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Numerical simulation of dynamic processes in viscoelastic medium

L. A. MERZHIEVSKIJ (NOVOSIBIRSK)

A MAXWELL-TYPE nonlinear model was proposed for a description of the state of deformation of viscoelastic bodies. A non-stationary one-dimensional process of deformation was solved. A method of "decomposition of the discontinuities" applied till now in a gas-dynamics was applied. The numerical procedure proposed was applied to the solution of the impact problem of two identical plates. The results of computations for two different impact velocities 1.8 km/s and 2 km/s were presented.

W pracy zaproponowano do opisu deformacji ciał lepko-sprężystych nieliniowy model typu Maxwella. Rozwiązano niestacjonarny, jednowymiarowy proces deformacji. Wykorzystano metodę „rokładu nieciągłości", stosowaną dotychczas w gazodynamice. Proponowany schemat numeryczny został zastosowany do rozwiązania zagadnienia zderzenia dwu identycznych płyt. Uzyskano rezultaty dla dwu różnych prędkości zderzenia: 1.8 km/sek i 2 km/sek.

Для описания деформации вязкоупругих тел в работе предложено использовать нелинейную модель Максвелловского типа. Рассмотрен нестационарный процесс одномерной деформации. Для численного решения использован метод „распада разрывов", применявшийся до сих пор только в газодинамике. Предложенная численная схема применена к решению задачи о соударения двух пластин из одинакового металла. Приведены результаты для двух разных скоростей соударения: 1,8 км/сек и 2 км/сек.

AT DIFFERENT stages of a deformation process the solids subjected to intensive dynamic loads, e.g. explosive ones, cover all the spectrum of states from liquid to elastic. For the mathematical description of such processe a relevant model, i.e. the system of equations, is needed. The solution of these equations, depending on the intensity of a load, must coincides with solutions of hydrodynamics equations, the theory of plasticity and elasticity. Maxwell equations for a viscoelastic body can be used as such universal equations. The paper by G. I. GUREVICH [1] should be apparently considered as the first attempt to construct the general model of strained solid on the basis of Maxwell equations. The concepts given in [1] were further developed in the monograph [2] in which the medium with nonlinear dependence of relaxation time on stresses and temperature at a linear relation of stresses and small strains, was studied. The Maxwell-type nonlinear model of strained body considered in this paper was developed by S. K. GODUNOV with collaborators [3-8]. The closed system of equations describing the model is of the form

$$\frac{\partial \varrho[E+{}^1/_2 u_i u_i]}{\partial t} + \frac{\partial[\varrho u_k(E+{}^1/_2 u_i u_i) - u_i \sigma_{ik}]}{\partial x_k} = 0, \tag{1}$$

$$\frac{\partial \varrho u_i}{\partial t} + \frac{\partial[\varrho u_i u_k - \sigma_{ik}]}{\partial x_k} = 0, \quad i = 1, 2, 3,$$

(1) [cont.]

$$\frac{\partial \varepsilon_{ii}}{\partial t}+u_k\frac{\partial \varepsilon_{ii}}{\partial x_k}-\frac{\partial u_i}{\partial x_i}+2\varepsilon_{ik}\frac{\partial u_k}{\partial x_i}=\varphi_{ii},\quad i=1,2,3,$$

$$\frac{\partial \varepsilon_{ij}}{\partial t}+u_k\frac{\partial \varepsilon_{ij}}{\partial x_k}-\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\varepsilon_{ik}\frac{\partial u_k}{\partial x_j}+\varepsilon_{jk}\frac{\partial u_k}{\partial x_i}=\varphi_{ij},$$

$$(i,j)=(1,2),(1,3),(2,3),$$

$$\sigma_{ik}=\varrho\left(\frac{\partial E}{\partial \varepsilon_{ik}}-2\varepsilon_{ij}\frac{\partial E}{\partial \varepsilon_{ik}}\right),\quad E=E(\varrho,D,\Delta,S),$$

$$\varrho=\varrho_0\sqrt{\det\|\delta_{ik}-2\varepsilon_{ik}\|},\quad \varphi_{ik}=\varphi_{ik}(\varepsilon_{pq},\tau),$$

$$\tau=\tau(\sigma,T),$$

$$\sigma=\frac{1}{\sqrt{2}}\sqrt{(\sigma_{11}+\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+2(\sigma_{12}^2+\sigma_{13}^3+\sigma_{23}^2)},$$

