III.

§ 1.

$a, b, c, k \,\varepsilon\, \mathrm{n} \,.\supset:$

1. $0 \,\varepsilon\, \mathrm{n}\, a$.
2. $a \,\varepsilon\, \mathrm{n} \times 1$.
3. $a \,\varepsilon\, \mathrm{n}\, a$.
4. $ab \,\varepsilon\, \mathrm{n}\, a$.
5. $a \,\varepsilon\, \mathrm{n}\, b \,.\, b \,\varepsilon\, \mathrm{n}\, a \,.=.\, a = \pm b$.
6. $a \,\varepsilon\, \mathrm{n}\, b \,.\, b \,\varepsilon\, \mathrm{n}\, c \,.\supset.\, a \,\varepsilon\, \mathrm{n}\, c$.
7. $a, b \,\varepsilon\, \mathrm{n}\, c \,.\supset.\, a + b, a - b \,\varepsilon\, \mathrm{n}\, c$.
8. $a \,\varepsilon\, \mathrm{n}\, b \,.\supset.\, ac \,\varepsilon\, \mathrm{n}\, bc$.
9. » $.\supset.\, ac \,\varepsilon\, \mathrm{n}\, b$.
10. $a \,\varepsilon\, b + \mathrm{n}\, k \,.\, b \,\varepsilon\, c + \mathrm{n}\, k \,.\supset.\, a \,\varepsilon\, c + \mathrm{n}\, k$.
11. » $.\, a' \,\varepsilon\, b' + \mathrm{n}\, k \,.\supset.\, a + a' \,\varepsilon\, b + b' + \mathrm{n}\, k$.
12. » $.\supset.\, ca \,\varepsilon\, cb + \mathrm{n}\, k$.
13. » $.\, a' \,\varepsilon\, b' + \mathrm{n}\, k \,.\supset.\, aa' = bb' + \mathrm{n}\, k$.
14. » $.\supset.\, a^m \,\varepsilon\, b^m + \mathrm{n}\, k$.
15. $ca \,\varepsilon\, cb + \mathrm{n}\, ck \,.\supset.\, a \,\varepsilon\, b + \mathrm{n}\, k$.

20. $a + b \,\varepsilon\, 2\mathrm{n} \,.=.\, a - b \,\varepsilon\, 2\mathrm{n} \,.=.\, a, b \,\varepsilon\, 2\mathrm{n} \,.\cup.\, a, b \,\varepsilon\, 2\mathrm{n} + 1$.
21. $a(a+1) \,\varepsilon\, 2\mathrm{n}$.
22. $a(a+1)(a+2) \,\varepsilon\, 6\mathrm{n}$.
23. $a(a+1)(2a+1) \,\varepsilon\, 6\mathrm{n}$.
24. $a(2a+1)(7a+1) \,\varepsilon\, 6\mathrm{n}$.
25. $(2a+1)^2 - 1 \,\varepsilon\, 8\mathrm{n}$.

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26. $a, b \,\varepsilon\, 2\mathrm{n} + 1 \,.\supset.\, a^2 - b^2 \,\varepsilon\, 8\mathrm{n}$.
27. $ab(a^2 + b^2)(a^2 - b^2) \,\varepsilon\, 30\mathrm{n}$.
28. $a \,\varepsilon\, \mathrm{n}\, b \,.\, m \,\varepsilon\, \mathrm{N} \,.\supset.\, a^m \,\varepsilon\, \mathrm{n}\, b^m$.
29. $m \,\varepsilon\, \mathrm{N} \,.\, a^m \,\varepsilon\, \mathrm{n}\, b^m \,.\supset.\, a \,\varepsilon\, \mathrm{n}\, b$.
30. $0! = 1$ . . . .
31. $a \,\varepsilon\, \mathrm{N} \,.\supset.\, a! = \Pi_{r=1}^{r=a}\, r = 1 \times 2 \times \ldots \times r$. } . . [Def.]
32. $(a+b)! \,\varepsilon\, \mathrm{N}\,(a!)(b!) \,.\, (a+b+c)! \,\varepsilon\, \mathrm{N}\,(a!)(b!)(c!)$.
33. $m \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}|\mathrm{Z}_m \,.\supset.\, (\Sigma_{r=1}^{r=m} fr)! \,\varepsilon\, \mathrm{N}\, \Pi_{r=1}^{r=m} [(fr)!]$.
34. $\mathrm{N}_0 = \mathrm{N} \cup \iota 0$ . . . . [Def.]

