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Multicriteria optimization of single-layer cable systems

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THE PAPER deals with multicriteria structural optimization of cable structures. First, a general formulation of multicriteria optimization problem is presented and discussed. Next, some applications concerning the single-layer cable systems are considered. Minimum weight and maximum of the lowest natural frequency of free vibration are taken as optimization criteria. The permissible stresses and displacements are taken as behavioral constraints. The optimization problem is solved by using nonlinear programming and selected methods of multicriteria optimization.

W pracy przedstawiono zagadnienie optymalizacji wielokryterialnej konstrukcji cięgnowych. Najpierw przedstawiono ogólne sformułowanie zagadnienia optymalizacji wielokryterialnej, a następnie rozważono zagadnienie optymalizacji wielokryterialnej konstrukcji cięgnowych jednopasowych. Jako kryteria optymalizacji przyjęto minimum ciężaru konstrukcji i maksimum najniższych częstości drgań własnych. Ograniczenia zachowawcze dotyczą naprężeń i przemieszczeń. Zagadnienie optymalizacji rozwiązano za pomocą programowania nieliniowego i wybranych metod optymalizacji wielokryterialnej.

В работе представлена задача многокритериальной оптимизации вантовых конструкций. Сначала представлена общая формулировка задачи многокритериальной оптимизации, а затем рассмотрена задача многокритериальной оптимизации вантовых однопоясных конструкций. Как критерия оптимизации приняты минимум веса конструкции и максимум самых низких частот собственных колебаний. Консервативные ограничения касаются напряжений и перемещений. Задача оптимизации решена при помощи нелинейного программирования и избранных методов многокритериальной оптимизации.

1. Multicriteria optimization in structural design

1.1. Introduction

THE RESULTS of single criterion (scalar) optimization of single and double-layer cable systems as well as cable nets under static loading have been presented in papers [25, 26, 27]. The present paper is concerned with multicriteria structural optimization of single-layer cable systems under static and dynamic loading. First, the multicriteria optimization approach in optimum structural design is discussed. Next, two-criteria optimization of single-layer cable systems is considered. Minimum weight and maximum of natural frequency of free vibration are used as optimization criteria. The permissible stresses and displacements are taken as the behavioral constraints.

The paper deals with the problem of formulating the multiobjective function and finding the set of compromise solutions and also with selecting a preferable solution.

Appendix contains a few methods for selecting a preferable solution from the set of compromise solutions.

1.2. Characteristic of multicriteria optimization approach

Optimum structural design usually involves a number of requirements that should be met at the same time to obtain the fully useful design. In the case of single criterion optimization, one of the requirements is selected as the criterion while the remaining ones are met by including them into the constraints set. But with such an approach, it is necessary to determine a priori the bounds which these requirements should fulfill. Multicriteria or multiobjective optimization enable us to take into account numerous criteria that are often mutually conflicting. It is then possible to find the compromise and preferable solutions which — although none of the criteria involved attains its extremum — guarantee meeting of all the requirements in the best way possible [1, 9, 10, 21]. The multicriteria optimization approach has been already discussed in many papers devoted to optimum structural design (see e.g. [3, 13, 16, 37]).

Some criteria of structural optimization, namely minimum volume or weight of a structure, minimum potential energy or maximum structural stiffness, minimum displacement at selected points or regions of the structure, maximum critical force, maximum of the lowest frequency of free vibration, maximum moment of inertia and maximum safety or reliability, are discussed in papers [28] and [30]. Two of these criteria, namely minimum weight and maximum of the lowest frequency, will be considered in the following to solve optimization problems of single-layer cable systems. In papers [11] and [12] ESCHENAUER discussed the optimization problem of space structure that supports radiotelescopes, assuming the following criteria: minimum weight and minimum displacements of the radiotelescope surface from its initial configuration under different loading states. In [44] SATTLER presented a survey of multicriteria optimization methods and their use for the optimization of a structure consisting of beam and truss elements. He assumed the minimum weight of the lattice structure and the minimum displacements of the beam surface under different loading states as the optimization criteria. KOSKI [33, 34, 35] formulated the multicriteria optimization problem of bar structures assuming the minimum weight and minimum displacement of selected structural nodes as the objective functions. STADLER [46—49] applied two optimization criteria, namely minimum mass and minimum strain energy and called thus determined shapes the natural shapes. In the general statement of the multicriteria optimization problems of structures given by BAIER in [4, 5], the structural weight and energy stored under various loading states were assumed to be the optimization criteria. CARMICHAEL [8] solved the multicriteria optimization problem by employing the method of constrained objective functions. The optimization problems of mechanical structures with a few objective functions are also treated by OSYCZKA [36, 37] and RAO [39]. The state-of-art of multicriteria optimization approach in optimum structural design has been presented in [28, 29, 30]. In the present paper a brief formulation of the multicriteria optimization problem will be presented.

