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Computer-based suport of multicriteria cooperative decisions – some problems and ideas

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COMPUTER-BASED SUPPORT OF MULITCRITERIA COOPERATIVE DECISIONS – SOME PROBLEMS AND IDEAS

Introduction

The paper deals with cooperation problems related to the decision situations in which several parties consider participation in a joint enterprise. Questions arise: When the cooperation is beneficial? What should be the fair engagement of the parties in the enterprise and the fair allocation of benefits among them? In the paper problems related to construction of computer-based systems supporting the decisions analysis made by the parties are discussed. Such systems can be built with application of the control theory methods, the mathematical modeling techniques, the optimization procedures and the modern advanced information technology.

It is assumed that each party has given own set of criteria, in general different, and has independent preferences over the criteria. Sovereignty of the decision makers representing the parties is assumed, i.e. the decision makers are fully responsible for the decisions they made. The computer-based system is only a tool aiding analysis of the decision situations and a tool facilitating the consensus seeking.

In practice, cooperation problems are solved through a negotiation process. Each party before the negotiations should derive its Best Alternative To Negotiated Agreement (abbreviated further as BATNA) - the concept introduced by Fisher, Ury [1]. In the negotiations a party can compare analyzed proposals to the derived BATNA and can evaluate its possible benefit from the cooperation.

The cooperation situations are modeled in the game theory: as so called bargaining problem for two and more players. The classical axiomatic theory of bargaining has been developed by Nash [23], Raiffa [27], Kalai, Smorodinsky [3], Roth [29], Thomson [31], and many others. The classical bargaining problem in the case of two and many issues is formulated in the theory in terms of utilities as a pair (S, d). Two parties (players) can reach any of the payoffs from the agreement set S, if they unanimously agree. The disagreement point d defines payoffs of the players in the case when they do not reach such an agreement. It is derived on the base of BATNA concept; in particular it can be the status quo point.

A solution of the bargaining problem is considered as a method to choose a point from the set S, accepted by rational players. Different solution concepts are proposed under different set of axioms (assumed properties describing feeling of fairness) the solution should fulfill. The argumentation for acceptation of the solution concept by the players is the following: if rational players agree on a selected set of axioms- principles and accept them as fair, why they should not accept the solution concept which fulfills the axioms.

In the paper the cooperative game model is used to describe the cooperation problems in the case of multicriteria payoffs of players. Solution concepts for the classical bargaining problem, like the egalitarian solution and those proposed by Nash, Raiffa-Kalai-Smorodisky are considered. The solution concepts extended on the multicriteria case can be used to derive mediation proposals generated in the system and presented to the parties for analysis in iterative procedures. The presented mediation support with use of the computer based system has been inspired by the single negotiation procedure frequently applied in international negotiations (see [28]).

The application area includes among others analysis of cooperation in the case of innovative activities, education systems and cost allocation problems. The references attached include among others selected papers dealing with computer-based support in negotiations [2, 5-9, 11-18], relating to the multicriteria decision analysis [10- 13, 16, 18, 25, 26, 34-36], to the utility function approach [4, 19-22, 29, 30, 32, 33] and to the game theory - mentioned above.

1. Idea of computer-based system

The proposed system includes a model representation, modules supporting unilateral analysis made by decision makers (DMs), module generating mediation proposals, as well as modules including an optimization solver, respective data bases, procedures enabling interactive sessions realizing mediation procedure and a graphical interface.

The model describing the decision situation of the parties is the base for decision analysis. The model is constructed by a system analyst with use of the gathered information according to the rules of the system sciences. It includes the specification of decision variables, exogenous variables, output quantities, criteria, model relations. The model parameters are identified on the base of the collected data. The model should be verified and validated. Therefore modules containing respective data base, model editor, procedures for estimation of model parameters and for model verification are included in the system.

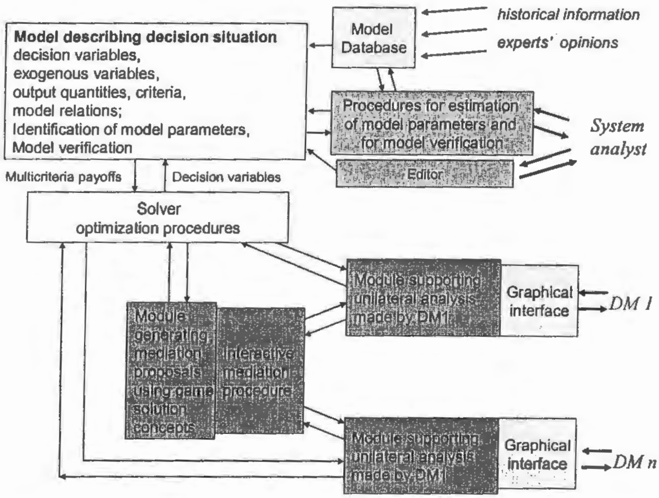

Fig. 1. General idea of computer-based system supporting cooperative decisions

The module supporting unilateral analysis enables each DM to obtain independently information about possible multicriteria payoffs for assumed scenario, and look for the preferred option. The analysis is made in an interactive way.