§ 2.

$a, b, c \,\varepsilon\, \mathrm{N} \,.\supset:$

1. $\mathrm{quot}\,(a, b) = \max\,[(\mathrm{N}_0) \cap \overline{x \,\varepsilon}\,(xb \leqq a)]$ . . . . [Def.]
2. $a < b \,.=.\, \mathrm{quot}\,(a, b) = 0$.
3. $a \geqq b \,.=.\, \mathrm{quot}\,(a, b) \,\varepsilon\, \mathrm{N}$.
4. $a \,\varepsilon\, \mathrm{N}\, b \,.\supset.\, \mathrm{quot}\,(a, b) = a/b$.
5. $\mathrm{rest}\,(a, b) = a - b\, \mathrm{quot}\,(a, b)$ . . . . . . [Def.]
6. $a \,\varepsilon\, \mathrm{N}\, b \,.=.\, \mathrm{rest}\,(a, b) = 0$.
7. $a \,-\varepsilon\, \mathrm{N}\, b \,.=.\, \mathrm{rest}\,(a, b) \,\varepsilon\, \mathrm{N}$.
8. $\mathrm{rest}\,(a, b) < b$.
9. $q, r \,\varepsilon\, \mathrm{N}_0 \,.\, a = bq + r \,.\, r < b \,.\supset.\, q = \mathrm{quot}\,(a, b) \,.\, r = \mathrm{rest}\,(a, b)$.
10. $\mathrm{quot}\,(ac, bc) = \mathrm{quot}\,(a, b)$.
11. $\mathrm{rest}\,(ac, bc) = c \times \mathrm{rest}\,(a, b)$.
12. $\mathrm{rest}\,(a + bc, b) = \mathrm{rest}\,(a, b)$.
13. $\mathrm{quot}\,(a, b) \,\varepsilon\, \mathrm{N} \,.\supset.\, a > 2\, \mathrm{rest}\,(a, b)$.
14. $a > b \,.\supset.\, \mathrm{quot}\,(a, c) \geqq \mathrm{quot}\,(b, c)$
15. $b > c \,.\supset.\, \mathrm{quot}\,(a, b) \leqq \mathrm{quot}\,(a, c)$
16. $a \,\varepsilon\, \mathrm{N} c \,.\supset.\, \mathrm{quot}\,(a + b, c) = \mathrm{quot}\,(a, c) + \mathrm{quot}\,(b, c)$.
17. $\mathrm{quot}\,\{\mathrm{quot}\,(a, b), c\} = \mathrm{quot}\,(a, bc)$.
18. $\mathrm{rest}\,(a, b) < \mathrm{quot}\,(a, b) \,.=:\, \mathrm{quot}\,[a, \mathrm{quot}\,(a, b)] = b \,.\, \mathrm{rest}\,[a, \mathrm{quot}\,(a, b)]$ $= \mathrm{rest}\,(a, b)$.
19. $\mathrm{rest}\,(a, b) = b - 1 \,.\, m \,\varepsilon\, \mathrm{N} \,.\supset.\, \mathrm{rest}\,(a^{2m}, b) = 1 \,.\, \mathrm{rest}\,(a^{2m-1}, b) = b - 1$.
20. $a - b \,\varepsilon\, \mathrm{N}\, c \,.=.\, \mathrm{rest}\,(a, c) = \mathrm{rest}\,(b, c)$.

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§ 3.

$a, b, c, d \,\varepsilon\, \mathrm{N} \,.\supset:$

1. $\mathrm{D}(a, b) = \max(a/\mathrm{N} \cap b/\mathrm{N})$ . . . . [Def.]

1'. $m \varepsilon 1 + \mathrm{N} . f \varepsilon \mathrm{N}/\mathrm{Z}_m . \supset . \mathrm{D}(f\mathrm{Z}_m) = \max[(f1)/\mathrm{N} \cap (f2)/\mathrm{N} \cap \ldots \cap (fm)/\mathrm{N}]$ [Def.]

2. $\mathrm{D}(a, b) = \mathrm{D}(b, a)$.