1.3. Formulation of the multicriteria optimization problem

The problem of multicriteria structural optimization is the generalization of a single-criterion optimization and it allows to get closer to the real conditions crucial for the selection of a design solution. The problem of multicriteria optimization can be formulated as follows:

$$\min_{\mathbf{x}\in\Omega} \mathbf{f}(\mathbf{x}),$$

where $\mathbf{f}:\Omega \to R^k$ is a vector objective function given by

$$\mathbf{f}^T(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\}$$

and $\Omega \subset R^n$ is a feasible domain defined by the equality and inequality constraints, i.e.

$$\Omega = \{\mathbf{x} \in R^n : \mathbf{h}(\mathbf{x}) = 0, \mathbf{g}(\mathbf{x}) \leqslant 0\}.$$

The components $f_i:\Omega \to R$, $i = 1, 2, \ldots, k$ are called the criteria of optimization and $\mathbf{x}$ is the vector of design variables. Since the particular components f_i of the objective vector are mutually conflicting, it is impossible to find the so-called ideal feasible solution $f_i = \min_{\mathbf{x}} f_i$, $i = 1, 2, \ldots, k$. The problem of multicriteria optimization can be solved in two stages. The first stage consists in determining the Pareto solution. In the second stage a preferable solution will be found. A vector $\overset{*}{\mathbf{x}} \in \Omega$ is called Pareto optimal if and only if there exists no $\mathbf{x} \in \Omega$ such that $f_i(\mathbf{x}) \leqslant f_i(\overset{*}{\mathbf{x}})$ for $i \in K$, $K = \{1, 2, \ldots, k\}$ with $f_i(\mathbf{x}) < f_i(\overset{*}{\mathbf{x}})$ for at least one i, $i \in K$. In other words, the above definition states that $\overset{*}{\mathbf{x}}$ is a Pareto optimal solution if there exists no feasible vector $\mathbf{x}$ which would decrease some criterion without causing a simultaneous increase in at least one criterion. There exist a number of methods which allow to generate the compromises set and they are discussed in numerous publications, e.g, [7, 10, 21, 28, 29]. They may be divided into two categories of non-preference techniques including Pareto optimization and preference techniques. In the second stage, a preferable solution is determined on the basis of the compromise set. A few methods for selection of a preferable solution are discussed in [21, 28, 29, 44]. A global criterion method, method of utility functions and method of constrained objective functions presented in Appendix are often used to select the preferable solution.

2. Two criteria optimization of single-layer cable systems

2.1. Characteristics of cable-suspended structures

This section deals with an optimization problem of single-layer cable systems which are often used as carrying elements in mechanical structures e.g. building machines as well as the load-carrying elements in electric power lines or hanging rope-ways. Single-layer cable systems are also used in large-span roofing structures as shown on Fig. 1.

Cable-suspended structures are substantially different from other kinds of structures because they are capable of assuming a variety of shapes under action of different loadings. That is why static and dynamic analysis of cable-suspended structures are different from those commonly known. The first difference is that the equilibrium conditions should be determined with the actual shape of the structure taken into account. The principle of structural rigidity cannot be used here. The second difference consists in the fact thas the principle of superposition is inapplicable to cable-suspended structures. This follows

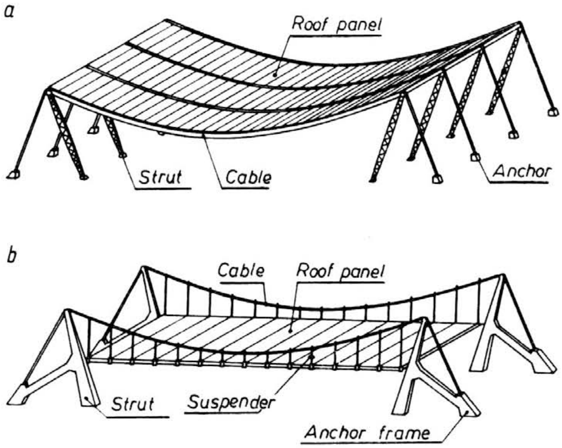

FIG. 1. Schematic diagrams of single-layer cable systems.

from geometric nonlinearity of cables caused by large changes in cables shapes due to varying loadings. The elongations of the large span cables can also result in major displacements and deformations of the structural shape. Then, to write the conditions of equilibrium and deformability, it is necessary to take into account all the loads acting on the structure which has no a priori determined shape. It is assumed that the cable can not resist bending and compression and constitutes a kinematically variable system. The dead weight of the cable can be neglected in comparison to the live loads acting on the cable. The physical nonlinearity of cables depends on their material behavior and construction of cables. However, for the sake of simplicity the stress-strain relationships can be assumed as linear because, within the range of working stresses, the behavior of cables obeys Hooke's law.

