The criteria are in general conflicting in the case of one decision maker as well as between them. We assume that each decision maker has his own preferences among his criteria, but his utility function is not known explicitly. A multicriteria bargaining problem is described by a set of decision makers called also players which are involved in a bargaining process, an agreement set being subset of the attainable payoffs in the criteria space of the players, and a distinguished outcome called a disagreement point. Any outcome in the agreement set can be the result of the bargaining process if the players reach unanimous agreement. In another case, the result is the disagreement point.

Formulation and analysis of different solution concepts to the bargaining problem with scalar payoffs of players have been presented in many papers including (Nash 1950, 1953, Kalai and Smorodinsky 1975, Roth 1979, Thomson 1980, Peters 1986, Moulin 1988) and others. The solutions do not transfer on the case with multicriteria payoffs of players in a simple way. Papers dealing with multicriteria payoffs of players in bargaining are relatively rare. In the papers (Kruś and Bronisz 1993, Kruś 1996, 2001, 2011)one can fine proposals of the solution concepts to multicriteria bargaining problem generalizing the Raiffa-Kalai-Smorodinsky solution and the lexicographic (Imai, 1983) solution concepts. The generalized Raiffa-Kalai-Smorodinsky solution concept has been applied in an interactive procedure supporting multicriteria bargaining process and implemented in the computer-based system MCBARG.

This paper presents theoretical results relating to a generalization of the Nash solution concept on the case of the multicriteria bargaining problem. Two types of solutions are proposed and analyzed: one shot solution and an iterative solution. Two theorems presenting properties of the solutions are formulated and proved.

2 Formulation of the multicriteria bargaining problem

A bargaining process is considered in the case of two decision makers negotiating conditions of possible cooperation. It is assumed that results of the cooperation are measured by a vector of criteria which is in general different for each decision maker. Criteria of the decision maker i, $i = 1, 2$ presenting his payoff are denoted by a vector
$y_i = (y_{i1}, y_{i2}, \dots y_{im^i}) \in I\!R^{m^i}$,
where m^i is a number of the criteria of the decision maker i, and $I\!R^{m^i}$ is a space of his criteria. The criteria of the both decision makers are denoted by $y = (y_1, y_2) \in I\!R^M$, where $M = m^1 + m^2$. The space $I\!R^M$ is the cartesian product of the citeria spaces of the both decision makers.

A partial ordering is introduced in the criteria spaces. Let $I\!R^m$ denote a space of criteria. Each of m criterions can be maximized or minimized.

However, to simplify the notation and without loss of generality we assume that the decision makers maximize all their criteria.

Let $z, y \in I\!R^m$, we say, that
a vector z **weakly dominates** y and denote $z \geq y$, when $z_i \geq y_i$ for $i = 1, 2, \ldots, m$,
a vector z **dominates** y and denote $z > y$, when $z_i \geq y_i, z \neq y$ for $i = 1, 2 \ldots, m$,
a vector z **strongly dominates** y and denote $z \gg y$, when $z_i > y_i$ for $i = 1, 2 \ldots, m$.

A vector $z \in I\!R^m$ is **weakly Pareto optimal** (weakly nondominated) in set $Y_0 \subset I\!R^m$ if $z \in Y_0$ and does not exist $y \in Y_0$ such, that $y \gg z$.
A vector $z \in I\!R^m$ jest **Pareto optimal** (nondominated) in set $Y_0 \subset I\!R^m$ if $z \in Y_0$ and does not exist $y \in Y_0$ such, that $y \geq z$.

A multicriteria bargaining problem is formulated as a pair (S, d), where the element $d = (d_1, d_2) \in S \subset I\!R^M$ is called a disagreement point, and the set S is an agreement set. The agreement set $S \subset I\!R^M$ is the subset of the set of attainable payoffs dominating the disagreement point d. The agreement set defines payoffs attainable by the both decision makers but under their unanimous agreement. If such an agreement is not achieved, the payoffs of all decision makers are defined by the disagreement point d. The problem consists in supporting the decision makers in reaching a nondominated solution, agreeable and close to their preferences.

The multicriteria bargaining problem is analyzed under the following general conditions:

C1 agreement set S is compact and convex,

C2 agreement set S is nonempty and includes at least one point $y \in S$ such, that $y \gg d$,

C3 disagreement point $d \in S_0$, additionally for any $y \in S$, we have $y > d$.