2'. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_n \,.\, g \,\varepsilon\, (\mathrm{Z}_n/\mathrm{Z}_n)\ \mathrm{sim} \,.\supset.\, \mathrm{D}(f\mathrm{Z}_n) = \mathrm{D}\big(f(g\mathrm{Z}_n)\big)$

3. $\mathrm{D}(a, 0) = \mathrm{D}(0, a) = a$ . . . . . [Def.]

4. $\mathrm{D}(a, -b) = \mathrm{D}(-a, b) = \mathrm{D}(-a, -b) = \mathrm{D}(a, b)$.

5. $\mathrm{D}(a, a) = a$.

6. $a \,\varepsilon\, \mathrm{N}b \,.\supset.\, \mathrm{D}(a, b) = b$.

7. $a > b \,.\supset.\, \mathrm{D}(a, b) = \mathrm{D}(b, a - b)$.

8. » $.\supset.\, \mathrm{D}(a, b) = \mathrm{D}(b, \mathrm{rest}(a, b))$.

8'. $\mathrm{D}(a, a + 1) = 1$.

9. $a, b \,\varepsilon\, \mathrm{N}c \,.\supset.\, \mathrm{D}(a, b) \,\varepsilon\, \mathrm{N}c$.

9'. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, (\mathrm{N}a)/\mathrm{Z}_n \,.\supset.\, \mathrm{D}(f\mathrm{Z}_n) \,\varepsilon\, \mathrm{N}a$

10. $\mathrm{D}(ac, bc) = c\mathrm{D}(a, b)$.

10'. $\mathrm{D}(a, b) = 1 \,.\supset.\, \mathrm{D}(ac, bc) = c$.

11. $\mathrm{D}(a, b) = 1 \,.=.\, (1 + \mathrm{N}) \cap (a/\mathrm{N}) \cap (b/\mathrm{N}) = \Lambda$.

12. $\mathrm{D}(a/\mathrm{D}(a, b),\ b/\mathrm{D}(a, b)) = 1$.

12'. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_n \,.\supset\therefore\, a = \mathrm{D}(f\mathrm{Z}_n) \,.=:\, (f\mathrm{Z}_n)/a \,\varepsilon\, \mathrm{KN} . \mathrm{D}\big((f\mathrm{Z}_n)/a\big) = 1$

13. $\mathrm{D}(a, b, c) = \mathrm{D}(\mathrm{D}(a, b), c)$.

14. $a \,\varepsilon\, 2\mathrm{N} \,.\, b \,\varepsilon\, 2\mathrm{N} + 1 \,.\supset.\, \mathrm{D}(a + b,\ a - b) = \mathrm{D}(a, b)$.

15. $a, b \,\varepsilon\, 2\mathrm{N} + 1 \,.\supset.\, \mathrm{D}(a + b,\ a - b) = 2\mathrm{D}(a, b)$.

16. $m, n \,\varepsilon\, \mathrm{N} \,.\supset.\, \mathrm{D}(a, b) = 1 \,.=.\, \mathrm{D}(a^m, b^n) = 1$.

17. $ab \,\varepsilon\, \mathrm{N}c \,.\, \mathrm{D}(a, c) = 1 \,.\supset.\, b \,\varepsilon\, \mathrm{N}c$.

18. $\mathrm{D}(a, c) = 1 \,.\supset.\, \mathrm{D}(ab, c) = \mathrm{D}(b, c)$.

19. $\mathrm{D}(a, c) = 1 \,.\, \mathrm{D}(b, c) = 1 \,.=.\, \mathrm{D}(ab, c) = 1$.

19'. $\mathrm{D}(a, c) = \mathrm{D}(b, c) = \mathrm{D}(a, d) = \mathrm{D}(b, d) = 1 \,.\supset.\, \mathrm{D}(ab, cd) = 1$.

20. $m \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_m \,.\supset\therefore\, \mathrm{D}(a, \Pi_1^m f) = 1 \,.=:\, r \,\varepsilon\, \mathrm{Z}_m \,.\supset_r.\, \mathrm{D}(a, fr) = 1$.

21. $a \,\varepsilon\, \mathrm{N}b \,.\, a \,\varepsilon\, \mathrm{N}c \,.\, \mathrm{D}(b, c) = 1 \,.\supset.\, a \,\varepsilon\, \mathrm{N}bc$.

