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The influence of sampling interval on accuracy of probabilistic attenuation correction for GPR signal

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INTRODUCTION

A new method has been proposed to correct the attenuation of ground-penetrating radar (GPR) signals using probabilistic signalprocessing based on Bayesian statistics (Chishima *et al.* 2013: 173–175). The advantages of this method are the selective amplification of GPR signals without the amplification of the noise and the automatic estimation of the medium attenuation coefficient. However, this method is easily affected by the sampling interval employed during data acquisition. In this paper, this problem is demonstrated by some results of numerical experiments, and a way to avoid the problem is discussed.

METHOD FOR ATTENUATION CORRECTION

The proposed attenuation correction method is briefly described below.

Let a list of sampled data of a GPR trace be a vector $\mathbf{d}=(d_{1,}\ d_{2,}\ \dots\ ,\ d_N)$, and let an ideal signal be a random variable vector $\mathbf{f}=(f_1,\ f_2,\ \dots\ ,\ f_N)$ that is not affected by either medium attenuation or noise, where each di, fi represents the amplitude value at time $i\Delta t$ (Δt: sampling interval). For the method, it is essential to search for $\mathbf{f}_{est}$ maximizing $p(\mathbf{f}|\mathbf{d})$, where $p(\mathbf{f}|\mathbf{d})$ is the

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Likelihood	Prior Distribution
$p(\boldsymbol{d}\vert\boldsymbol{f}) = p(\boldsymbol{d}\vert\boldsymbol{f},\alpha,\sigma_n) = \frac{\exp\left(-\frac{1}{2\sigma_n{}^2}\Sigma_{i=1}^{N}(d_i - e^{-\alpha i \Delta t} f_i)^2\right)}{Z_{lik}}$	$p(\boldsymbol{f}) = p(\boldsymbol{f}\vert\sigma_f) = \frac{exp\left(-\frac{1}{2\sigma_f{}^2}\Sigma_{i=1}^{N-1}(f_i - f_{i+1})^2\right)}{Z_{pri}}$

Table 1. Likelihood function and prior distribution modeled for the proposed attenuation correction method. Z_{lik} and Z_{pri} are a normalization constant

conditional probability distribution for **f** given **d**. In Bayesian statistics, the p(**f**|**d**) is referred to as the posterior distribution and can be constructed using the Bayes formula p(**f**|**d**) = p(**d**|**f**) p(**f**)/p(**d**), where p(**d**) equals ∫p(**d**|**f**)p(**f**)d**f**.

The term p(**d**|**f**) on the right side of Bayes formula is called the likelihood function of **f** (hereafter likelihood). Another term, p(**f**), is called prior distribution, and represents prior knowledge or information about **f**. Table 1 shows the likelihood and prior distribution modeled for the attenuation correction. The likelihood is based on an assumption of the forwarding process from **f** to **d**, where **f** is affected by the medium attenuation, and the noise is then added to the attenuated **f**. For the prior distribution, a multidimensional Gaussian distribution was adopted so that as the differences between adjacent variables f_i and f_{i+1} decrease, p(**f**) will increase. This is because **f** does not fluctuate widely between the adjacent variables.

In modeling the p(**f**) and the p(**d**|**f**), three parameters, σ_f, σ_n, and α, were employed, which represent the standard deviation (SD) of (f_i- f_{i+1}), the SD of noise, and the attenuation coefficient [ns^{-1}], respectively. These parameters are called hyper-parameters and determine the shapes of each distribution. The hyper-parameters are estimated using the maximum likelihood method (MLM): considering p(**d**|σ_f, σ_n, α) to be a likelihood function of the hyper-parameters, and then searching for (σ_f', σ_n', α') maximizing ∫p(**d**|**f**, σ_n, α)p(**f** |σ_f)d**f** which is equivalent to p(**d**|σ_f, σ_n, α). Finally, the corrected signal for **d**, $\mathbf{f}_{est}$, is searched for by maximizing p(**f**|**d**, σ_f', σ_n', α').

