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Horizontal-vertical response spectra for El Centro 1940 earthquake

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A SINGLE-DEGREE-OF-FREEDOM structure subjected to horizontal and vertical earthquake ground accelerations is considered. The corrected accelerograms of El Centro 1940 earthquake are used. For a range of values of parameters, maximum horizontal responses are evaluated. Particular attention is given to the amplification effects of the vertical ground acceleration. Combined horizontal-vertical response spectra curves are developed. The results are compared with some peak response estimates and reasonable agreement is observed. A procedure for developing site-dependent smooth spectra is also outlined.

Rozważono konstrukcję z jednym stopniem swobody poddaną poziomym i pionowym przyspieszeniom wywołanym trzęsieniem ziemi. Wykorzystano skorygowane akcelerogramy trzęsienia El Centro 1940. Wyznaczono maksymalne reakcje poziome dla pewnego zakresu parametrów konstrukcji. Szczególną uwagę zwrócono na efekt wzmocnienia przyspieszeń pionowych. Opracowano wykresy dla widm drgań złożonych. Porównanie otrzymanych wyników z ocenami maksymalnych reakcji wykazuje dobrą zgodność. Omówiono również metodę przygotowania widm ciągłych uwzględniających położenie miejsca pomiaru.

Рассмотрена конструкция с одной степенью свободы, подвергнутая горизонтальным и вертикальным ускорениям, вызванным землетрясением. Использованы исправленные акцелерограммы землетрясения Эль Центро 1940. Определены максимальные горизонтальные реакции для некоторого интервала параметров конструкции. Особенное внимание обращено на эффект усиления вертикальных ускорений. Разработаны диаграммы для спектров сложных колебаний. Сравнение полученных результатов с оценками максимальных реакций показывает хорошее совпадение. Обсужден тоже метод подготовки непрерывных спектров, учитывающих положение места измерения.

1. Introduction

Use of response spectra for aseismic design of structures has attracted considerable attention in the past three decades. The technique provides an inexpensive and reasonably reliable alternative to response time history analysis. Furthermore, response spectra may be modified to meet the strength requirements in a particular seismic environment.

The procedure for constructing design response spectra was introduced by Housner [1], and Newmark *et al.* [2, 3]. These response spectra were developed from an ensemble of accelerograms that represents a variety of seismological, geological, and local soil conditions. To include the effect of the site conditions in the design response spectra several studies were carried out by Seed *et al.* [4], Mohraz [5], Wang and Yun [6], Mostaghel and Ahmadi [7, 8].

Use of design response spectra for the simultaneous action of three earthquake components were described in several codes [9, 10]. Further studies in this direction were carried out by Chu *et al.* [11], Penzien and Watabe [12] Roseblueth and Contreras

[13], ANAGNOSTOPULOS [14], and WILSON and BUTTON [15]. In these works, the techniques for combining the response spectra of various components were discussed.

In spite of considerable progress in developing appropriate design response spectra, one aspect of the response amplification is not fully considered as yet. Namely, the enhancement of lateral deflection due to the action of an axial excitation through the so-called $P-\Delta$ effect is not included in the design response spectra. In the present work a weightless column carrying a heavy mass at the top is considered as the working structural model. The North-South and vertical (NS+V) accelerograms of El Centro 1940 earthquake are used as base excitations. Response spectra for the horizontal deflection of the mass are evaluated, which include the effects of the verical earthquake excitation. The influences of the damping coefficient and load factor are examined. It is shown that the vertical ground acceleration amplifies the peak horizontal responses. The amount of amplification is quite modest as long as the static load is a small fraction of the Euler buckling load. When the static load is close to the Euler buckling load, vertical ground acceleration significantly enhances the horizontal responses.

A Liapunov technique for evaluating response bound is also briefly described. The results are used to develop approximate estimates for the peak responses under combined horizontal-vertical (HV) earthquake excitations. These estimates depend on the site periods and peak HV ground accelerations. The estimated maximum velocity responses are compared with the HV response spectra of (NS+V) El Centro 1940 earthquake and reasonable agreement is observed.

2. Equation of motion

Consider a single-degree-of-freedom structure consisting of a massless column which supports a lumped mass, m, at the top as shown in Fig. 1. Under seismic excitations the equation for the lateral deflection of the mass is given as [16, 17]

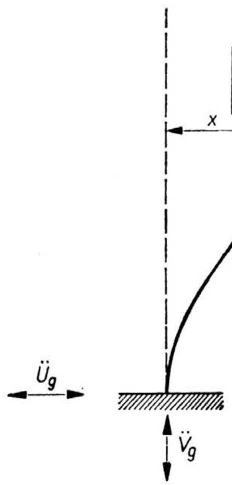

FIG. 1. Single-degree-of-freedom column under horizontal-vertical base excitations.

$$\ddot{x}+2\xi_1\omega\dot{x}+\omega_1^2\left(1-\frac{mg+m\ddot{v}_g}{P_{cr}}\right)x = -\ddot{u}_g, \tag{2.1}$$

where x is the lateral deflection of the mass, ξ_1 is the damping coefficient, ω_1 is the natural frequency of the load-free column, g is the acceleration of gravity, P_{cr} is the critical buckling load of the column and $\ddot{u}_g$ and $\ddot{v}_g$ are horizontal and vertical earthquake ground accelerations.