Let **B** denote class of all multicriteria bargaining problems satisfying the above conditions.

Let us see, that when looking for a solution of the multicriteria bargaining problem we have to consider two decision problems: the first one - the solution should be related to the preferences of all decision makers, the second - it should satisfy fairness rules accepted by the decision makers. The first problem relates to multicriteria decision making. The multicriteria decision support is proposed with application of the reference point method developed by A.P. Wierzbicki (Wierzbicki 1986), (Wierzbicki, Makowski, Wessels 2000). In relation to the second problem, solution concepts of the multicriteria bargaining problem are proposed having properties that can be accepted by rational decision makers.

Using the reference point approach the decision maker can explore set of Pareto optimal points in the objective space. In consecutive iterations the decision maker assumes reference points in the objective space. The computer based system derives respective Pareto optimal outcomes. This approach is proposed here to be applied for analysis of the multicriteria bargaining problem by each decision maker independently. Such Pareto optimal points generated for a given decision maker we call as individually nondominated. The individually nondominated points are generally not attainable. The individually nondominated point is an outcome that could be achieved by a decision maker if he would have full control of the moves of the other decision maker. We assume that the decision maker finishing such an exploration will select his preferred Pareto optimal outcome. Composition of the preferred Pareto optimal points of the both decision makers we call as an utopia point relative to the players aspirations, denoted as RA utopia. The RA utopia point significantly differs from the ideal (utopia) point. The selected preferred Pareto optimal points of the decision makers carries information about their preferences.

Definition 1 *For any multicriteria bargaining problem (S, d), a point $y^i \in S$ is individually nondominated by decision maker i, $i = 1, 2$, if $y^i \geq d$ and there is no $z \in S$ such that $z \geq d$, $z_i > y^i_i$. A point u is an utopia point relative to the aspiration of decision makers (RA utopia point) if for each decision maker $i = 1, 2$, there is an individually nondominated point y^i such that $u_i = y^i_i$.*

The set of all RA utopia points of the multicriteria bargaining problem (S, d) will be denoted $U(S, d)$.

Definition 2 *A solution to a multicriteria bargaining problem* (S, d) *is a function* $F : B \times \mathbb{R}^M \longrightarrow \mathbb{R}^M$ *which associates to each problem* (S, d) *and each RA utopia point* $u \in U(S, d)$, *a point of* S, *denoted* $f(S, d)$.

3 One shot solution concept

Let both decision makers have made multicriteria analysis in their spaces of criteria and have indicated their preferred nondominated payoffs $y^1, y^2 \in S$ defining the RA utopia point.

Let us construct a two dimensional hyperplane H^2 defined by the points d, y^1, y^2. Each point $y \in H^2$ may be defined as

$$y = d + a_1(y^1 - d) + a_2(y^2 - d).$$

Let A denote mapping from H^2 to $\mathbb{R}^2$ defined by $A(y) = A[d + a_1(y^1 - d) + a_2(y^2 - d)] = (a_1, a_2)$. A two person bargaining problem $(A(S^H), A(d))$ can be considered on the hyperplane with scalar payoffs of decision makers.

We formulate axioms defining required properties of the solution to the bargaining problem (S, d) in an analogical way as in the case of the classic Nash solution concept formulated for scalar payoffs of players with an additional individual rationality axiom.

(A1) Pareto-optimality
$y^N = f^N(S, d)$ is Pareto-optimal in the set S,

(A2) Individual rationality
For every bargaining game (S, d), $y^N = f^N(S^H, d) \geq d$.

(A3) Symmetry
We say, that bargaining problem (S, d) is symmetric, if $d_1 = d_2$ and $(x_1, x_2) \in S$, then $(x_2, x_1) \in S$. A solution fulfills the symmetry property, if for the symmetric problem, $f_1^N(S^H, d) = f_2^N(S^H, d)$.

(A4) Independence of equivalent preference representation (Independence of positive affine transformation of criteria)

$Lf^N(S^H, d) = f^N(LS^H, Ld)$, where L is a positive affine mapping.

The property means that the solution should not depend on the scale in which the player's criteria are measured. If a given player's objectives are measured by a criteria vector $y_i = (y_{i,1}, \ldots, y_{i,mi})$ then are also measured by any criteria vector $v_i = a_i y_i + bi$, where a_i and b_i are real numbers, and a_i is positive. If some player's criteria are changed from y_i to $_v$i then the solution should yield the same underlying outcome.