The purpose of optimization in the design of cable systems is to find the best shape of cable structure according to minimum weight criterion and/or maximum of the lowest frequency of free vibrations. The first criterion comes from economical consideration. The second one is derived from the experience that the most dangerous for dynamically loaded structures is usually the lowest natural frequency (e.g. in the case of wind loading). The dynamic analysis of cable systems is closely connected with its static solution. The cable shape and static internal force coming from static loading have a large influence on the natural frequency of free vibrations and their amplitudes as well as on the dynamic internal force. In the classical theory of elastic vibrations of structures such phenomenon does not occur.

2.2. Basic relationships of static and dynamic response

2.2.1. Large sag inextensible cables. SAXON and CAHN [43] have considered the in-plane free vibrations of an inextensible cable fixed at the rigid supports which are situated at the same level (Fig. 2). They have shown, in form of a diagram (Fig. 3), the relationship between the natural frequencies of free vibrations which are determined by means of parameter Λ_n/π and the sag of the cable which can be calculated on the basis of an angle

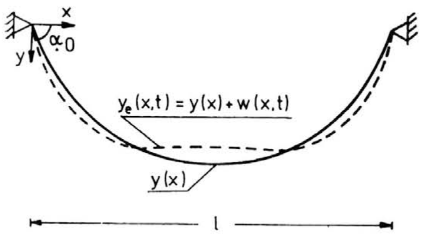

FIG.2. A symmetric in-plane free vibration.

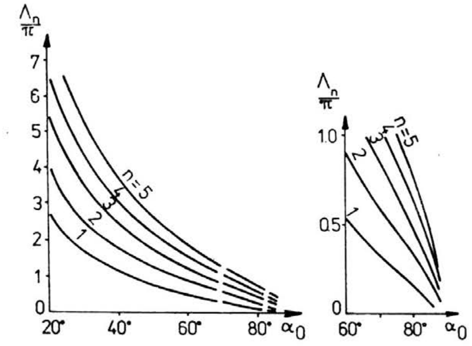

FIG. 3. Diagram of the relationship between the parameter Λ_n/Π describing the natural frequency and the angle α_0 (after [43]).

α_0 between tangent line to cable shape at the supports and horizontal line. This relationship is a monotonically decreasing function and maximal natural frequencies occur for the very small cable sags.

On the other hand, it has been proved that the minimum weight of single-layer cable systems corresponds to the large cable sags [23, 24]. This can be seen in Fig. 4 showing

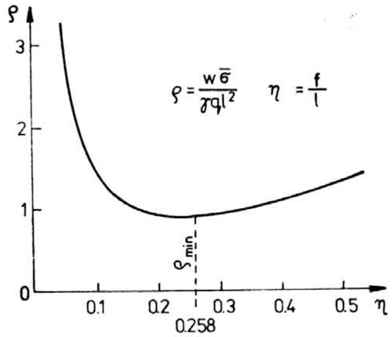

FIG. 4. Diagram of cable weight ϱ versus cable sag η.

the diagram of cable weight ϱ with respect to cable sag $\eta = f/l$ with the optimal value of $\eta = 0.258$: (i.e. a rather large cable sag). From the comparison of results discussed above it can be observed that the objective functions, minimum weight and maximum of the lowest frequencies of free vibrations, are in conflict. It means that a compromise solution should be found. In order to get such a solution it is necessary to formulate and solve the multiobjective optimization problem for single-layer cable systems.