Equation (2.1) may be restated as

$$\ddot{x}+2\xi\omega_0\dot{x}+\omega_0^2(1-\beta\ddot{v}_g)x = -\ddot{u}_g, \tag{2.2}$$

where

$$\omega_0^2 = \omega_1^2\left(1-\frac{mg}{P_{cr}}\right), \quad \xi = \xi_1\omega_1/\omega_0, \quad \beta = m/(P_{cr}-mg). \tag{2.3}$$

Here ω_0 is the natural period of the column under static load and ξ is the effective damping coefficient. The parameter β may be restated as

$$\beta = \frac{\gamma}{g(1-\gamma)}, \quad \gamma = \frac{mg}{P_{cr}}, \tag{2.4}$$

where γ is the load factor.

The horizontal deflection of the column subjected to combined horizontal and vertical (HV) base excitation is governed by Eq. (2.2). When the effects of vertical acceleration are negligible (i.e. $\beta_0\ddot{v}_g \ll 1$), the solution to Eq. (2.2) is given by

$$x(t) = -\int\limits_0^t h(t-\tau)\ddot{u}_g(\tau)\,d\tau, \tag{2.5}$$

where the impulse response is defined as

$$h(t) = \frac{1}{\omega_d}e^{-\xi\omega_0 t}\sin\omega_d t \tag{2.6}$$

with

$$\omega_d^2 = \omega_0^2(1-\xi^2). \tag{2.7}$$

When $\beta_0\ddot{v}_g$ is not negligible, a closed form solution to Eq. (2.2) is not available.

3. Response spectra

The maximum values of relative displacement S_d, relative velocity S_v and absolute acceleration response S_a are of primary interest for aseismic design. These represent the peak strain, kinetic energy and force exerted on the structure, respectively. The corresponding plots versus natural period (or frequency) are referred to as the response spectra. Accordingly,

$$\begin{aligned} S_d(T_0, \xi, \gamma) &\equiv \operatorname{Max}_t |x(t)|, \\ S_v(T_0, \xi, \gamma) &\equiv \operatorname{Max}_t |\dot{x}(t)|, \\ S_a(T_0, \xi, \gamma) &\equiv \operatorname{Max}_t |\ddot{x}(t)+\ddot{u}_g(t)|. \end{aligned} \tag{3.1}$$

where $T_0 = 2\pi/\omega_0$ is the natural period of the structure and γ is the load factor defined in Eq. (2.4).

Pseudovelocity and pseudoacceleration response spectra are defined as

$$PS_v = \omega_0 S_d, \quad PS_a = \omega_0^2 S_d. \tag{3.2}$$

For simple linear structures with a small damping coefficient and negligible axial force ($\gamma = 0$), it follows that

$$PS_v \approx S_v, \quad PS_a \approx S_a. \tag{3.3}$$

When the effects of vertical acceleration are included, approximations given by the relations (3.3) are no longer accurate.

Here we are concerned with developing response spectra for the column shown in Fig. 1. The NS and vertical accelerograms of El Centro 1940 earthquake are used.

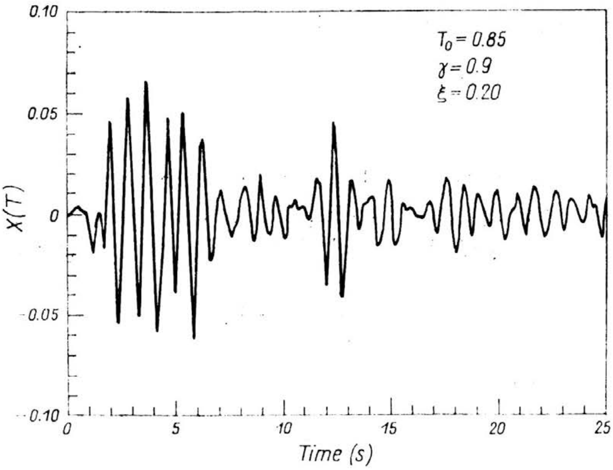

FIG. 2. A typical displacement response time history.