(A5) Independence of irrelevant alternatives

Let (S, d) and (P, d) be bargaining problems such that $P \subset S$ and $f^N(S^H, d) \in P$.

Then $f^N(S^H, d) = f^N(P^H, d)$.

The last axiom means that if decision makers have agreed solution $f^N(S^H, d)$ in bargaining problem (S^H, d), then decreasing of the agreement set S to a set P which includes the solution, i.e. $f^N(S, d) \in P$, should not change the final payoffs of the players.

Axiom A1 assures efficiency of the solution in set S. The solution is individually rational according to axiom A2. Axiom A3 means that both decision makers are treated in the same way. Axiom A4 prevent possible manipulation of decision makers by changing scales measuring their payoffs, i.e. any decision maker will not benefit by changing scales measuring his payoffs.

Theorem 1 *There is a unique solution to the multicriteria bargaining problem $(S, d) \in \mathbf{B}$, satisfying the axioms A1 - A5. It is defined by*

$$f_N(S^H, d) = \arg\max_{y \in S^H} ||y_1 - d_1|| \cdot ||y_2 - d_2||,$$

where: S^H is the intersection of the set S by the hyperplane H^2 defined by the disagreement point d and the preferred points y^1, y^2 of the players, $y = (y_1, y_2)$, $y_1 \in R^{m_1}$, $y_2 \in R^{m_2}$, $||.||$ is a distance measured on the hyperplane H^2.

Thus the solution is a function which selects the unique outcome which maximize product of improvements of players outcomes in comparison to the disagreement point d. It can be treated as a generalization of the Nash solution for the multicriteria bargaining problem.

The generalized Nash solution concept expresses preferences of the players defined by the preferred payoffs indicated after multicriteria analysis done independently by each of them. In the following we show that the solution $f_N(S^H, d)$ is well defined and satisfies the axioms A1 - A5. It is necessary also to show that any solution f satisfying the axioms must be identical with $f_N(S^H, d)$.

Proof

The function $f_N(S^H, d)$ for given hyperplane defined by the preferred points of the players maximizes the geometric average of the gains, since the geometric average is a continuous, convex function. S is a compact set, so also the set S^H defined by the intersection of the set S and the hyperplane H^2 constructed for any preferred points y^1, y^2 of players is compact. The sets S and S^H are convex whereas the geometric average is a strictly convex function. Therefore this maximum is achieved at a unique point of S^H and also of S. What more there are no points in S dominating this point. That means the solution $f_N(S^H, d)$ is well defined and is Pareto optimal in S (satisfies the axiom A1).

The agreement set S includes elements dominating the disagreement set. The geometric average achieve a positive value at its maximum in the set S^H. Therefore $f_1^N(S^H, d) \geq d_1$ and $f_2^N(S^H, d) \geq d_2$, so the solution is individually rational as it is in the axiom A2.

Let (S^H, d) be a symmetric bargaining game with the solution $y = f^N(S^H, d)$. Let v be any permutation of y, i.e. $v_i = y_j$ and $v_j = y_i$ for some components i and j. At the element v of S the average gains are equal to those at y. As it was shown previously, the maximum average is achieved in a unique point of S. Therefore $v = y$. In the case of an arbitrary permutation v we have $y_i = y_j$ for all i, j, what proofs that the axiom A3 is satisfied.

Let (S^a, d^a) be a bargaining game derived from an arbitrary game (S, d) by a change of preference representation as in the statement of the axiom A4. Then a point v in the set S^a has coordinates $v_i = a_i y_i + b_i$, where $y = (y_1, y_2)$ is the corresponding point in the set S. So $(v_1 - d_1^a)(v_2 - d_2^a) = (a_1 y_1 + b_1 - a_1 d_1^a - b_1)(a_2 y_2 + b_2 - a_2 d_2^a - b_2) = a_1 a_2 (y_1 - d_1)(y_2 - d_2)$. The constant $a_1 a_2$ does not effect where the geometric average achieves its maximum. Therefore we have $L[f^N(S^H, d)] = f^N(L(S^H), L(d))$ where the transformation L is defined by the relation $a_i(\cdot) + b_i$ for $i = 1, 2$.