22. $m \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_m : s, t \,\varepsilon\, \mathrm{Z}_m \,.\, s \mathrel{-=} t \,.\supset_{s,t}.\, fs \,\varepsilon\, a/\mathrm{N} \,.\, \mathrm{D}(fs, ft) = 1 :$
$\supset\therefore\, a \,\varepsilon\, \mathrm{N}(\Pi_1^m f)$.

23. $\mathrm{D}(a, b) = 1 \,.\supset.\, \mathrm{N} \cap \overline{x \,\varepsilon}\, (\mathrm{D}(a, x)\, \mathrm{D}(b, x) = x) = \mathrm{N} \cap (ab)/\mathrm{N}$.

24. $\mathrm{D}(a, b) = 1 \,.\supset.\, \mathrm{N} \cap (ab)/\mathrm{N} = (\mathrm{N} \cap a/\mathrm{N}) \times (\mathrm{N} \cap b/\mathrm{N})$.

25. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_n \,.\supset.\, \mathrm{N} \cap (f1)/\mathrm{N} \cap \ldots \cap (fn)/\mathrm{N} = \mathrm{N} \cap \big(\mathrm{D}(f\mathrm{Z}_n)\big)/\mathrm{N}$

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§ 4.

$a, b, c \,\varepsilon\, \mathrm{N} \,.\, \supset :$

1. $\mathrm{m}(a, b) = \min(a \times \mathrm{N} \frown b \times \mathrm{N})$ . . . . [Def.]

1'. $m \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_m \,.\, \supset .\, \mathrm{m}(f\mathrm{Z}_m) = \min[(f1) \times \mathrm{N} \frown (f2) \times \mathrm{N} \frown \ldots$
$\frown (fm) \times \mathrm{N}]$ . . . . [Def.]

2. $\mathrm{m}(a, a) = a$.

3. $\mathrm{m}(a, b) = \mathrm{m}(b, a)$

3'. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_n \,.\, g \,\varepsilon\, (\mathrm{Z}_n/\mathrm{Z}_n)\,\mathrm{sim} \,.\, \supset .\, \mathrm{m}(f\mathrm{Z}_n) = \mathrm{m}\big(f(g\mathrm{Z}_n)\big)$.

4. $a \,\varepsilon\, \mathrm{N}\, b \,.\, \supset .\, \mathrm{m}(a, b) = a$.

5. $\mathrm{m}(a, b) = ab/\mathrm{D}(a, b)$.

6. $\mathrm{D}(a, b) = 1 \,.\, \supset .\, \mathrm{m}(a, b) = ab$.

7. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_n : r, s \,\varepsilon\, \mathrm{Z}_n \,.\, \supset_{r,s} .\, \mathrm{D}(fr, fs) = 1 : \supset .\, \mathrm{m}(f\mathrm{Z}_n) = \Pi_1^{\,n} f$.

8. $c \,\varepsilon\, \mathrm{N}\, a \,.\, c \,\varepsilon\, \mathrm{N}\, b \,.\, \supset .\, c \,\varepsilon\, \mathrm{N}\mathrm{m}(a, b)$.

9. $\mathrm{N}a \frown \mathrm{N}b \frown \mathrm{N}c = \mathrm{N}\mathrm{m}(a, b, c)$.

10. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, \mathrm{N}/\mathrm{Z}_n \,.\, \supset .\, (f1) \times \mathrm{N} \frown \ldots \frown (fn) \times \mathrm{N} = \big(\mathrm{m}(f\mathrm{Z}_n)\big) \times \mathrm{N}$.

11. $\mathrm{m}(a, b, c) = \mathrm{m}(\mathrm{m}(a, b), c)$.

12. $\mathrm{m}(a, b, c) = abc\mathrm{D}(a, b, c) / [\mathrm{D}(a, b)\mathrm{D}(a, c)\mathrm{D}(b, c)]$.

§ 5.

1. $\mathrm{Np} = (1 + \mathrm{N}) \frown \overline{x \,\varepsilon}\, [(1 + \mathrm{N}) \frown (x/(1 + \mathrm{N})) = \Lambda]$. [Def.]

2. $2, 3, 5, 7, 11, \ldots \,\varepsilon\, \mathrm{Np}$.

3. $a \,\varepsilon\, 1 + \mathrm{N} \,.\, \supset .\, \min[(1 + \mathrm{N}) \frown (a/\mathrm{N})] \,\varepsilon\, \mathrm{Np}$.