2.2.2. Flat sag extensible cables. The in-plane free vibrations of the extensible flat sag cables fixed at supports situated at the same level have been considered by ANANIEV [2] and later by RZHANICYN [41]. HAJDUK and OSIECKI [18, 19], IVOVITCH [22], POPOV and RASTORGUJEV [38] have developed the dynamic analysis of single-layer cable systems. In [18] and [19] a nomogram (Fig. 5) for determining the values of the lowest (first) natural frequencies for symmetric in-plane free vibrations is presented. It can be seen

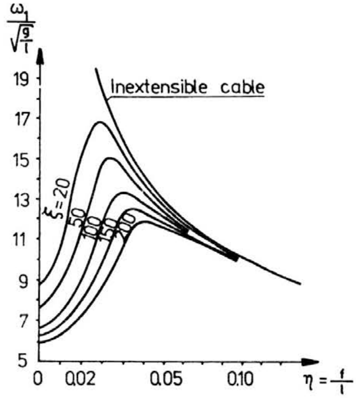

FIG. 5. Nomogram for determination of the lowest frequency for symmetric in-plane free vibration of the cable (after [18]).

from Fig. 5 that maximal values of the lowest natural frequencies occur for the small sag cables, i.e. $\eta = 0.03$—0.05 corresponds to the interval (20—200) of the parameter $\xi = ql/A$, respectively.

On the other hand, the same extensible flat sag cables were optimized according to minimum weight criterion in [24]. It is been shown that optimal values of cable sags determined according to the minimum weight criterion occur always on the boundary of the feasible domain (Fig. 6), which was determined by the permissible sag $(\bar{f})$, stress $(\bar{\sigma})$ and displacement $(\bar{w})$ constraints.

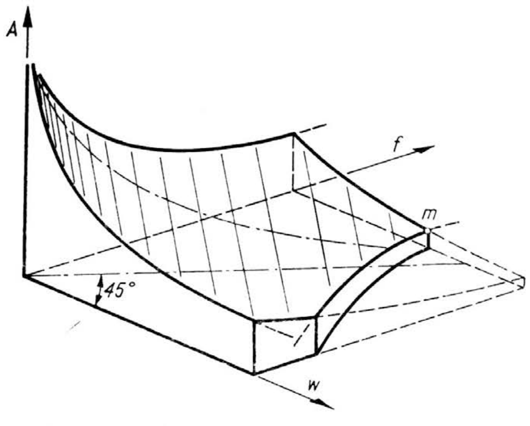

FIG. 6. Diagram of cross-section area A versus cable sag f and displacement w.

A similar conclusion as for the large sag cables can be drawn also for the flat sag cables by comparing the results discussed above: it means that a set of compromise solutions should be found by use of the multiobjective optimization approach.

2.3. Formulation of two criteria optimization problem

2.3.1. A general problem formulation. In general, the multiobjective optimization of single-layer cable systems can be formulated in the following way. Find cable sag $\eta = f/l$

and cross-sectional area A (design variables) and eventually the material properties (e.g modulus of elasticity E) for given cable span l, static loading $q(x)$, dynamic loading $p(x, t)$ and permissible stresses $\bar{\sigma}$ which minimize the weight of the single-layer cable system

$$\min_{\eta\in\Omega} W = \int_0^s A(\eta, E, q)\, ds_1 \tag{2.1}$$

and maximize the lowest natural frequency of the in-plane free vibrations

$$\max \omega_1 = \omega_1(\eta, A, E, q), \quad \text{where} \quad \Omega = \{\eta : 0.01 \leqslant \eta \leqslant 0.1\} \tag{2.2}$$

and satisfy the following set of constraints [18]:

The static equilibrium equations

$$\begin{aligned} &\frac{d}{dx}[T(x)\sin\alpha_0(x)] + q_y(x) = 0, \\ &\frac{d}{dx}[T(x)\cos\alpha_0(x)] + q_x(x) = 0. \end{aligned} \tag{2.3}$$

The dynamic equilibrium equations

$$\begin{aligned} \frac{m}{\cos\alpha_0(x)}\frac{\partial^2 w(x,t)}{\partial t^2} &= \frac{\partial}{\partial x}\left[T(x)\frac{\dfrac{\partial w(x,t)}{\partial x} - \varepsilon(x,t)y'(x)}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] \\ &\quad + \frac{\partial}{\partial x}\left[N(x,t)\frac{\dfrac{\partial w(x,t)}{\partial x} + y'(x)}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] + p_y(x,t), \\ \frac{m}{\cos\alpha_0(x)}\frac{\partial^2 u(x,t)}{\partial t^2} &= \frac{\partial}{\partial x}\left[T(x)\frac{\dfrac{\partial u(x,t)}{\partial x} - \varepsilon(x,t)}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] \\ &\quad + \frac{\partial}{\partial x}\left[N(x,t)\frac{1+\dfrac{\partial u(x,t)}{\partial x}}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] + p_x(x,t). \end{aligned} \tag{2.4}$$