Let the agreement sets S and P be such that $P \subset S$. Let the preferences of players be the same. That means the hyperplane H^2 describing the preferences is still valid. Intersections of the sets on the hyperplane obviously satisfy relation $P^H \subset S^H$. The function f^N for the set S^H obtains its maximum in the point $f^N(S^H, d)$. It is always at least as large as the maximum which it achieves on any subset $P^H \subset S^H$. Therefore the solution fulfils the axiom A5.

In the following it is shown that any solution $f(S^H, d)$ satisfying the axioms A1 - A5 has to be identical to $f^N(S^H, d)$. We can observe that if a solution $f(S^H, d)$ satisfies the axioms A1 and A3 then it coincides with $f^N(S^H, d)$ for any symmetric game (S^H, d). As the set S is symmetric, convex and compact, the solution is unique and equal to the Pareto optimal point at which $||y_1 - d_1|| \cdot ||y_2 - d_2||$ is maximized. That means $f(S^H, d) = f^N(S^H, d)$. $\diamond$

A construction of the solution to the multicriteria bargaining problem, which is based on the Nash idea is presented in Fig. 1. In this example decision maker 1 has two criteria $y_{1,1}$ and $y_{1,2}$ respectively, decision maker 2 has only one criterion $y_{2,1}$. Let point y^1 be defined according to preferences of the first decision maker. The preferred point y^2 of the second one is defined by the maximal attainable value of his payoff. Hyperplane H^2 is defined by points d, y^1 and y^2. Arrows drown on the figure present improvement directions leading to the selected nondominated payoffs. The Nash cooperative solution $y^N = f^N(S, d)$ to the bargaining problem (S, d) is defined as the point of set the S maximizing product of the payoffs increases for decision

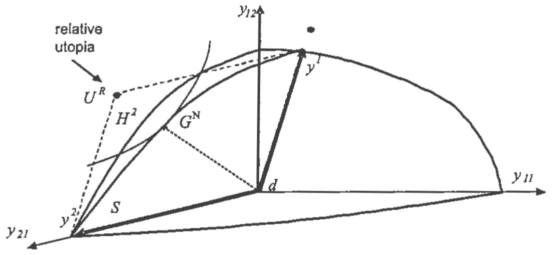

Figure 1: Construction of the generalized Nash solution to the multicriteria bargaining problem

makers 1 and 2 on the hyperplane H^2. The point fulfills the axioms A1 - A5 for final payoffs $y \in \mathbb{R}^2$ under assumptions that preferences of decision makers are expressed by points y^1 i y^2.

4 Iterative solution concept

The iterative solution concept is formulated under a general assumption that the solution of the multicriteria bargaining problem (S, d) is looked for in some number of rounds $t = 1, 2, \ldots, T$, in which outcomes d^t are determined. The final outcome d^T, admissible and accepted by the players is the solution of the problem. It has been assumed that the process d^t should fulfill the following postulates:

P1

It starts at the disagreement point: $d^0 = d$, and all outcomes belong to the agreement set: $d^t \in S$ for $t = 1, 2, \ldots, T$.

P2

The process is progressive, i.e. $d^t \gg d^{t-1}$ for $t = 1, 2, \ldots, T$.

P3

The final outcome is Pareto optimal, i.e. d^T (equal to $\lim_{t\to\infty} d^t$ if $T{=}\infty$) is a Pareto optimal in the set S.

P4

The acceptable demands are limited according to the principle of α-limited confidence. Let α^{ti} , $0 < \alpha^{ti} \leq 1$, be a confidence coefficient at the round t assumed by the player i. Then the demands $d^t - d^{t-1}$ are limited by: $d^t - d^{t-1} \leq \alpha^t(f^t(d^{t-1}) - d^{t-1})$ for $t = 1, 2, \ldots, T$, where α^t is a minimal confidence coefficient in the round t, i.e. $\alpha^t = min\{\alpha^{t1}, \alpha^{t2}, \alpha^t_{max}\}$, α^t_{max} is a maximum value of the confidence coefficient in the round t, such that the outcome d^t belongs to the set S, $f^t(d^{t-1})$ is a solution of the bargaining problem (S^t, d^{t-1}), the set $S^t = \{y \in I\!R^M : y \in S, y > d^{t-1}\}$.