$$D=\frac{1}{2}(d_1^2+d_2^2+d_3^2),\quad \Delta=d_1d_2d_3,$$

$$d_i=\ln\frac{k_i}{\sqrt{k_1k_2k_3}},\quad k_i=(1-2\varepsilon_i)^{-1/2},$$

where t is the time, x— the Eulerian space coordinates, ϱ_0, ϱ — the initial and current density of the medium, u_i — the components of the velocity vector, σ_{ij}, ε_{ij} — the components of stress and strain tensors, ε_i — the main values of strain tensors, σ — the intensity of tangential stresses, D, Δ the invariants of the strain tensor (as and ϱ), T, S — temperature and entropy, E — the density of internal energy per mass unit, τ — the time of relaxation of tangential stresses, φ_{ij} — the functions describing relaxation of tangential stresses in the medium. The following form of functions φ_{ij} is proposed in [3]:

$$\varphi_{ij}=-\frac{1}{\tau}\left(\varepsilon_{ij}-\frac{\varepsilon_{pq}\varrho\varepsilon_{qp}}{\varrho\varepsilon_{11}+\varrho\varepsilon_{22}+\varrho\varepsilon_{33}}\delta_{ij}\right)=\frac{1}{2\tau}\left[\delta_{ij}-2\varepsilon_{ij}+\frac{3\varrho}{\varrho\varepsilon_{11}+\varrho\varepsilon_{22}+\varrho\varepsilon_{33}}\delta_{ij}\right],$$

$$\varrho\varepsilon_{ij}=\frac{\partial\varrho}{\partial\varepsilon_{ij}}.$$

This expression is invariant with respect to the choice of the orthogonal coordinate system and does not break the continuity equation and provide an increase in entropy. The dependences $E=E(\varrho,D,S)$ (the variant of the energy equation where as determining values only two invariant strain tensors are used), $\tau=\tau(\sigma,T)$ which are necessary for the solution of specific problems, are given in [5, 6]. The functions are constructed in the form of interpolating formulae having experimental data, with the function for relaxation time being constructed on the basis of data on the dependence of the dynamic yield limit σ_* upon strain velocity. These formulae close completely the system of equations describing the model considered. Due to the awkwardness of the system of equations, the solution of specific problems requires the use of a computer and hence the development of numerical calculation methods. It is also of interest to find a more general approach to the construction of the relaxation time function since available experimental data on the dependence of σ_* upon strain velocity are attributed to the narrow range of materials and to the restricted range of change in determining parameters. Naturally, the formulated problems are most conveniently solved with the plane one-dimensional variant of the considered model (1):

$$\frac{\partial \varrho}{\partial t} + \frac{\partial(\varrho u)}{\partial x} = 0,$$

$$\frac{\partial(\varrho u)}{\partial t} + \frac{\partial(\varrho u^2 - \sigma_1)}{\partial x} = 0,$$

$$\frac{\partial \varrho(E+u^2/2)}{\partial t} + \frac{\partial[\varrho u(E+u^2/2) - \sigma_1 u]}{\partial x} = 0,$$

$$\frac{\partial(\varrho\beta)}{\partial t} + \frac{\partial(\varrho u\beta)}{\partial x} = \varrho\frac{\alpha-\beta}{\tau}, \tag{2}$$

$$\alpha = \ln k_1, \quad \beta = \gamma = \ln k_2 = \ln k_3, \quad \varrho = \frac{\varrho_0}{k_1 k_2 k_3},$$

$$E = E(\varrho, D, S), \quad \tau = \tau(\sigma, T).$$

Here stress and strain tensors take a diagonal form since the selected coordinate axes coincide with their main axes; σ_1 is the normal stress on the area perpendicular to the axis x.

Let us consider the following procedure of constructing the dependence of τ on determining parameters [9, 10]. Let us determine the strain tensor, which in the given case is characterized by the components ε_i, $i = 1, 2, 3$, as the sum of tensors of elastic ε_i^y and plastic ε_i^n strains:

$$\varepsilon_i = \varepsilon_i^y + \varepsilon_i^n,$$

and this is true for small strains. The basic meaning of the tensor of plastic strain ε^n is determined by the relation

$$\varepsilon^n = (\varepsilon_1^n - \varepsilon_2^n)/2 = 3\varepsilon_1^n/4 = -3\varepsilon_2^n/2.$$

In the variables used

$$\varepsilon^n = {}^1/_4\,[\exp(-2\beta) - \exp(-2\alpha)].$$

It is known that stresses grow at the expense of a general strain and relax at the expense of a plastic one. It is natural to assume that relaxation time is inversely proportional to the velocity of plastic strain $\dot{\varepsilon}^n$,

$$\tau \sim (\dot{\varepsilon}^n)^{-1}. \tag{3}$$

The plastic strain is accomplished by the movement of dislocations, and its velocity is connected with the characteristics of the dislocation field by the Orovan relation

$$\dot{\varepsilon}^n = \frac{d\varepsilon^n}{dt} = bNv, \tag{4}$$

where N, v are the density of movable dislocations per volume unit and their mean velocity, respectively, b — the Burgers vector modulus. The values N and v, in turn, are the functions of medium state. There are several different expressions for these functions constructed on the basis of plausible reasoning and corrected using experimental data. The variant of dependences, suggested by GILMAN [11], became most common

$$N = N_0 + M\varepsilon^n, \quad v = v_0 \exp[-(H_0 + H\varepsilon^n)/\sigma], \tag{5}$$