The geometric nonlinear equation

$$\varepsilon(x,t) = \left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right]\cos^2\alpha_0(x) + \left[\frac{\partial w}{\partial x}\frac{dy(x)}{dx} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\cos^2\alpha_0(x). \tag{2.5}$$

One of the following physical equations depending on the structural material behavior taken into consideration

a) Hooke's law

$$N(x,t) = EA\varepsilon(x,t). \tag{2.6a}$$

b) Linear rheological laws

$$\Pi(t)N(x,t) = A\Gamma(t)\varepsilon(x,t).$$

$\Pi(t)$ and $\Gamma(t)$ are linear differential operators with respect to time depending on the rheological model of the material:

For Voigt–Kelvin material:

$$\Pi(t) = 1, \Gamma(t) = E+\bar{\eta}\frac{\partial}{\partial t},$$

where $\bar{\eta}$ — coefficient of internal damping; it has been obtained

$$N(x, t) = EA\varepsilon(x, t)+\bar{\eta}A\frac{\partial \varepsilon}{\partial t}. \tag{2.6b}$$

For the standard model:

$$\Pi(t) = 1+\frac{\bar{\eta}}{E}\frac{\partial}{\partial t}, \quad \Gamma(t) = E+\left(1+\frac{E}{E'}\right)\bar{\eta}\frac{\partial}{\partial t}$$

it has been found

$$N(x, t)+\frac{\bar{\eta}}{E'}\frac{\partial N}{\partial t} = EA\varepsilon(x, t)+\left(1+\frac{E}{E'}\right)\bar{\eta}A\frac{\partial \varepsilon}{\partial t}. \tag{2.6c}$$

c) Plastic deformability

$$N(x, t) = \begin{cases} EA\varepsilon(x, t) & \text{for} \quad \varepsilon \leqslant \varepsilon_e, \\ A[E\varepsilon(x, t)+\phi(\varepsilon-\varepsilon_e)] & \text{for} \quad \varepsilon > \varepsilon_e, \end{cases} \tag{2.6d}$$

where ε_e is the elastic limit deformation and ϕ is the post-elastic behavior function for the cable (e.g. Ramberg–Osgood law for postelastic material behavior).

d) Rigid cable

$$\varepsilon(x, t) = 0, \tag{2.6e}$$

and the mechanical constraints concerning allowable stresses and displacements, e.g.

$$\sigma_{\max} \leqslant \bar{\sigma}, \tag{2.7}$$

$$w_1(x, t)_{\max} \leqslant \bar{w}_1 \quad \text{or} \quad w_2 = \int_0^l w(x, t)dx \leqslant \bar{w}_2. \tag{2.8}$$

The above system of nonlinear partial differential equations was derived on the basis of continous model of mass distribution in dynamic analysis of single-layer cable systems. A solution of such a system of nonlinear equations cannot be found easily. But this system of equations can be linearized for the flat sag cables and elastic material behavior. In what follows, the system of linearized equations will be used to solve the multiobjective optimization problem for single-layer cable systems.

2.3.2. Problem formulation for the extensible flat sag cables. The multiobjective optimization problem of single-layer cable systems with the assumptions of the flat sags and Hooke's law for material behavior can be formulated as follows. Find cable sag $\eta = f/l$ and cross-sectional area A for the given cable span l, static loading $q(x)$, modulus of elasticity E, dynamic loading $p(x, t)$ and permissible stress $\bar{\sigma}$ such that the weight of a single-layer cable system

$$W = \gamma As = \gamma Al\left(1+\frac{8}{3}\eta^2\right) \tag{2.9}$$

is minimized. γ is the bulk density of the cable material. It was assumed here that the catenary can be replaced by a parabolic curve of second order

$$y = 4f\left(\frac{x}{l} - \frac{x^2}{l^2}\right); \tag{2.10}$$

because of its flatness. In this case the approximate length of cable is

$$s = l\left(1 + \frac{8}{3}\eta^2\right). \tag{2.11}$$

In addition, the first natural frequency of the in-plane free vibrations

$$\omega_1 = 2\alpha_1 \sqrt{\frac{H_{st}}{ql}} \sqrt{\frac{g}{l}} \tag{2.12}$$

has to be maximized.