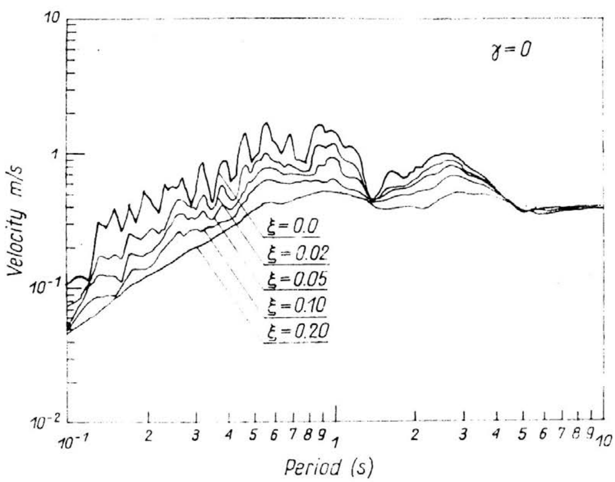

FIG. 3. Response spectra for El Centro 1940 earthquake.

A fourth-order Runge–Kutta scheme is employed for numerical integration. For a range of values of parameters, the time histories of the response are evaluated. A typical displacement response for a duration of 25 seconds is shown in Fig. 2.

Various types of response spectra as defined by Eqs. (3.1) and (3.2) may be constructed. Here the results for the velocity response spectra $S_v(T_0, \xi, \gamma)$ are reported. For several values of ξ, γ and T_0 the peak values of $\dot{x}(t)$ are evaluated from the corresponding response time histories. The resulting response spectra are plotted in Figs. 3–8. Figure 3 shows the response spectra of the NS-component of El Centro 1940 earthquake in the absence of vertical excitation. S_v for increasing values of γ are shown in Figs. 4–6. In Figs. 7 and 8, for fixed values of the damping coefficient, the HV response spectra for different values of the load factor are plotted. The amplification effects of the vertical ground acceleration may now be studied.

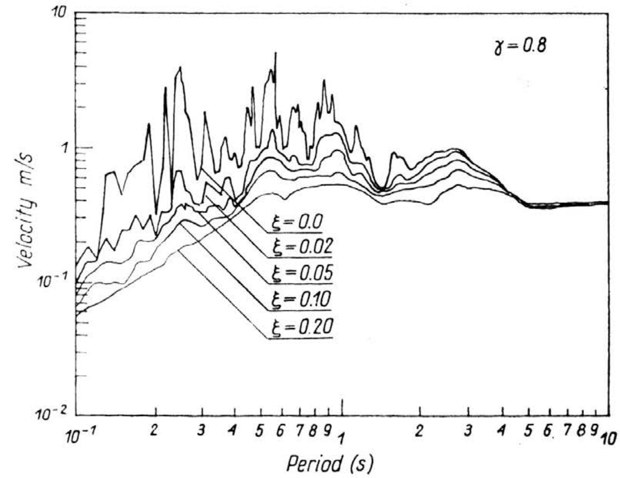

FIG. 4. HV reponse spectra for El Centro 1940 earthquake.

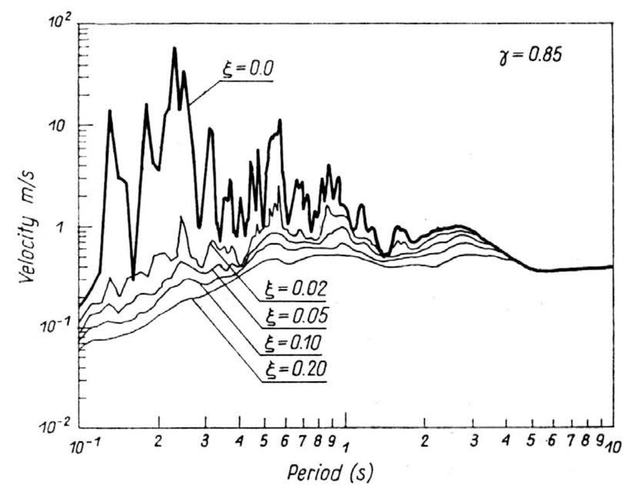

FIG. 5. HV response spectra for El Centro 1940 earthquake.

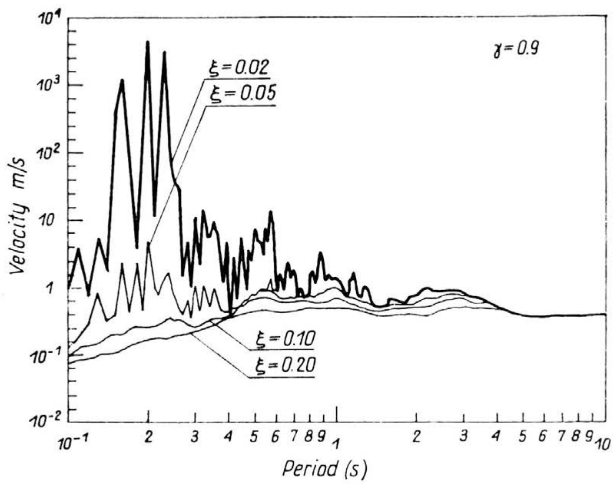

FIG. 6. HV response spectra for El Centro 1940 earthquake.