4. » $.\, \supset .\, \mathrm{Np} \frown (a/\mathrm{N}) -= \Lambda$.

5. » $: x \,\varepsilon\, 1 + \mathrm{N} \,.\, x^2 \leqq a \,.\, x \,\varepsilon\, a/\mathrm{N} \,.\, =_x \Lambda \,.\!\cdot\!.\, \supset .\, a \,\varepsilon\, \mathrm{Np}$.

5'. » $: x \,\varepsilon\, \mathrm{Np}$. » » » »

6. $b \,\varepsilon\, \mathrm{Np} \,.\, a -\varepsilon\, \mathrm{N}b \,.\, \supset .\, \mathrm{D}(a, b) = 1$.

7. » $.\, a < b \,.\, \supset .\, \mathrm{D}(a, b) = 1$.

8. $a, b \,\varepsilon\, \mathrm{Np} \,.\, a -= b \,.\, \supset .\, \mathrm{D}(a, b) = 1$.

9. $a \,\varepsilon\, \mathrm{Np} \,.\, bc \,\varepsilon\, \mathrm{N}a \,.\, \supset .\, b \,\varepsilon\, \mathrm{N}a \,.\, \cup .\, c \,\varepsilon\, \mathrm{N}a$.

10. » $.\, n \,\varepsilon\, \mathrm{N} \,.\, b^n \,\varepsilon\, \mathrm{N}a \,.\, \supset .\, b \,\varepsilon\, \mathrm{N}a$.

11. » » $.\, a^n \,\varepsilon\, \mathrm{N}b \,.\, \supset .\, b \,\varepsilon\, a^{\mathrm{N}}$.

12. $a \,\varepsilon\, \mathrm{Np} \,.\, b -\varepsilon\, \mathrm{N}a \,.\, \supset .\, b^{a-1} - 1 \,\varepsilon\, \mathrm{N}a$.

13. » $.\, \supset .\, (a - 1)! + 1 \,\varepsilon\, \mathrm{N}a$.

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14. $a \varepsilon 1 + \mathrm{N} . \supset . \min \{(1+\mathrm{N}) \cap (a! + 1)/\mathrm{N}\} \varepsilon \mathrm{Np} \cap (a + \mathrm{N})$.

15. $\max \mathrm{Np} = \Lambda$.

16. $\mathrm{num}\, \mathrm{Np} = \infty$.

17. $x \varepsilon \mathrm{N} . x < 17 . \supset . x^2 - x + 17 \varepsilon \mathrm{Np}$.

18. » $. x < 41 . \supset . x^2 - x + 41 \varepsilon \mathrm{Np}$.

19. $a, b \varepsilon 1 + \mathrm{N} . \supset . \mathrm{mp}(b, a) = \max (\mathrm{N}_0 \cap \overline{x \varepsilon} (a \varepsilon \mathrm{N}b^x))$ [Def.]

20. » $. \supset . \mathrm{mp}(b, a) \varepsilon \mathrm{N}_0$.

21. » $c \varepsilon \mathrm{N} . \mathrm{D}(b, c) = 1 . \supset . \mathrm{mp}(b, a) = \mathrm{mp}(b, ac)$.

22. $a \varepsilon 1 + \mathrm{N} . b \varepsilon \mathrm{Np} \cap a/\mathrm{N} . \supset . \mathrm{D}(a/b^{\mathrm{mp}(b,\, a)}, b) = 1$.

24. $a, b \varepsilon \mathrm{N} . \supset .\therefore a \varepsilon \mathrm{N}b . = : x \varepsilon \mathrm{Np} . \supset_x . \mathrm{mp}(x, b) \leq \mathrm{mp}(x, a)$.

25. $a \varepsilon 1 + \mathrm{N} . (a-1)! + 1 \varepsilon \mathrm{N}a . \supset . a \varepsilon \mathrm{Np}$.