This corresponds to the symmetric in-plane eigenmode

$$X_1(x) = C_1\left(1 - \cos\alpha_1 \frac{2x}{l} - \tan\alpha_1 \sin\alpha_1 \frac{2x}{l}\right), \tag{2.13}$$

where C_1 is a constant and α_1 can be determined from the following transcendental equation

$$\tan\alpha_1 - \alpha_1 + \alpha_1^3 \frac{H_{st}}{16\eta^2 EA} = 0. \tag{2.14}$$

The optimal solution should satisfy the following system of inequality and equality constraints:

Side constraints

$$0.01 \leqslant \eta \leqslant 0.1, \quad A > 0. \tag{2.15}$$

Static governing equation

$$\eta = \frac{1}{8}\sqrt{\left(\frac{ql}{H_{st}}\right)^2 - 24\frac{H_{st}}{EA}}. \tag{2.16}$$

Stress constraint

$$\sigma_{\max} = \frac{H_{st} + H_d}{A\cos\alpha_{\max}} = \frac{H_{st} + H_d}{A}(1 + 16\eta^2) \leqslant \bar{\sigma}, \tag{2.17}$$

where

$$H_d(t) = \frac{EA}{l}\int_0^l \frac{\partial w}{\partial x} y'(x)\, dx$$

with

$$w(x, t) = \phi(t) \cdot X_1(x).$$

Dynamic displacement constraint

$$w_{\max} = \phi(t) \cdot X_1(x) \leqslant \bar{w}, \tag{2.18}$$

where $\phi(t)$ is determined by the dynamic magnification factor regarding only the steady-state response. The last two constraints arise from the dynamic response and can be calculated on the basis of the dynamic loading represented e.g. by

$$p(x, t) = p_0 \sin(\overline{\omega} t). \tag{2.19}$$

Substituting Equation (2.16) into (2.9) gives the following cable weight function

$$W = \gamma A l \left[1 + \frac{1}{24}\left(\frac{ql}{H_{st}}\right)^2 - \frac{H_{st}}{EA}\right]. \tag{2.20}$$

2.4. Solution of the optimization problem

2.4.1. Determination of the sets of the feasible and compromise solutions. In order to solve the multiobjective optimization problem it is necessary to determine the set of feasible solutions in the design space (A, η) and the set of compromise solutions in the objective space (ω_1, W). The sets of feasible and compromise solutions should satisfy the constraints (2.14)—(2.18) as given above.

To find the sets of feasible and compromise solutions two problem formulations have been checked. In the first formulation we maximize the first natural frequency of free vibrations for a given cable weight $W = \text{const}$ and take the constraints (2.14)—(2.18) into account. The second one deals with the minimization of cable weight for a given frequency ω_1 considering the same group of constraints (2.14)—(2.18). Both formulations give the same sets of feasible and compromise solutions as shown in Figs. 7 and 8. The

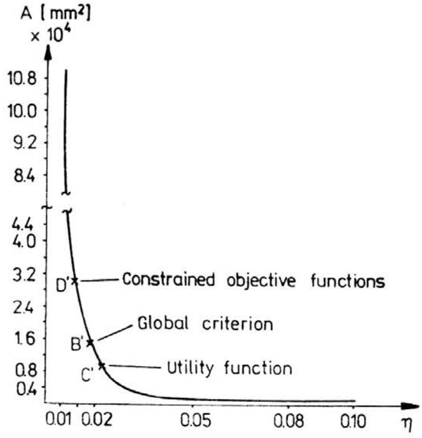

FIG. 7. Representation of the design variables space.

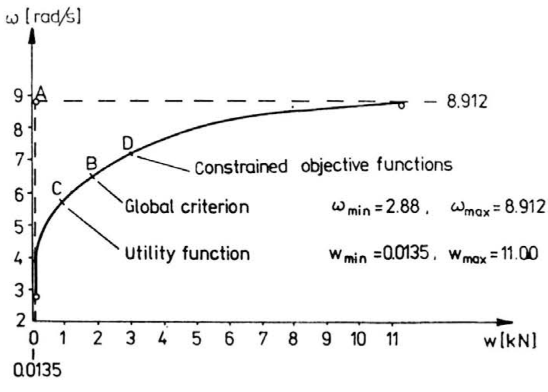

FIG. 8. Representation of the objective function space.

optimization problems were solved with two different methods. The first one uses the method of Lagrangian multipliers [45] and require the gradients of the objective functions and the constraints. The gradients were evaluated numerically.

The second algorithm is based on the evolution strategy [40] which works with random numbers and it can learn itself by the improvements of the objective function. This algorithm does not require any evaluation of gradients.

The two algorithms were used to compare their capability in solving the optimization problem described above and to increase the probability of attaining a global optimum.

2.4.2. Choosing a preferable solution. The set of compromise solutions shown in Figure 8 contains a number of solutions. It has to be decided which one should be taken as the preferable solution. There exist a few methods of choosing a preferable solution from the set of compromise solutions (see Appendix).