In practical implementation of this method, an iterative algorithm has been constructed by incorporating belief propagation into the EM (expectation-maximization) algorithm. The belief propagation is a mathematical technique that is employed to calculate efficiently the posterior distribution. The EM algorithm is a popular numerical calculation algorithm that is used for hyper-parameter estimation with the MLM.

EXPERIMENTS

In the experiments, three signals that were acquired at different Δt values, from a synthetic GPR trace were used. First, a GPR trace was numerically made by the finite-difference time-domain (FDTD) method for simulation of electromagnetic wave propagation. Figure 1a shows a model of the FDTD simulation. The transmitting and receiving points, which are regarded as two imaginary GPR antennae, are 0.02 m above ground, and the distance between the two points is 0.4 m. A boundary is at a depth of 0.4 m in the ground. The upper part is assumed to

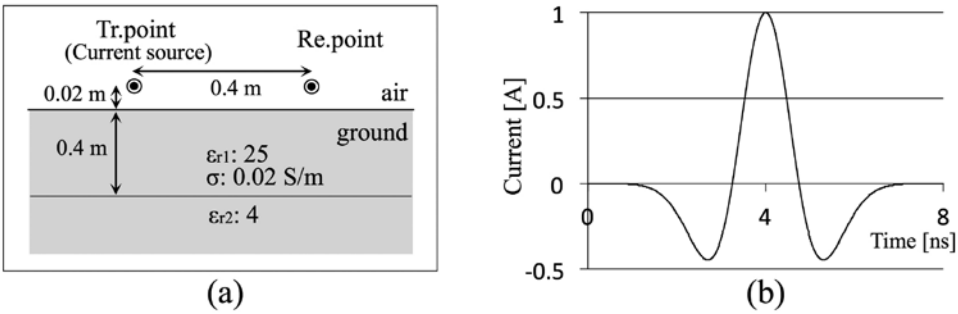

Fig. 1. (a) Model of FDTD simulation and (b) input signal at Tr.point in the model

be loamy wet soil with a relative permittivity, ε_{r1}, of 25 and a conductivity, σ, of 0.02 S/m. The lower part is assumed to be granite with ε_{r2}=4. Figure 1b shows the signal input at the transmitting point as a current source. The center frequency is 300 MHz and the amplitude peak is at 4 ns. For sufficient precision, the FDTD simulation was performed with a time step of 0.01 ns. Then, the FDTD simulation result was quantized to 8-bit data (with amplitude values raging from -128 to 127 in integer form), and white Gaussian noise (average 0, variance 2^2) was added to the 8-bit data, as shown in Figure 2a. The reflection signal from the ground surface is at around 4 ns, and the one from the boundary is at around 18 ns. The three signals considered in the experiments were made by resampling the signal in Figure 2a with Δt = 0.05, 0.2, and 0.4 ns.

Figure 2b, c, and d show the correction results, $\mathbf{f}_{est}$'s, for the signals at deferent Δt values, overlapping an ideal **f** simulated by FDTD, which is a signal without both medium attenuation and noise. As shown in Figure 2c for Δt = 0.2 ns, the corrected reflection signal from the boundary appears to be very close to the ideal **f**. In Figure 2d for Δt = 0.4 ns, the corrected reflection signal and the noise after 20 ns appear to have been much more amplified than in Figure 2c. Fundamentally, a sampling interval of 0.4 ns is not appropriate for actual GPR surveys using a 300 MHz antenna due to the coarse sampling points. In contrast, the sampling points for Δt =0.05 ns would be sufficiently dense, but the method has not worked: the signal at Δt = 0.05 ns in Figure 2c has not been amplified and appears to be almost the same as Figure 2a.

DISCUSSION

The reason for which the reflection signal at Δt = 0.05 ns was not amplified is considered as follows: For any time-domain discrete signals, when the sampling interval becomes shorter, the distribution of the differences between the adjacent values, f_i and f_{i+1}, gradually deviates from the Gaussian distribution; ultimately, the values are only 0 or 1.