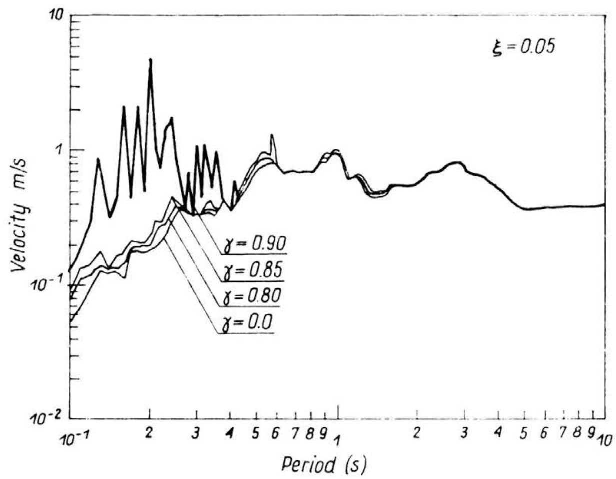

FIG. 7. HV response spectra for El Centro 1940 earthquake.

Figures 3–8 show that the amplitudes of the response spectra curves in the low period region (between 0.1s to 1s) increase rapidly with the load factor for $\gamma \geqslant 0.8$ and small damping coefficients. When γ is small or ξ is large (10% or 20%), the amplification effects of the vertical acceleration are negligible.

The results also show that the structural damping significantly modifies the amplification of horizontal responses. For instance, for zero damping the maximum values of S for $\gamma = 0$ and $\gamma = 0.85$ are 1.8 m/s and 98 m/s, respectively. That is, the vertical acceleration amplifies the response by a factor of fifty. The same maxima for a damping coefficient of 0.20 are 0.5 m/s and 0.54 m/s. That is, an increase of about ten percent.

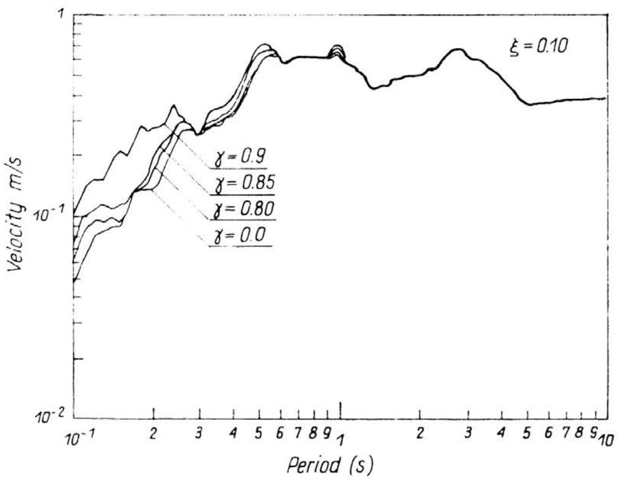

FIG. 8. HV response spectra for El Centro 1940 earthquake.

4. Site-dependent horizontal-vertical response spectra

In this section a formal bound on the response of the column subjected to combined horizontal and vertical earthquake ground accelerations is briefly described. The results are used to develop a procedure for constructing site-dependent design response spectra including the vertical base excitation.

In [17–19], using a Liapunov method, several bounds on the peak responses of structures were obtained. For a lightly damped column subjected to HV ground accelerations as described by Eq. (2.2), the expression for the bound as developed in [17] is given by

$$U(t) \leqslant \int_0^t |\ddot{u}_g(\theta)| \exp\left\{-\xi\omega_0(t-\theta) + \frac{\beta\omega_0}{2} \int_0^t \ddot{v}_g(\tau)\,d\tau\right\} d\theta, \tag{4.1}$$

where

$$U(t) = [\omega_0^2(1+\xi^2)x^2 + (\dot{x}+\xi\omega_0 x)^2]^{1/2}. \tag{4.2}$$

Consider the case when only the knowledge of peak horizontal and vertical ground accelerations $\ddot{u}_{gm}$ and $\ddot{v}_{gm}$ and site periods for horizontal and vertical vibrations are available. To estimate the bound given by the inequality (4.1), in [17] it was assumed that the peak accelerations act on the structure for some effective time durations. This assumption is now refined by assuming that

$$\ddot{u}_g(t) = \begin{cases} \ddot{u}_{gm}, & 0 \leqslant t \leqslant t_u, \\ 0, & \text{otherwise} \end{cases} \tag{4.3}$$

and

$$\ddot{v}_g(t) = \begin{cases} \ddot{v}_{gm}, & 0 \leqslant t \leqslant t_v, \\ 0, & \text{otherwise.} \end{cases} \tag{4.4}$$

That is, we assume that the peak response is reached if the constant peak accelerations $\ddot{u}_{gm}$ and $\ddot{v}_{gm}$ act on the column for effective time durations of t_u and t_v, respectively.