Using a global criterion to find a preferable solution we have $k = 2$ and we choose $p = 2$. The following data have been taken in numerical solutions: $q = 1$ kN/m, $E = 200$ kN/mm^2, $\bar{\sigma} = 1{,}2$ kN/mm^2, $p_0 = 0.001$ kN/m, $\bar{\omega} = 1$ rad/s, $\delta = 0.06$, where δ is the logarithmic decrement of damping. The objective functions are $f_1(x) = W(\eta, A)$ and $f_2(x) = \omega_1(\eta, A)$. The ideal solution satisfying the constraints (2.14)—(2.18) was found numerically and shown in Fig. 8 as the point A within the coordinates: $W_{min} = 0.0135$ kN, $\omega_{max} = 8.91$ rad/s. It does not belong to the set of compromise solutions. The preferable solution was found numerically by minimization of the distance function with $p = 2$; [6] i.e.

$$\min_{\eta\in\Omega} F^{(2)} = [(W - W^{id})^2 + \mu^2(\omega_1 - \omega_1^{id})^2]^{1/2}, \tag{2.21}$$

where $\mu = 1$ kN s/rad. The preferable solution obtained by global criterion is shown in Figure 8 as the point B ($W^{pr} = 1.60$ kN, $\omega^{pr} = 6.46$ rad/s) corresponding to the point B' in Fig. 7 ($\eta = 0.0185$, $A = 15990$ mm^2).

Next we determine the preferable solution using the utility function method in the form of (A.3) with weighting factors $w_1 = 0.5$ and $w_2 = 0.5$. To find the preferable solution it is necessary to minimize the utility function

$$U(\mathbf{f}) = \sum_{i=1}^{2} w_i f_i(\mathbf{x}) = w_1 W(\eta, A) - w_2 \omega_1(\eta, A) \tag{2.22}$$

subject to the constraints (2.14)—(2.18). The preferable solution found numerically is shown in Fig. 8 as the point C ($W^{pr} = 0.97$ kN, $\omega_A^{pr} = 5.93$ rad/s) corresponding to the point C' on Fig. 7 ($\eta = 0.023$, $A = 9700$ mm^2). The preferable solution can also be found by employing the method of constrained objective functions. A first natural frequency of free vibrations has been chosen as objective function which should be maximized, i.e.

$$\max_{\eta\in\Omega} \omega_1 = 2\alpha_1 \sqrt{\frac{H_{st}}{ql}} \sqrt{\frac{g}{l}}, \tag{2.23}$$

subject to (2.14)—(2.18) and additional constraints concerning the cable weight

$$\underline{W} \leqslant W(\eta, A) \leqslant \overline{W}. \tag{2.24}$$

The lower and upper limits of cable weight can be established on the basis of the permissible interval of cable sags which was taken as follows

$$\underline{\eta} \leqslant \eta \leqslant \overline{\eta}, \tag{2.25}$$

i.e. $\underline{W} = W(\overline{\eta})$ and $\overline{W} = W(\underline{\eta})$. We have taken $\underline{\eta} = 0.015$ and $\overline{\eta} = 0.1$. The preferable solution is shown in Fig. 8 as the point D ($W^{pr} = 3.00$ kN, $\omega^{pr} = 7.176$ rad/s) corresponding to the point D' in Fig. 7 ($\eta = 0.015$, $A = 30000$ mm^2).

3. Conclusions

The following conclusions might be drawn on the basis of the results presented above:

The optimal cable sags obtained separately according to minimum weight and maximum first frequency of free vibrations occur in different parts of the feasible domain.

In order to satisfy the two conflicting criteria mentioned above, the multicriteria optimization problem has been formulated and the sets of feasible and compromise solutions have been found.

The advantage of multicriteria optimization approach is of getting much more information about the optimal solution than from the single criterion optimization.

