Arch. Mech., 38, 3, pp. 281–287, Warszawa 1986

On stability of a column under circulatory load

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THE PAPER is devoted to the generalization of the results obtained for the optimal segmentation of Beck's column to the case of column with viscoplastic hinge joints. The problem is solved by using the transfer matrix technique. For some parameters of the system qualitative change of the shape of characteristic curves has been observed with a considerable discontinuous rice of critical load.

Pracę poświęcono uogólnieniu rezultatów optymalnej segmentacji columny Becka na przypadek lepkosprężystego podparcia kolumny oraz połączeń sprężystymi przegubami. Zagadnienie rozwiązano, wykorzystując technikę macierzy przeniesienia. Przy pewnych parametrach układu zaobserwowano jakościową zmianę kształtu linii charakterystycznych, której towarzyszy istotny, nieciągły wzrost obciążenia krytycznego.

Работа посвящена обобщению результатов оптимальной сегментации колонны Бека на случай вязкоупругого опирания колонны и соединений упругими шарнирами. Задача решена, используя технику матрицы преобразования. При некоторых параметрах системы наблюдалось качественное изменение формы характеристических линий, которому сопутствует существенный, разрывный рост критической нагрузки.

1. Introduction

VARIOUS generalizations of Beck's problem have received considerable attention in recent years. The stability problem of inelastic columns may be said to constitute a special branch of the broader area of problems concerned with the dynamic stability of structures. Several investigators as ANDERSON, KAR [1], WAHED [2] have studied the influence of damping on the stability of a cantilever beam resting on an elastic foundation and subject to a follower force at its free end. In most cases, however, the critical load values were obtained by approximate methods without estimation of accuracy of the received results. Moreover, when dampers are used as structural members or viscoelastic supports acting at several distinct points on the column, it is possible to obtain an exact solution [3]. As shown in [3] the critical force for the case of a single damper is independent of the damping coefficient, but depends mainly on the position of the damper. This paper is devoted to the generalization of the above results to the case of Beck's column with inelastic supports and some hinge-joints. The problem is solved by using the transfer matrix technique.

2. Formulation of the problem

In the following the structure principally shown in Fig. 1 will be considered. It consists of a segmented column supported by various types of inelastic supports or hinge-joints.

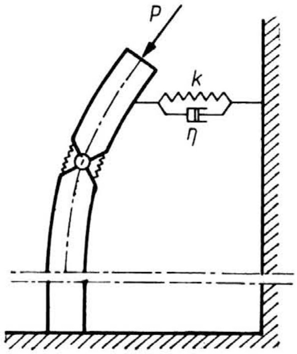

FIG. 1. Column under circulatory load.

The segment-joints or the supports are located at positions $x_1, x_2, \ldots, x_n$ and are characterized by the parameters $\varkappa_1, \ldots, \varkappa_n$, respectively, where

(2.1) $$\varkappa_j = \varkappa_j(k_j, \eta_j, \omega), \quad j = 1, 2, \ldots, n;$$

the parameters k_j, η_j refer to stiffness and damping of the j-th joint or support, ω is a circular frequency imposed on the system. For the case of a simple structure, particularly when it is subjected to distributed follower load, good results can be obtained using LEIPHOLTZ' generalization of the adjointness principle [4].

In the case of a more complicated structure with discontinuities of stiffness, cross-section or foundation at several distinct points, the exact solution can be obtained by using the transfer matrix technique [3, 5].

The simplest form of equation of motion of a uniform segment reads

(2.2) $$EI\frac{\partial^4 y}{\partial x^4} + P\frac{\partial^2 y}{\partial x^2} + \varrho A\frac{\partial^2 y}{\partial t} = 0$$

with EI bending stiffness, P longitudinal force, ϱ density, A cross-sectional area.