Using the relations (4.3) and (4.4) in the inequality (4.1), we find

$$U \leqslant \Gamma \ddot{u}_{gm} t_u, \tag{4.5}$$

where

$$\Gamma = \frac{\left[1-\exp\left\{-\left(\zeta\omega_0 - \frac{1}{2}\beta\omega_0\ddot{v}_{gm}\right)t_v\right\}\right]\exp\{-\zeta\omega_0(t_u-t_v)\} + \frac{1}{\zeta\omega_0 t_u}[1-\exp\{-\zeta\omega_0(t_u-t_v)\}]}{\left(\zeta-\frac{1}{2}\beta\ddot{v}_{gm}\right)\omega_0 t_u}. \tag{4.6}$$

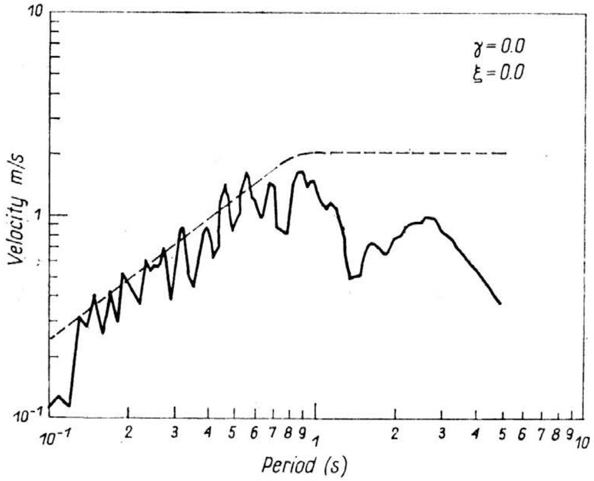

FIG. 9. Estimated and actual response spetra.

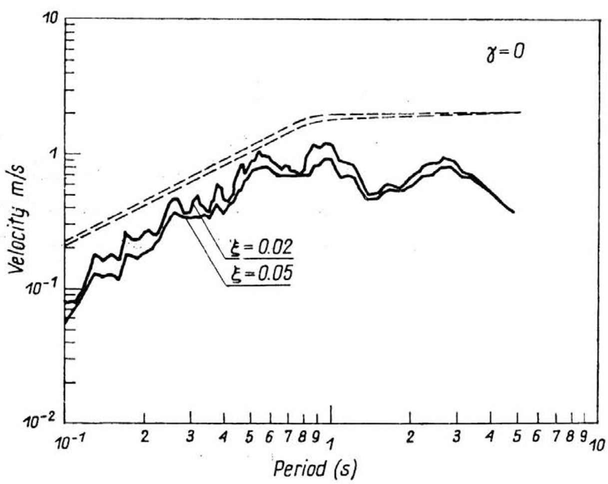

FIG. 10. Estimated and actual response spectra.

The factor Γ shows the effects of the damping coefficient and vertical ground acceleration on the peak horizontal response estimate. When $t_v = t_u$ or $v_g = 0$, the results reduce to those obtained in [17].

For the effective time durations, we assume that

$$t_u = \mathrm{Min}(T_{pu}, T_0), \tag{4.7}$$

and

$$t_v = (1-\xi^{1/4})^4 \mathrm{Min}(T_{pv}, \eta T_0), \tag{4.8}$$

where $T_0 = 2\pi/\omega_0$ is the natural period of the column under the dead load, and T_{pu} and T_{pv} are the site periods for horizontal and vertical vibrations, respectively. Here η is a coefficient which is assumed to be given as

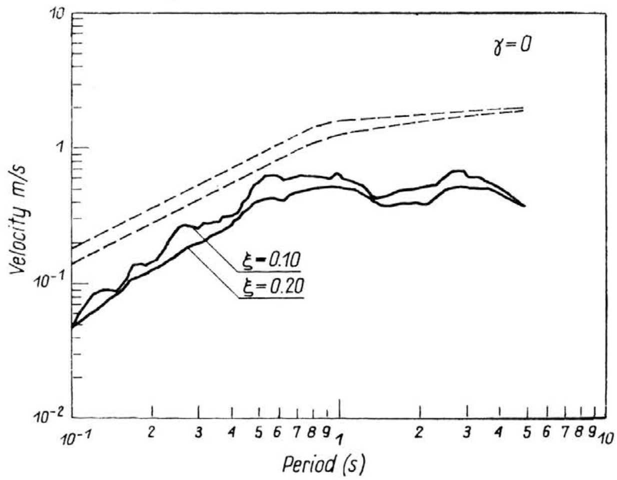

FIG. 11. Estimated and actual response spectra.