P5

Each player is assumed to behave in rational way, trying to maximize his outcomes in each round according to his preferences expressed with use of the nondominated outcome selected in his criteria space after the unilateral analysis. Let y_i^{it} be preferable nondominated outcomes selected by the player i, $i = 1, 2$, in his criteria space and $y^i \in S$ be the respective point in multicriteria space of the both players. The players rationality can be formulated that for any d^t, at each round t, there is no such outcome $y \in S$, $y > d^t$ which fulfils the condition $y - d^{t-1} \leq \alpha^t(f^t(d^{t-1}) - d^{t-1})$, where $f^t(d^{t-1}) = f^{Nt}(S^H, d^{t-1})$ and S^H is the intersection of set S by the hyperplane H^2 generated in $I\!R^M$ by the points d^{t-1} and y^1, y^2, selected by the players in the round t .

Theorem 2 *For any multicriteria bargaining problem (S, d), for any confidence coefficients α^{ti} such that $0 < \alpha_{min} \leq \alpha^{ti} \leq 1$, $t = 1, 2, \ldots, T$, and for the solution $f^t(d^{t-1})$ defined by the generalized Nash solution concept to the multicriteria bargaining problem (S^t, d^{t-1}), there is a unique process d^t satisfying the postulates P1 - P5. The process is defined by: $d^0 = d$,*
$d^t = d^{t-1} + \alpha^t \cdot [f^t(d^{t-1}) - d^{t-1}]$, for $t = 1, 2, \ldots, T$, where T is a minimal number of t for which $d^t = d^{t-1}$ or $T = \infty$.

Proof From the postulates P3, P4 and P5 it follows that the sequence d^t, $t = 1, 2, \ldots$ is defined in the unique way. The sequence is monotonically increasing and limited, so it is convergent.

The outcome $f^t(d^{t-1})$ is Pareto optimal in S, due to properties of the generalized Nash solution concept.

Let the limit $\lim_{t\to\infty} d^t$ be denoted by d_{lim}. From the above definition of the process d^t, it follows that $d_{lim} \in S$.

Let d_{lim} be not Pareto optimal in S. Then for any round t, the following relations hold:
$||d^t - d^{t-1}|| = ||\alpha^t[f^t(d^{t-1}) - d^{t-1}]|| \geq \alpha_{min}||f^t(d_{lim}) - d_{lim}|| = \delta > 0.$

That means that the sequence $\{d^t\}_{t=0}^{\infty}$ is not convergent. It is in contradiction to the assumption. Therefore it has been shown, that d_{lim} is Pareto optimal in the set S. $\diamond$

5 Algorithm

The iterative solution concept can be basis for construction of an algorithm which can be implemented in a computer-based system. The system is treated as a tool which supports multicriteria analysis made by decision makers and derives mediation proposals.

Let $d^t \in S$ denote a vector of payoffs in round t for $t = 1, 2, \ldots$, and $d^0 = d$. Let $S^t = \{y : y \in S, y > d^{t-1}\}$.

Each decision maker i has the following parameters to control process of multicriteria analysis and derivation of mediation proposal: reference points $r_i^t \in I\!R^{m^i}$, indicated preferred payoff nondominated in set S and confidence coefficient $\alpha_i^t \in (\delta, 1]$, where δ is a relatively small positive number $\delta > 0$.

On the basis of the reference points assumed by given decision maker attainable nondominated payoffs are derived, analyzed further by him. He is asked to indicate the preferred payoff. Each decision maker has access only to information in his own space of criteria. He does not know criteria nor attainable payoffs of the second decision maker.

Each decision maker can reduce improvement of his payoff, and at the same time of payoff of the second decision maker, in given round assuming relatively small value for the confidence coefficient.

Step 1. Set $t = 1$.

Step 2. System invites decision makers $i = 1, 2$ to make independently analysis of their nondominated payoffs in multicriteria bargaining problem (d^{t-1}, S^t).

Step 2.1 System presents to decision maker i information about the ideal point I_i^t, and the status quo point d_i^{t-1} in the decision maker criteria space. The ideal point is derived as $I_i^t = (I_{i,1}^t, I_{i,2}^t, \dots, I_{i,m_i}^t)$, where $I_{i,j}^t = \max y_{i,j} : y = (y_1, y_2) \in S^t \wedge y_{3-i} = d_{3-i}$.