The system generates also mediation proposals. The mediation proposals are derived with use of selected solution concepts of the theory of cooperative games and on the base of the preferences expressed by DMs. The mediation proposals are generated and presented to the DMs in a special mediation procedure.

Optimization techniques are utilized in the system: in the procedures of multicriteria analysis in the modules supporting individual unilateral analysis and in the module generating mediation proposals to calculate game solution concepts. The respective optimization procedures are included in the solver module.

2. Model

To describe the cooperation situation an extension of the classical bargaining problem is considered in the case of n decision makers (DMs) called further players. Each DM (player) $i = 1, \ldots, n$, has defined:

- a vector of decision variables $x_i \in \boldsymbol{R}^{ki}$, where ki is number of the variables of the player i,
- a vector of criteria (to be maximized) $y_i \in \boldsymbol{R}^{mi}$, where mi is number of criteria of the player i.

A mathematical model describing the decision situation is given, defining:

- a set of admissible decisions $X^0 \subset R^K$, where $R^K = R^{k1} \times \ldots \times R^{kn}$ is the space of decisions of all the players,

- a space of payoffs of all the players $R^M = R^{m1} \times \ldots \times R^{mn}$, it is the Cartesian product of the multicriteria spaces of the players' payoff,
- a function $F : X^0 \rightarrow R^M$ defining vectors of the players' payoffs for given values of decision variables. In the case of continuous function F and compact set X^0 , the set of attainable payoffs $Y^0=F(X^0)$ is also compact.

Let each player has his own reservation point $d_i \in R^{mi}$ assumed in his multicriteria space on the base of the BATNA concept. Then the Multicriteria Bargaining Problem (MBP) can be defined by the disagreement point $d=(d_1, \ldots d_n) \in R^M$, and the agreement set S consisting of the points of the set $Y^0 \subset R^M$ dominating the point d. Each point of the agreement set can be reached if all the players unanimously agree, i.e the problem consists in selection of the point from the set S, which could be accepted by all the players.

Remarks to the problem formulation

- Each DM (player) has his own set of criteria, in generally different.
- Set of attainable payoffs is considered in the space R^M being Cartesian product of individual multicriteria spaces of the players.
- Set of attainable payoffs $Y^0 \subset R^M$ is in general not given explicitly.
- Multicriteria payoffs of each player can be derived by computer-based system for given values of the decision variables of all the players, using model relations.

An example of multicriteria bargaining problem is presented in Fig. 2 in the case of two players. Player 1 has criteria y_{11} and y_{12}, player 2 has only one criterion y_{21} In the three-dimensional space of criteria an agreement set S and a disagreement point d are shown.

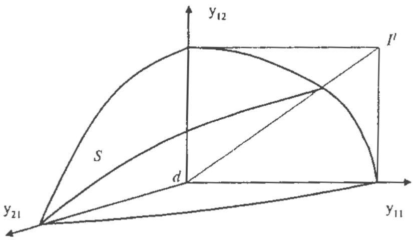

Fig. 2. An example of multicriteria bargaining problem

The disagreement point d is based on BATNA of each player. In general case derivation of the disagreement point may also require additional multicriteria analysis made by each player. The agreement set S is defined by model relations, and in general is not known explicitly. The ideal point in the criteria space of the player 1 is also shown denoted by I^{1}.

3. Unilateral analysis

Each player starts from unilateral interactive multicriteria analysis of the problem. During the analysis he can obtain information about possible outcomes for different assumptions about his preferences. He has also to make assumptions about the conterplayers' outcomes or counterplayers' preferences. The analysis can be made applying the reference point approach developed by Wierzbicki [31-33] with use of the order approximation functions. According the approach each player assumes reference points in the space of his criteria and the system generates respective outcomes which are Pareto optimal in the set S. For some number of reference points assumed by a player, a characterization of the Pareto frontier of the set S can be obtained.

Outcomes characterizing the Pareto frontier in the case of i-th player are derived by:

$$\max_{x \in X_0}[s\ (y_i(x), y_i^*)] \tag{1}$$

where:

y^* is a reference point assumed by the player in the space $\boldsymbol{R}^{mi}$,

$y_i(x)$ defines vector of criteria of the i-the player, which are dependent on the vector x of decision variables, by the model relations,

$s(y,y^*)$ is the order approximating achievement function.