Therefore, if an input signal is known, it is important to verify whether the differences between the adjacent values at Δt in the input signal are normally distributed. This may be done using a statistical test of normality, such as the Shapiro-Wilk test. Actually, in the case of the input signal

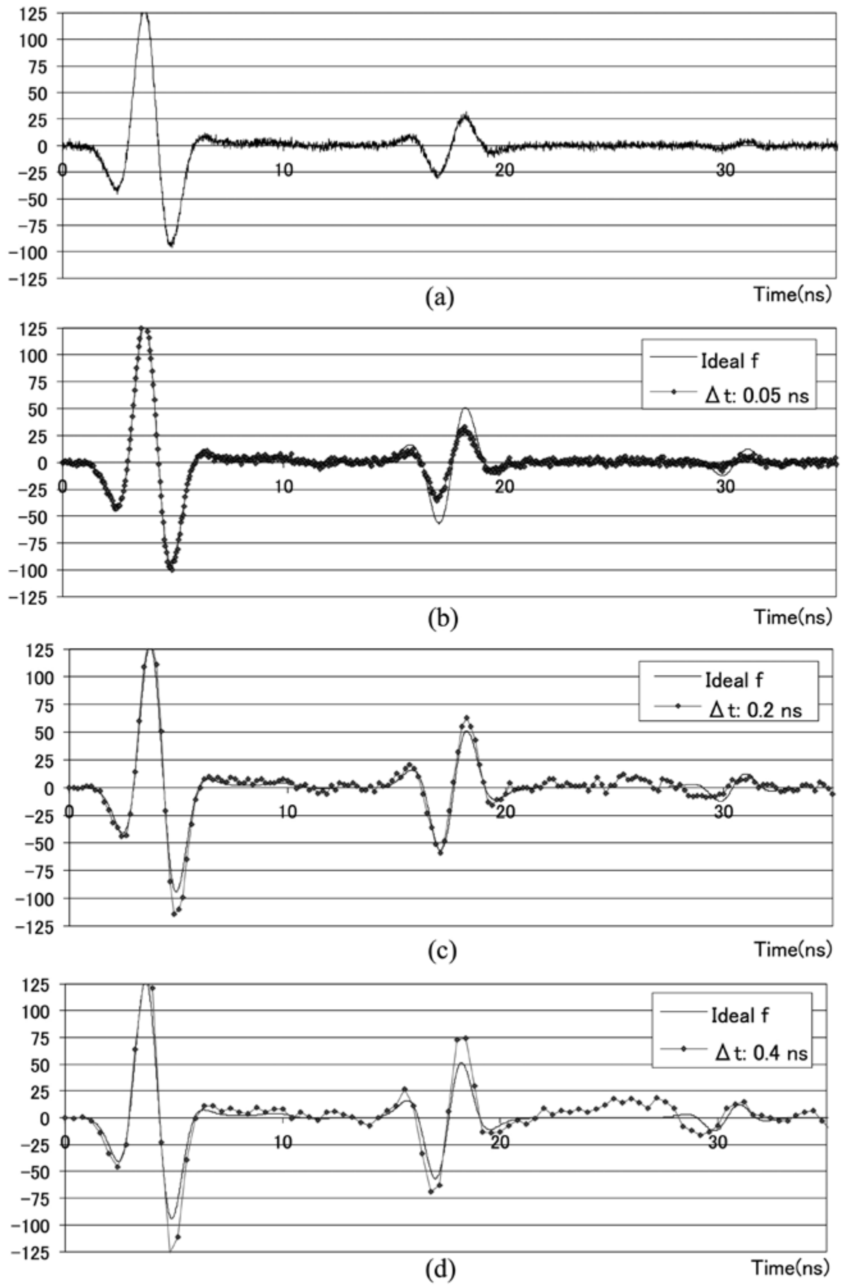

Fig. 2. (a) Simulated signal by FDTD with 8-bit quantization and noise addition, and (b), (c), (d) the results of correction for the signal at different Δt

shown in Figure 1a, the Shapiro-Wilk test revealed that the normality of the distribution of the differences between the adjacent values at $\Delta t = 0.2$ ns was accepted with a *p*-value 0.2502, and the normality at $\Delta t = 0.05$ ns was rejected with a *p*-value 0.0004 (5% significance level).

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