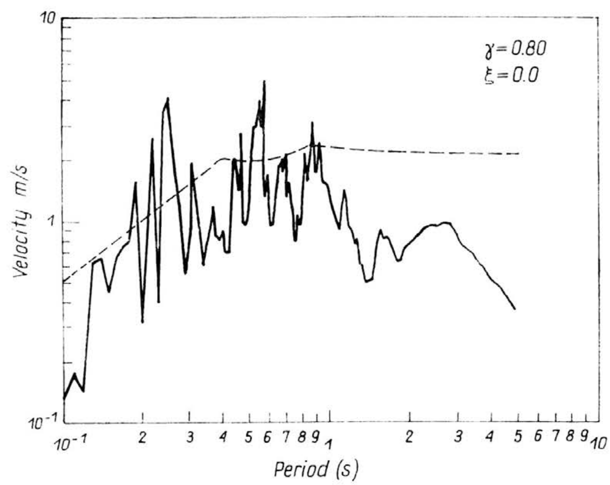

FIG. 12. Estimated and actual response spectra.

$$\eta = \begin{cases} 0.2, & \bar{\gamma} \leqslant 1, \\ 0.2+30(\bar{\gamma}-1), & \bar{\gamma} \geqslant 1, \end{cases} \tag{4.9}$$

where

$$\bar{\gamma} = \gamma\left(1+\frac{\ddot{v}_{gm}}{g}\right) = \frac{m(g+\ddot{v}_{gm})}{P_{cr}}. \tag{4.10}$$

The assumptions (4.7)–(4.10) are modified forms of those of [17]. Using the effective time duration t_u as given by the assumption (4.7) leads to the proper shape of the response spectra for low to moderate natural periods in the absence of a vertical excitation. In particular, for rigid structures with T_0 below the site period a constant peak acceleration response is obtained. This is consistent with the commonly used design response spectra

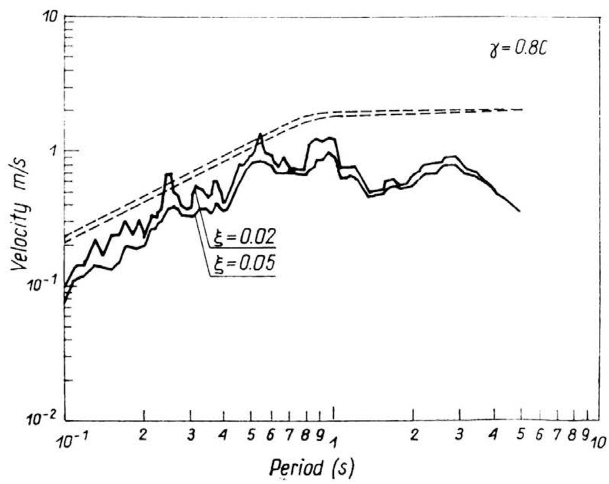

FIG. 13. Estimated and actual response spectra.

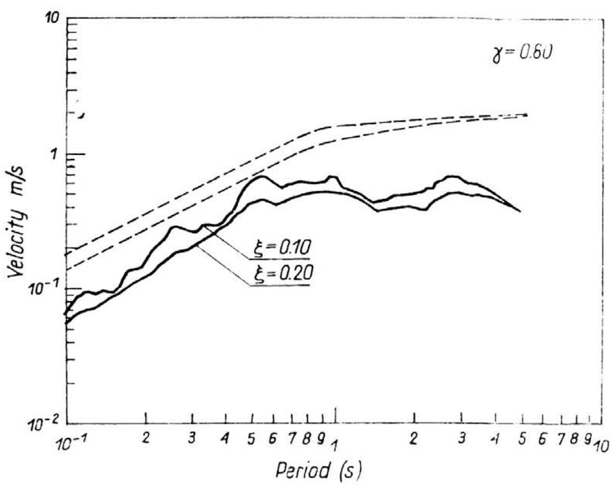

FIG. 14. Estimated and actual response spectra.

of [1–10]. A similar form with some modifications is used in prescribing t_v in Eq. (4.8). The amplification effect of $\ddot{v}_g$ appears to be a strong function of ξ and γ. The influences of these parameters are empirically included in Eqs. (4.8).