The function

$$s(y_i,\ y_i^*) = \min_{1 \leq j \leq mi} [a_j(y_{ji}-y^*_{ji}) + a_{mi+1} \textstyle\sum_{i=1}^{mi} a_j(y_{ji}-y^*_{ji})\ ], \tag{2}$$

states an example of the achievement function suitable in this case, where $y_i^* \in R^{mi}$ is a reference point, $a_j,\ 1 \leq j \leq mi$, are scaling coefficients, and $a_{mi+1} > 0$ is a small parameter.

The assumed reference points and the obtained Pareto outcomes are stored in a data base, so that a characterization of the Pareto frontier can be made and analyzed by the player.

Fig. 3 presents results of the unilateral, interactive analysis made by the player 1 in his criteria space for two different assumptions about the second player outcomes: 1-st - for the counterplayer's outcomes assumed on the level of *d*, and 2-nd - for the conterplayer's aspirations assumed by the player 1.

Using the reference point approach the player can generate a number of such characterizations of the Pareto frontier. At the end, the player is asked to indicate the preferred outcome.

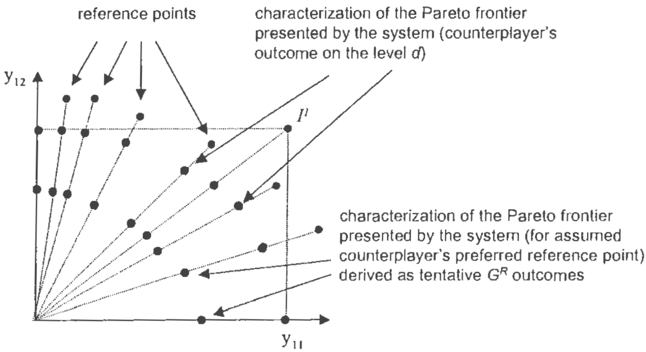

Fig. 3. Characterizations of the Pareto frontier obtained during unilateral analysis

The unilateral analysis is made by each player. Information about the indicated preferred outcomes of all the players are collected.

4. Interactive procedure supporting mediation process

The procedure has been proposed under inspiration of the Single Negotiation Text (SNT) procedure frequently applied in international negotiations. The SNT procedure formulated by Roger Fisher and described among others by Raiffa [28], is applied to break crisis situations which appear in hard positional negotiations. According to the procedure opponents should not discuss tasks independently nor formulate and consider counterproposals. They obtain and analyze, in consecutive rounds, proposals prepared by the mediator. In each round they work on the same text. On the base of their opinions and suggestions, the mediator prepares improved proposal being analyzed in the next round.

The proposed procedure consists of sequence of rounds $t=1,2,\ldots T$. Rules of the procedure can be point out as follows:

- in each round each player supported by the computer based system makes interactive unilateral analysis in his criteria space and indicates a required improvement direction of his outcome according to his preferences among the criteria,
- the computer-based system generates consecutive mediation proposals on the base of the improvement directions indicated by all players,
- each player analyzes the proposals and corrects required improvements of his outcome and the system generates new improved mediation proposal.

The consecutive mediation proposal d^t is generated in the round t on the base of the players indications, according to the scheme:

$$d^0=d,$$
$$d^t=d^{t-1}+\alpha^t\cdot[G^t-d^{t-1}], \text{ for } t=1,2, \quad T, \qquad (3)$$

where $\alpha^t=\min\{\alpha^{t1}, \ldots,\alpha^{tn}\}$, α^{ti} is so called confidence coefficient assumed by the player i in the round t, $0<\varepsilon<\alpha^{ti}<1$, G^t is the game solution calculated in the round t, for example the Raiffa solution, generalized on multicriteria case.

In the Cartesian product of multicriteria spaces of the players' payoffs a point is derived which is a composition of the preferred outcomes indicated by the players after the unilateral analysis. This point denoted by U^R in Fig. 4 is called the relative utopia point. It relates to aspirations of the players. In fact it is derived according to the players preferences expressed after the unilateral analysis. In general it is different than the ideal point defined by the maximal values of criteria in the S set.