25'. $n, a \varepsilon 1 + \mathrm{N} . f \varepsilon (\mathrm{Np} / \mathrm{Z}_n) \mathrm{sim} . f\mathrm{Z}_n = \mathrm{Np} \cap a / \mathrm{N} . s \varepsilon \mathrm{Z}_{n-1} . \supset . \mathrm{Np} \cap \left[a / \Pi_{r=1}^{r=s} (fr)^{\mathrm{mp}(fr,\, a)}\right] / \mathrm{N} = f\mathrm{Z}(s+1, n)$

31. $u \varepsilon \mathrm{KN} . f \varepsilon (u/\mathrm{Z}_{\mathrm{num}\, u}) \mathrm{sim} . \supset . \Sigma u = \Sigma_{r=1}^{r=n} fr$

32. » $. \supset . \Pi u = \Pi_{r=1}^{r=n} fr$

33. $n \varepsilon (\mathrm{N} \cup \iota \infty) . f \varepsilon (\mathrm{N}/\mathrm{Z}_n) \mathrm{sim} . \supset . \Sigma_r fr = \Sigma_{r=1}^{r=n} fr$

34. » $. \supset . \Pi_r fr = \Pi_{r=1}^{r=n} fr$ [Def.]

31'. $u \varepsilon \mathrm{K(KN)} . f \varepsilon (u/\mathrm{Z}_{\mathrm{num}\, u}) \mathrm{sim} . \supset . \Sigma u = \Sigma_{r=1}^{r=n} fr$

32'. » $. \supset . \Pi u = \Pi_{r=1}^{r=n} fr$

33'. $n \varepsilon (\mathrm{N} \cup \iota \infty) . f \varepsilon ((\mathrm{KN})/\mathrm{Z}_n) \mathrm{sim} . \supset . \Sigma_r fr = \Sigma_{r=1}^{r=n} fr$

34'. » $. \supset . \Pi_r fr = \Pi_{r=1}^{r=n} fr$ [Def.]

35. $u \varepsilon \mathrm{KN} . \supset . \min_1 u = \min u$

36. » $n \varepsilon \mathrm{N} . \supset . \min_{n+1} u = \min (u \cap (\min_n u + \mathrm{N}))$ [Def.]

37. » » $. \supset . u_n = \min_n u$

38. $(\mathrm{Np})_1 = 2 . (\mathrm{Np})_2 = 3 . (\mathrm{Np})_3 = 5 . (\mathrm{Np})_4 = 7 . (\mathrm{Np})_5 = 11 \ldots$

39. $a \varepsilon \mathrm{N} + 1 . \supset : \mathrm{mp}(x, a) = 0 . = . x - \varepsilon a/\mathrm{N}$.

41. $a \varepsilon 1 + \mathrm{N} . \supset . a = \Pi_r \left[(\mathrm{Np})_r^{\mathrm{mp}((\mathrm{Np})_r,\, a)} \right]$

42. » $. \supset . \mathrm{N} \cap a/\mathrm{N} = \Pi_r \left[(\mathrm{Np})_r^{\mathrm{Z}(0,\, \mathrm{mp}((\mathrm{Np})_r,\, a))} \right]$

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43. $n \varepsilon 1 + \mathrm{N} \,.\, f \varepsilon \mathrm{N}/\mathrm{Z}_n \,.\supset.\, \mathrm{D}(f\mathrm{Z}_n) = \Pi_r\left[(\mathrm{Np})_r^{\min\{ \mathrm{mp}((\mathrm{Np})_r, f\mathrm{Z}_n)\}} \right]$

44. » $.\supset.\, \mathrm{m}(f\mathrm{Z}_n) = \Pi_r\left[(\mathrm{Np})_r^{\max\{ \mathrm{mp}((\mathrm{Np})_r, f\mathrm{Z}_n)\}} \right]$

45. » $.\supset:\, \mathrm{D}(f\mathrm{Z}_n) = 1 \,.=.\, \mathrm{Np} \frown (f1)/\mathrm{N} \frown \ldots \frown (fn)/\mathrm{N} = \Lambda$.

§ 6.