Equation (4.5) together with Eqs. (4.6)–(4.10) provide an estimate for the peak horizontal response of the column. For light dampings, using the approximation $\omega_0 x \simeq \dot{x}$, from Eq. (4.2) it follows that

$$\operatorname*{Max}_t U(t) \simeq 2 \operatorname*{Max}_t |x(t)| = \sqrt{2} S_v. \tag{4.11}$$

Equations (4.11) and (4.8) are used to construct smooth site-dependent HV response spectra. The results for El Centro 1940 earthquakes (NS+V) are shown in Figs. 9–17.

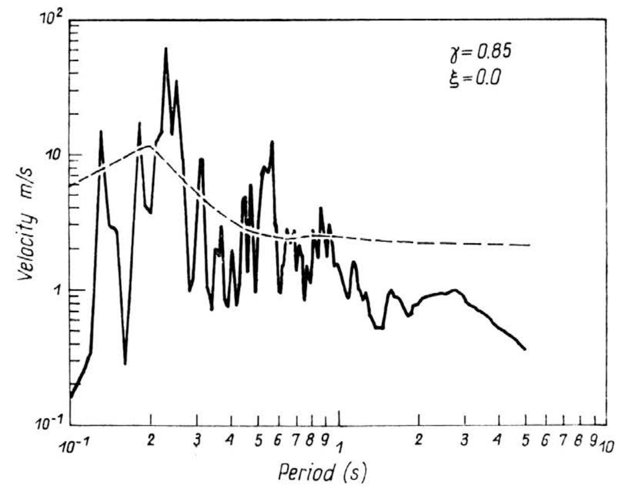

FIG. 15. Estimated and actual response spectra.

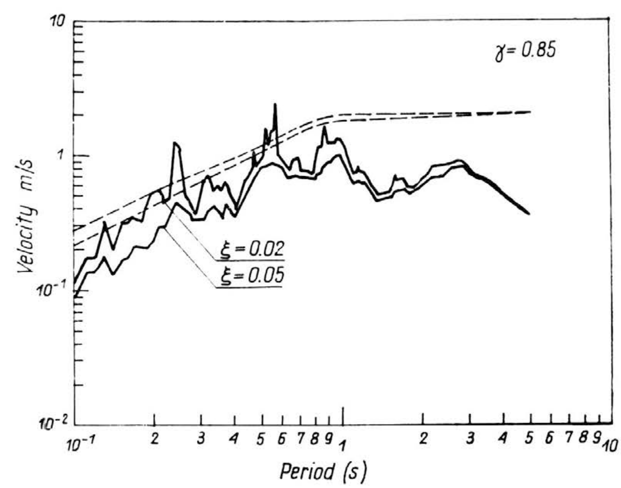

FIG. 16. Estimated and actual response spectra.

The site periods of $T_{pu} = 0.85$ s and $T_{pv} = 0.4$ s and peak ground accelerations of $\ddot{u}_{gm} = 0.348$ g and $\ddot{v}_{gm} = 0.21$ g are used in these analyses.

In Figs. 9–11 the site-dependent spectra are plotted by dashed lines for $\gamma = 0$ and several values of the damping coefficient. The actual response spectra of the NS-component of El Centro 1940 earthquake are shown by the solid lines in these figures. In Figs. 12–17 the predicted HV response spectra are compared with the actual ones for several values of ξ and γ. It appears that for $\xi = 0$, Eq. (4.8) somewhat underestimates the peak responses for highly rigid structures. However, for dampings more than two percent, the predicted HV response spectra agree with reasonable accuracy with the actual ones. These results imply that Eqs. (4.5) and (4.11) may be used as a basis for constructing site-dependent HV design response spectra.