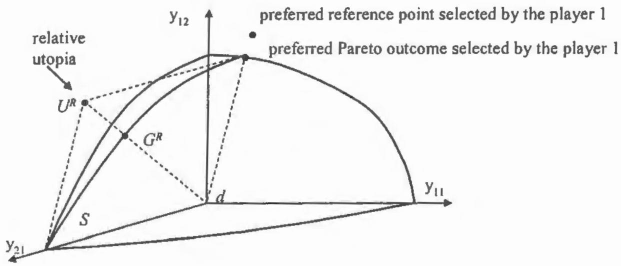

Fig. 4. Relative utopia and generalized Raiffa solution

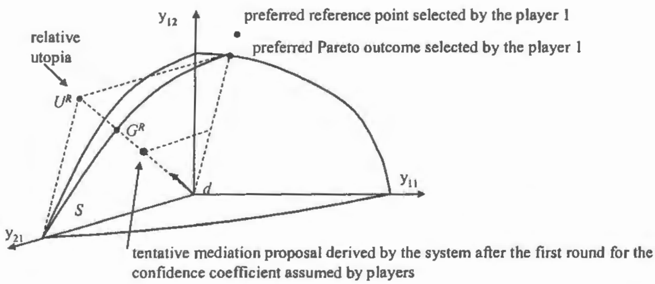

Fig. 5. Tentative mediation proposal

The generalized Raiffa solution G^R is the maximal point in S, on the line linking the disagreement point d and the relative utopia U^R. Each player assuming the confidence coefficient relatively small, less then 1, can limit increase of the payoffs of all the players in the given round as it is presented in Fig. 5. A tentative mediation proposal is derived according to the formula (3).

The tentative mediation proposal derived in the round t is treated as the disagreement point d^t in the next round $t+1$. Next, unilateral analysis is made by each player who explores the set of points belonging to S and dominating the d^t point. It is illustrated by Fig. 6.

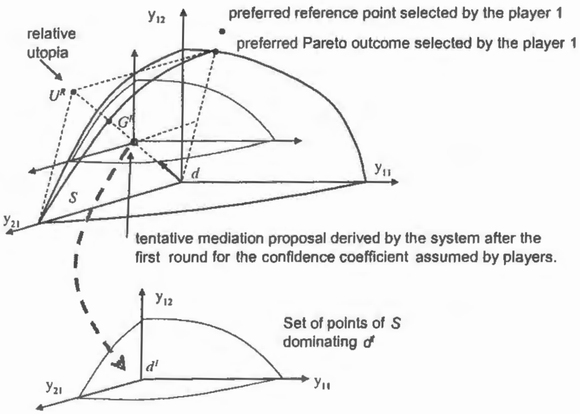

Fig. 6. The mediation proposal is treated as the disagreement point in the problem analyzed in the next round.

The preferred outcome selected by a player defines a direction in his multicriteria space. The directions of all players define a hyperplane in the Cartesian product of the spaces. The relative utopia and generalized

Raiffa solutions lay in the hyperplane. Other theoretical solutions of the game theory laying on the hyperplane can be considered based for example on the egalitarian concept or Nash cooperative solution concept.

The egalitarian solution maximizes gain of equal coordinates. It satisfies axioms of weak Pareto optimality, symmetry, strong monotonicity.

The Nash cooperative solution maximizes the product of the payoffs increases. It satisfies the axioms of Pareto optimality, symmetry, scale invariance, independence of irrelevant alternatives (see Nash [23]).

The generalized Raiffa solution concept mentioned before satisfies axioms of weak Pareto optimality, symmetry, scale invariance, restricted monotonicity (see Kruś, Bronisz [12] and Kruś [16]). The solution can be only weakly Pareto optimal even if the set S is convex. It means that a payoff of some player can be improved without decreasing of the payoffs of other players. Application of the reference approach and the achievement function of the form (2) partially solves the problem. Maximization of the function for $x \in S$ and decreasing parameter $a_{n+1} \rightarrow 0_+$ results in the lexico-graphic order applied to two separate terms of the function. For further discussion of the reference point method and the lexicographic ordering see [10, 26].

5. FINAL REMARKS

In the paper a computer-based system supporting cooperative decisions is proposed. The model based approach is applied. The model of cooperation problem is formulated with use of the game theory ideas.

The system supports multicriteria analysis of the problem made independently by parties with use of the reference point approach. Each player assuming reference point in the space of his criteria can generate with use of the system a set of outcomes characterizing Pareto frontier of possible outcomes. It is made by solving maximization problems with specially constructed achievement functions. The system generates also Pareto optimal compromise outcomes. They are derived taking into account the information on the parties preferences expressed in a special interactive procedure. The outcomes satisfy axioms of cooperative solutions formulated in the theory of games generalized on the multicriteria case. They can be treated as mediation proposals aiding the players in looking for the consensus. Parties using the system can understand the nature of the cooperation problem, can learn what their real preferences among the criteria are, can analyze possible outcomes, and can make the final decision about cooperation consciously.

The paper continues the line of research presented in the references [11-18]. It is a part of the research including development of methods and computer experiments in the case of different cooperation problems. The research includes decision situation described by the multicriteria bargaining problems, but also by the multicriteria noncooperative games, the multicriteria cooperative games with and without side payments. In the research the utility function approach, being an alternative to the direct multicriteria analysis, is also developed. In particular concepts proposed by R. Kulikowski [19-22] are applied to support decision analysis taking into account the presence of risk. The concepts extend ideas developed in the papers [4, 24, 30, 32, 33]. They are applied among others in the case of financial analysis [19, 21], analysis of innovative activities [15, 20] and analysis of education decisions [22].

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