Appendix. Methods for selecting a preferable solution

A.1. Global criterion method

The global criterion method can be used to solve the following problem: find the minimum of the vector objective function

$$\min f_j(\mathbf{x}), \quad j = 1, 2, \ldots, k$$

satisfying the constraints

$$h_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, s, \quad g_i(\mathbf{x}) \leqslant 0, \quad i = 1+s, \ldots, m.$$

The first step consists in finding the ideal solution, that is, the vector $f_j(\mathbf{x}^{id})$, $j = 1, 2, \ldots, k$ which satisfies the minimum condition of each objective function $f_j(\mathbf{x})$ considered independently of the remaining ones. Then, the global criterion is formulated by requiring the distance between the optimal and ideal points

$$F^{(p)} = \Big[\sum_{j=1}^{k} |f_j(\mathbf{x}) - f_j(\mathbf{x}^{id})|^p\Big]^{1/p}, \quad 1 \leqslant p < \infty \tag{A.1}$$

to be minimum and satisfy the constraints

$$h_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, s, \quad g_i(\mathbf{x}) \leqslant 0, \quad i = 1+s, \ldots, m.$$

There exist the following cases:

$$\begin{aligned} p = 1, \quad & F^{(1)} = \sum_{j=1}^{k} |f_j(\mathbf{x}) - f_j(\mathbf{x}^{id})|, \\ p = 2 \quad & F^{(2)} = \Big[\sum_{j=1}^{k} |f_j(\mathbf{x}) - f_j(\mathbf{x}^{id})|^2\Big]^{1/2}, \\ p \to \infty \quad & F^{(\infty)} = \max_{j=1,2,\ldots,k} |f_j(\mathbf{x}) - f_j(\mathbf{x}^{id})|. \end{aligned} \tag{A.2}$$

The optimal vector is the one that minimizes the global criterion. The optimal solution depends substantially on the parameter p. For example, BOYCHUK and OVCHINNIKOV [7] propose to assume $p = 1$, while SALUKVADZE [42] suggests to put $p = 2$. If the particular functions $f_j(\mathbf{x})$ involve different units, then they are multiplied by coefficients $\mu_j = 1$ that include the corresponding units so that the expressions $\mu_j f_j(\mathbf{x})$ become dimensionless.

A.2. Method of utility function

By employing this method, the problem of multicriteria optimization is formulated as follows: find the minimum of the function

$$\min U = \min U(f_1, f_2, \dots, f_k)$$

subject to the constraining conditions

$$h_i(\mathbf{x}) = 0, \quad i = 1, 2, \dots, s, \quad g_i(\mathbf{x}) \leqslant 0, \quad i = 1+s, \dots, m.$$

The function $U(f_1, f_2, \dots, f_k)$ is called the utility function. It must be defined by analysing the intended objectives that are to be attained by making use of the solution of the optimization problem. In many cases, it may be difficult to define this function. This problem was treated by, among others, FARQUHAR [14], FISHBURN [15], HUBER [20] and KEENEY and RAIFFA [31, 32].

The solution of the problem is the contact point between the compromise set and the contour lines of the function U (for details see e.g. CHANKONG and HAIMES [9]). The utility function $U(f_1, f_2, \dots, f_k)$ can take various forms. It is most frequently additive and disjunctive with respect to the objective function, that is

$$U(f_1, f_2, \dots, f_k) = U_1 f_1 + U_2 f_2 + \dots + U_k f_k.$$

In a particular case, prioritization factors of individual objective functions can be given and then

$$U(f_1, f_2, \dots, f_k) = \sum_{j=1}^{k} w_j f_j(\mathbf{x}). \tag{A.3}$$

Other form of the utility function can be e.g.

$$U(f_1, f_2, \dots, f_k) = \prod_{j=1}^{k} U_j f_j.$$

The advantage of this method is its simplicity and the reduction of the problem of multicriteria optimization to the optimization with a single objective function. The principal difficulty lies in the determination of a utility function.

A.3. Method of constrained objective functions

This method is applicable provided that it is possible to determine the maximum values to be attained by the particular objective function. If this is possible, the problem of multicriteria optimization can be formulated as follows: find

$$\min f_r(\mathbf{x}),$$

subject to the constraints

$$h_i(\mathbf{x}) = 0, \quad i = 1, 2, \dots, s, \quad g_i(\mathbf{x}) \leqslant 0, \quad r = 1+s, \dots, m$$

and

$$f_j(\mathbf{x}) \leqslant u_j, \quad j = 1, 2, \dots, k, \quad j \neq r.$$

In a version of this method, the objective function has its lower and upper bound limits, that is, the additional constraints

$$f_j(\mathbf{x}) \geqslant l_j, \quad j = 1, 2, \ldots, k, \quad j \neq r$$

are produced.

In using this method, the main difficulty consists in finding such constraints l_j and u_j that would ensure the attainment of particular objectives and the existence of a non-empty objective region. One can also vary these and perform trade-off analysis as in the surrogate worth trade-off method (see e.g. HAIMES and HALL [17]). Moreover, it is necessary to make a decision which one of the objective functions should be selected as a criterion in solving the problem.

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