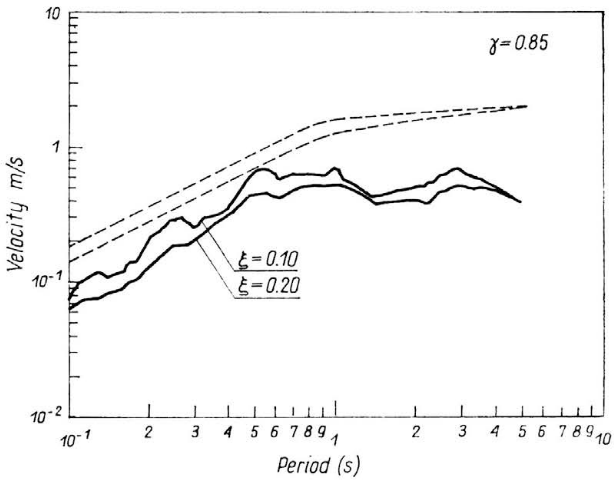

FIG. 17. Estimated and actual response spectra

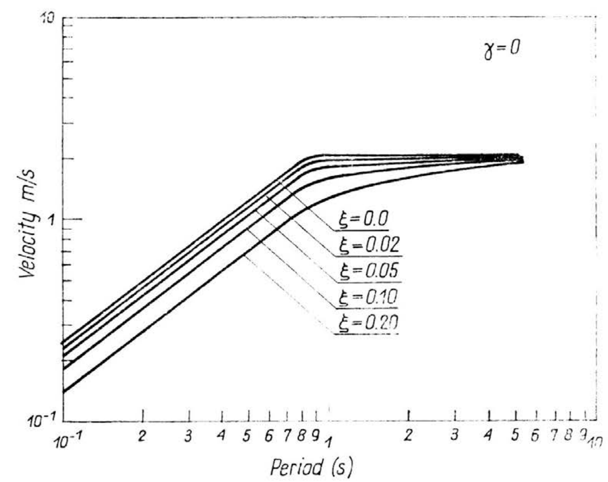

FIG. 18. Estimated site-dependent response spectra.

Figures 18–20 show several smooth site-dependent HV response spectra for the conditions of El Centro 1940 earthquake. It appears that the amplification effects of the vertical excitation become significant only for lightly damped columns which are close to their buckling loads.

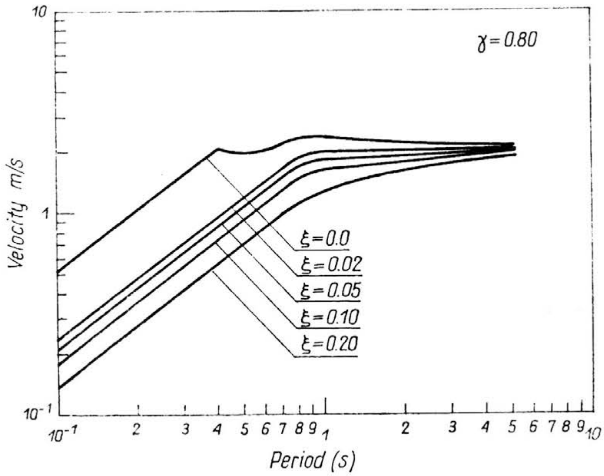

FIG. 19. Estimated HV site-dependent response spectra.

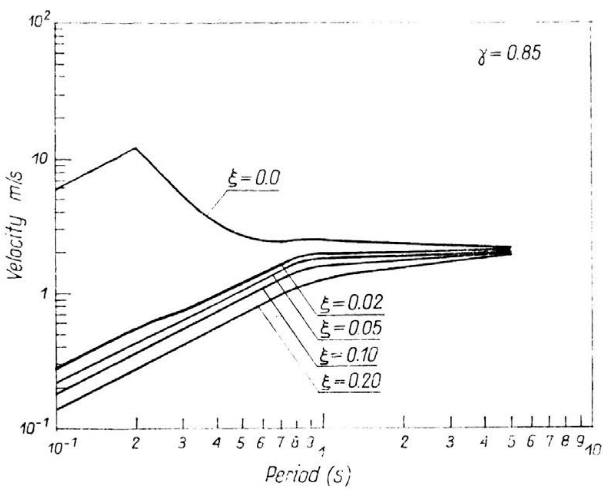

FIG. 20. Estimated HV site-dependent response spectra.

The procedure outlined here may also be used for constructing HV response spectra for secondary systems, pipings and equipments where the effects of axial excitations could become quite substantial.

5. Conclusions

Response of structures subjected to combined HV earthquake base excitations is considered. Particular attention is given to the influences of axial excitations through the $P-\Delta$ effect. A simple column is used as the working example. NS+V response spectra for the El Centro 1940 earthquake for several values of the damping coefficient and load factor are developed.

The results indicate that the vertical acceleration always amplifies the peak horizontal responses. The amount of amplification is a strong function of the damping coefficient and load factor. Substantial amplifications are observed for lightly damped columns which carry a static load which is more than 80% of their Euler buckling load. It is conjectured that the $P-\Delta$ effect is significant when $\gamma = m(g+\ddot{v}_{gm})/P_{cr} \geqslant 1$. That is, when the column under a constant acceleration of $\ddot{v}_{gm}$ is statistically unstable. When γ is small or damping is large, the amplification effects of the vertical acceleration are quite modest.

Based on a Liapunov bound on the response, a procedure for estimating the peak responses under HV earthquake excitation is developed. Several smooth site-dependent HV response spectra for the conditions of El Centro 1940 earthquake are constructed. The results are compared with the actual ones and reasonable agreement is observed.

Basing on the results presented in this study, it is concluded that for linearly elastic structures which carry static loads well below their buckling loads the amplification effects of the vertical acceleration are small. However, for tower-like structures with small damping the amplification may become significant. Furthermore, for secondary systems (like pipings and equipments) the axial excitations may amplify the lateral deflections to an extent. It is also conjectured that for structures which reach their yield limits during an earthquake the amplification effects of the vertical acceleration may become quite substantial.

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