The boundary conditions for the case of clamped end are

(2.3) $$y = 0, \quad \frac{\partial y}{\partial x} = 0.$$

For the case of free end with tangential force we have

(2.4) $$\frac{\partial^2 y}{\partial x^2} = 0, \quad \frac{\partial}{\partial x}\left(EI\frac{\partial^2 y}{\partial x^2}\right) = 0,$$

and for the free end of Reut's case it follows

(2.5) $$EI\frac{\partial^2 y}{\partial x^2} + Py = 0, \quad \frac{\partial}{\partial x}\left(EI\frac{\partial^2 y}{\partial x^2} + Py\right) = 0.$$

The exact solution for this segment of constant mass and stiffness distribution has the form

(2.6) $$y(x, t) = e^{i\omega t}(A_1 \operatorname{sh}\lambda_1 x + A_2 \operatorname{ch}\lambda_1 x + A_3 \sin\lambda_2 x + A_4 \cos\lambda_2 x),$$

where

$$\lambda_{1/2} = \left(\frac{\mp P}{2EI} + \sqrt{\left(\frac{P}{2EI}\right)^2 + \frac{\varrho A\omega^2}{EI}}\right)^{1/2}. \tag{2.7}$$

Since all dependent variables y, φ, M, Q have a similar constitutive form (2.8), the state vector $\mathbf{S}$ and the partial transfer matrix $\mathbf{T}_i$ can be expressed as follows. The state vector $\mathbf{S}$ is given by

$$\begin{gathered}\mathbf{S} = [y, \varphi, M, Q]^T = [y, y', EIy'', -EIy''']^T,\\ \mathbf{S}_{i+1}^0 = \mathbf{T}_i\mathbf{S}_i^0, \quad \mathbf{S}_j^0 = \mathbf{S}_j(x_j = 0).\end{gathered} \tag{2.8}$$

The transfer matrix for the segment is defined in [6]. Non-zero elements of the transfer matrix for a support sensitive to deflection are

$$t_{ii} = 1, \quad t_{41} = \varkappa_d \tag{2.9}$$

and in the case of a support sensitive to rotation

$$t_{ii} = 1, \quad t_{32} = \varkappa_r. \tag{2.10}$$

For a hinge-joint the non-zero elements are

$$t_{ii} = 1, \quad t_{23} = \varkappa_h. \tag{2.11}$$

The transfer matrix for the whole structure can be expressed by

$$\mathbf{T} = \mathbf{T}_n\mathbf{T}_{n-1} \ldots \mathbf{T}_2\mathbf{T}_1. \tag{2.12}$$

Satisfying the boundary conditions, we get a characteristic equation as relation between force and frequency,

$$\Phi(P, \omega) = 0. \tag{2.13}$$

It is to be noted that in the case of a dissipative structure, the characteristic equation (2.13) is of a complex form. In order to get the critical values, one may use the MIKHAYLOV stability criterion as a similar one [6].

3. Results of numerical calculations

Using such a method, the influence of viscosity of a support on the critical load can be achieved. In general, viscosity influences the stability boundary if the postcritical behaviour of the structure has an oscillating form (flutter), while in the case of a divergent type of instability the viscosity has no influence. As an illustrative example let us consider a column supported at the end by a support of stiffness k and damping coefficient η. The results are shown in Fig. 2. It can be seen that for the case of a damper, acting at the end of the column, the critical force takes a smaller value than in the case without damper ($k = 0$). In the range $0 < k < k_0$ the critical values increase with increasing stiffness for both $\eta = 0$ and $\eta > 0$, but the value of critical force is independent of the value of η. Thus P_{cr} as function of k jumps to a lower value a $k = k_0$ and further decreases with increasing k.

For the case $k > k_0$, the critical force takes the same value for $\eta = 0$ and $\eta > 0$, because in this range of k the instability is of a divergent type.

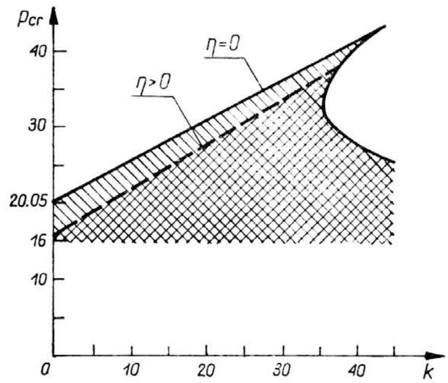

FIG. 2. Critical load versus spring stiffness.