Step 2.2 Decision maker i writes values of components of his reference point r_{ij}^t, $j = 1, 2, \dots, m^i$.

Step 2.3 System derives the nondominated solution in set S, according to the reference point approach and stores the solution in a data base.

Step 2.4 The decision maker analyzes generated nondominated payoff (payoffs). If he has enough information to select the preferred payoff, he indicates it as $\widehat{y}_i$ and assumes value for the confidence coefficient α_i^t. He signals finishing of the unilateral analysis phase.

Step 2.5 Has decision maker i finished unilateral analysis?
If no - go to Step 2.2, to generate next nondominated payoff. If yes - system writes the preferred nondominated payoff indicated by the decision maker $\widehat{y}_i$ as well as assumed value of confidence coefficient α_i^t to a data base.

Step 3. System checks whether both decision makers have finished their unilateral analysis, selected their preferred payoffs and defined values of the confidence coefficients. If no - system waits as long as they will finish generation and analysis of payoffs in Steps 2.1-2.5.

Step 4. System derives points $y^1 = (\widehat{y}_1, d_2^{t-1})$ and $y^2 = (d_1^{t-1}, \widehat{y}_2)$. Hyperplane H^2 is defined on this basis.

Step 5. System derives mediation proposal $d^t = (d_1^t, d_2^t)$ at round t,

$$d^t = d^{t-1} + \alpha^t [G^t - d^{t-1}],$$

where $G^t = \arg\max_{y \in S^H} ||y_1 - d_1^{t-1}|| \cdot ||y_2 - d_2^{t-1}||$,
$\alpha^t = min\{\alpha_1^t, \alpha_2^t\}$, $0 < \rho < \alpha_i^t \leq 1$ for $i = 1, 2$.

Step 6. System presents mediation proposal - payoffs d_i^t to decision makers $i = 1, 2$ respectively.

Step 7. System checks the cooperative solution of the round. Is it Pareto optimal in set S?
If yes - end of the procedure.
If no - set number of next round $t = t + 1$ and go to Step 2.

In the algorithm a sequence of bargaining problems (S^t, d^{t-1}) is formulated and analyzed. Decision makers make in each round independent analysis of nondominated payoffs in set S^t using reference points. Then each of them selects his preferred payoff. This is made in Steps 2.1-2.5. The selected payoffs and confidence coefficients assumed by decision makers are used by the system to derive a mediation proposal which is proposed to the decision makers in a given round (Steps 4-7). Proposed construction of the mediation proposal assures that the proposal is consistent with preferences of all decision makers in the given round. Decision makers using confidence coefficients can inflow on the number of following rounds of the procedure. They can again analyze Pareto optimal frontier of set S in these rounds and correct previously indicated preferences. The mediation proposal derived by the system according to ideas of the Nash cooperative solution, defines distribution of the cooperation benefits which fulfills axioms A1 - A5 describing fair play rules.

The sequence of the mediation proposals derived in the procedure converges to the Pareto optimal element in set S, according to the properties of the iterative solution.

6 Conclusions

The paper deals with multicriteria bargaining problem. Two decision makers are involved in a bargaining process. Each decision maker valuates results of the bargaining with use of his own vector of criteria. The bargaining problem is formulated in the space being the cartesian product of the criteria spaces of the decision makers.

Theoretical results relating to a generalization of the Nash solution concept on the case of the multicriteria bargaining problem are presented. Two types of solutions are proposed and analyzed: one shot solution and an iterative solution. Two theorems presenting properties of the solutions are formulated and proved.

It has been shown that the proposed one shot solution concept satisfies the axioms of Pareto optimality, individual rationality, symmetry, independence of positive affine transformation of criteria, independence of irrelevant alternatives. The solution satisfying the axioms is unique. The solution is constructed with use of the preferred outcomes selected individually by the decision makers after multicriteria analysis of attainable payoffs.

The iterative solution is proposed as a result of a multiround bargaining process. In each round the decision makers make multicriteria analysis of attainable payoffs and select preferred outcomes. An outcome of the round is derived in the direction defined by the one shot solution, but the improvement of payoffs is limited by so called confidence coefficient assumed by the decision makers. It has been shown that the process is convergent to a Pareto optimal outcome in the agreement set and that the final solution satisfies the axioms mentioned above. The iterative solution states a theoretical background for construction of an algorithm, which can be implemented in a computer-base decision support system.

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