$a, b, c \,\varepsilon\, \mathrm{N} \,.\supset:$

1. $\mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}) = \Pi_r[\mathrm{mp}((\mathrm{Np})_r, a) + 1]$
2. $a \,\varepsilon\, \mathrm{N}^2 \,.=.\, \mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}) \,\varepsilon\, 2\mathrm{N} + 1$
3. $a \,\varepsilon\, \mathrm{N}^2 :=: b \,\varepsilon\, \mathrm{Np} \frown a/\mathrm{N} \,.\supset_b.\, \mathrm{mp}(b, a) \,\varepsilon\, 2\mathrm{N}$
4. $[\Pi(\mathrm{N} \frown a/\mathrm{N})]^2 = a^{\mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N})}$
5. $\mathrm{num}\left[\overline{(x, y)\,\varepsilon}\,(x, y \,\varepsilon\, \mathrm{N} . xy = a . \mathrm{D}(x, y) = 1 . x < y)\right] = a^{\mathrm{num}\,(\mathrm{Np} \frown a/\mathrm{N}) - 1}$
6. $\mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}) \,\varepsilon\, 2\mathrm{N} \,.\supset.\, \mathrm{num}\left[\overline{(x, y)\,\varepsilon}\,(x, y \,\varepsilon\, \mathrm{N} \,.\, xy = a) \,.\, x < y)\right] =$
   $(\mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}))/2$
7. $\mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}) \,\varepsilon\, 2\mathrm{N} + 1 \,.\supset.\, \mathrm{num}\left[\overline{(x, y)\,\varepsilon}\,(x, y \,\varepsilon\, \mathrm{N} . xy = a . x \leqq y)\right] =$
   $(\mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}) + 1)/2$.
8. $n = \mathrm{num}\,(\mathrm{N} \frown a/\mathrm{N}) \,.\, r \,\varepsilon\, \mathrm{Z}_n \,.\supset.\, (\mathrm{N} \frown a/\mathrm{N})_r \times (\mathrm{N} \frown a/\mathrm{N})_{n-r+1} = a$

11. $\pi a = \mathrm{Z}_a \frown \overline{x\,\varepsilon}\,[\mathrm{D}(a, x) = 1]$ . . . . [Def.]
12. $\varphi a = \mathrm{num}\,(\pi a)$ . . . . [Def.]
13. $\varphi 1 = 1 \,.\, \varphi 2 = 1 \,.\, \varphi 3 = 2 \,.\, \ldots$
14. $\varphi a \,\varepsilon\, \mathrm{N}$.
15. $\mathrm{D}(a, b) = 1 \,.\, b' \,\varepsilon\, \pi a \,.\supset.\, \mathrm{rest}\,(ab', b) \,\varepsilon\, \pi a$.
16. $\mathrm{D}(a, b) = 1 \,.\, b', b'' \,\varepsilon\, \pi a \,.\, b' \neq b'' \,.\supset.\, \mathrm{rest}\,(ab', b) \neq \mathrm{rest}\,(ab'', b)$.
17. $\mathrm{D}(a, b) = 1 \,.\supset.\, a^{\varphi b} - 1 \,\varepsilon\, \mathrm{N}b$.
18. $\mathrm{D}(a, b) = 1 \,.\supset.\, \varphi(ab) = (\varphi a)(\varphi b)$.
19. $n \,\varepsilon\, 1 + \mathrm{N} \,.\, f \,\varepsilon\, (\mathrm{N}/\mathrm{Z}_n)\,\mathrm{sim} : r, s \,\varepsilon\, \mathrm{Z}_n \,.\supset_{r,s}.\, \mathrm{D}(fr, fs) = 1 :\supset\therefore \varphi(\Pi(f\mathrm{Z}_n))$
    $= \Pi(\varphi(f\mathrm{Z}_n))$
20. $\Sigma_r \varphi((\mathrm{N} \frown a/\mathrm{N})_r) = a$
21. $\varphi a = \Pi_r[1 - 1/(\mathrm{Np} \frown a/\mathrm{N})_r]$
22. $\mathrm{D}(a, b) = 1 \,.\supset.\, (\mathrm{N} \frown \overline{x\,\varepsilon}\,(a^x - 1 \,\varepsilon\, \mathrm{N}b)) \neq \Lambda$
23. » $.\supset: x \,\varepsilon\, [\min\,(\mathrm{N} \frown \overline{x\,\varepsilon}\,(a^x - 1 \,\varepsilon\, \mathrm{N}b))] \times \mathrm{N} \,.=.\, a^x - 1 \,\varepsilon\, \mathrm{N}b$
24. $a \,\varepsilon\, b - \mathrm{N} \,.\, \mathrm{D}(a, b) = \mathrm{D}(b, c) = 1 \,.\supset.\, \mathrm{N} \frown \overline{x\,\varepsilon}\,(\mathrm{rest}\,(ac^x, b) = a) =$
    $[\min\,(\mathrm{N} \frown \overline{y\,\varepsilon}\,(b^y - 1 \,\varepsilon\, \mathrm{N}c))] \times \mathrm{N}$

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