A more complicated case was considered in [6] where the material of support was elastic-plastic. Due to the nonlinearity, in such a case various configurations of a stable and unstable limiting cycle are possible.

Let us now consider the behaviour of the column with an elastic hinge-joint with stiffness characterized by a parameter α,

$$\alpha = \left. \frac{L^2 E_1 I_1}{E I L_1^2} \right|_{L_1 \to 0}. \tag{3.1}$$

It is interesting to note that for the case of stiffness of an elastic hinge-joint smaller than the stiffness of the whole column ($\alpha < 1$) the critical value is greater than in the case of a uniform column without the hinge-joint (Fig. 3).

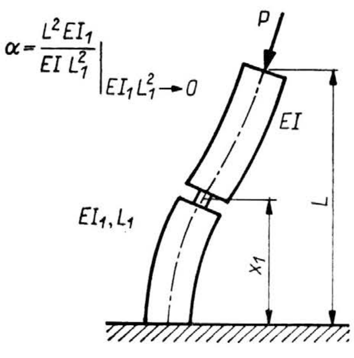

FIG. 3. Column with hinge-joint.

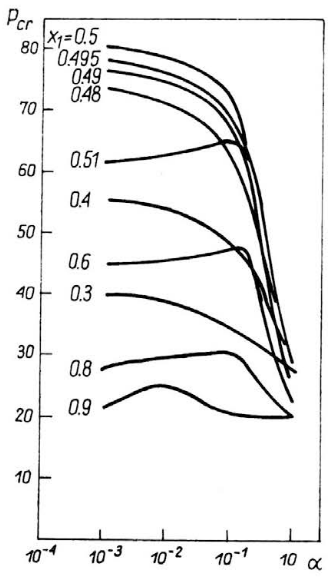

FIG. 4. Critical load versus hinge-joint stiffness.

The critical forces versus the stiffness of the hinge-joint for various values of x_1 describing the position of the joint are shown in Fig. 4. It can be seen that for $x_1 = 0.5$ the critical value of load increases rapidly in the range $\alpha \in (0.1, 1.0)$ with decreasing stiffness taking for $\alpha = 0.001$ a value about four times greater than for the case without a joint $(\alpha \to \infty)$.

For the values of $x_1 \in (0.5, 1.0)$ one can observe that there exists an optimal stiffness of the elastic joint for which the critical force reaches a maximum.

The configuration of the characteristic curves for various joint locations x_1 and a joint stifffness $\alpha = 0.001$ is shown in Fig. 5.

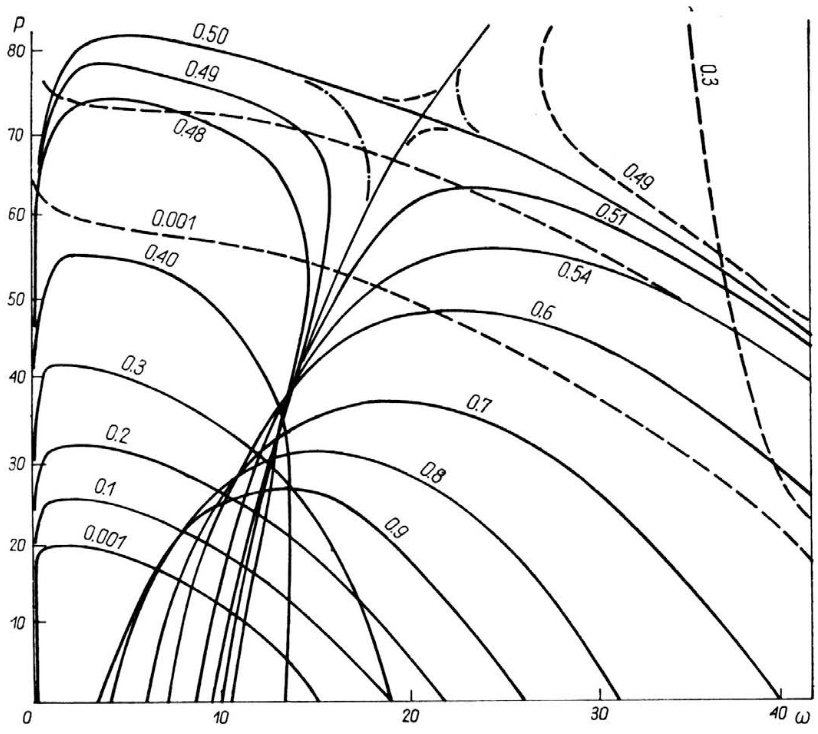

FIG. 5. Force-frequency plane. Characteristic curves for various joint locations ($\alpha = 0.001$).

It is interesting to observe that for the joint location $x_1 \in (0.5, 1.0)$ we have a classical shape of characteristic curves with an increasing critical value if the position x_1 tends from $x_1 = 1.0$ to $x_1 = 0.5$.

For $x_1 = 0.5$ we observe an intersection of the characteristic curves and a jump of the critical force from $P_{cr} \simeq 71$ to the value $P_{cr} \cong 81$ with a discontinuous decreasing of critical frequency from $\omega \cong 20$ to $\omega \cong 4.0$. Further change of the joint position in the direction of the damped end causes a decrease of the critical force and critical frequency. For the case of the column with an elastic joint of stiffness $\alpha = 0.001$ located at the position $x_1 = 0.001$, the critical load is smaller than in the case without a joint. The critical load as a function of dimensionless joint location is shown in Fig. 6 for some values of α. Similarly

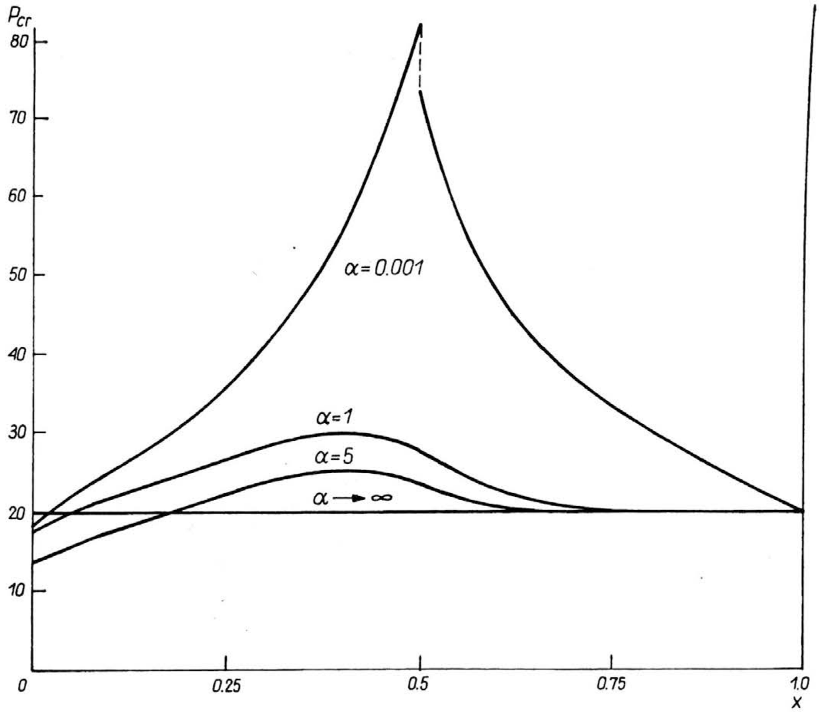

FIG. 6. Critical load versus joint location for various joint stiffnesses.

as in the case of viscoelastic support, there is a significant influence of damping on the critical load in case of an inelastic hinge-joint. This problem will be discussed in a separate paper.

4. Final remarks

The stability problem of a cantilever column with a local loss of rigidity or supported at a selected point and subjected to a follower force has been considered.

Considerable increase and decrease of the critical load has been observed depending on the parameters of support or hinge-joint. The characteristic curves are found to be sometimes very sensitive to small variations of the design variables. For this reason it might be regarded as theoretically feasible to make the critical load higher than that in the results obtained so far by using a hinge joint. It is interesting to know if such a phenomenon may occur also in the case of a supertangential load [7]. This problem as well as a limiting case to Ziegler's model will be disussed in a separate paper.

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Received June 10, 1985.