N_0 is the initial density of dislocations, M — the constant of multiplication, H_0 — characteristic stress of braking, H — the coefficient of strain hardening. JOHNSON and BERKER [12] describe the process of elastic precursor decay in aluminium, using the functions

$$N = N_0[1+(Q\varepsilon^n/bN_1)]\exp(-\lambda\varepsilon^n), \quad v = v_0\frac{\delta^m}{1+\delta^m}, \quad \delta = \frac{\sigma}{\sigma_0}, \tag{6}$$

where $Q, N_1, \lambda, \sigma_0, m$ are some constants. The expression for τ can be obtained with the help of Eqs. (3), (4), and (5) or (6). In the present paper the relations (5) were used, then

$$\tau = \varkappa\frac{\exp[(H_0+H\varepsilon^n)/\sigma}{bv_0(N_0+M\varepsilon^n)}, \tag{7}$$

where the proportionality coefficient $\varkappa$ is still to be determined. It should be noted that the expression obtained does not depend on the temperature of the medium. In fact, N_0, M, H_0, H, $\varkappa$ assumed to be constant values, are the functions of temperature, and this makes it possible to hope for the construction of a more complete dependence of τ on the parameters of medium state. However, the functions $N_0(t), \ldots$ are unknown at present. The value $\varkappa$ was chosen from the condition of coincidence of experimental dependence of the dynamic yield limit σ_* on the strain velocity $\dot{\varepsilon}$ with the analogous dependence calculated, using Eq. (7). The problem of calculation of such dependencies is considered in [8] for the function $\tau(\sigma, T)$ given in [6]. The uniaxial strain of a thin rod with the initial length l_0, directed along its longitudinal axis, is considered in the mentioned reference. Let one of its ends be fixed at the point $X = 0$, and the other move with the velocity $V(t)$, the velocity of different cross-sections is distributed linearly along the length, i.e.

$$v(x, t) = V(t)x/l(t),$$

$$l(t) = l_0 + \int_0^t (t)dt.$$

The strain velocity

$$\dot{\varepsilon} = \frac{\partial v}{\partial x} = \frac{V(t)}{l(t)}.$$

Only small strains are considered, then

$$\dot{\varepsilon} = V(t)/l_0.$$

From the condition of medium isotropy and problem symmetry

$$\beta = \gamma,$$

and since the rod is thin, then

$$\sigma_2 \equiv \sigma_3 \equiv 0.$$

With accepted assumptions the system (2) takes the form

$$\frac{d\alpha}{dt} = \dot{\varepsilon} - \frac{d_1}{\tau}, \qquad \frac{dS}{dt} = \frac{4b_0^2}{T}\frac{D}{\tau}, \tag{8}$$

where b_0 is the velocity of propagation of transverse waves calculated according to the formula

$$b_0 = \sqrt{\left(\frac{\partial E}{\partial D}\right)_{\rho S}}.$$

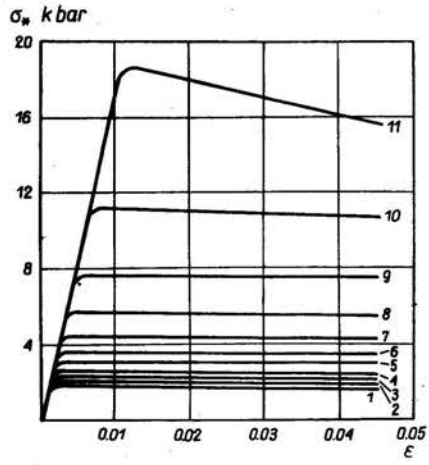

FIG. 1.

Figure 1 gives the curves of the dependence $\sigma_1(\varepsilon)$, $\varepsilon = \Delta l/l$, calculated according to the system (8), using Eq. (7), and the dependence $E = E(\varrho, D, S)$ from Eq. (5),-the material of samples is steel containing 0.2% carbon, the initial temperature of the samples $T =$ $= 293°$K. The curves *1–11* correspond to $= 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, 10^{3}, 10^{4}, 10^{5}, 10^{6}, 10^{7}$ 1/s. Here and in all other figures the values of stresses are given in kbar. The curves show that in the accepted model the value of stress at strain constant velocity during the whole process of determation does not exceed some value which it is natural to compare with the yield limit value. The comparison of calculated dependence $\sigma_*(\dot{\varepsilon})$ with experimental [13] is made in Fig. 2 (curves *1* and *2*, respectively) from which it follows that the

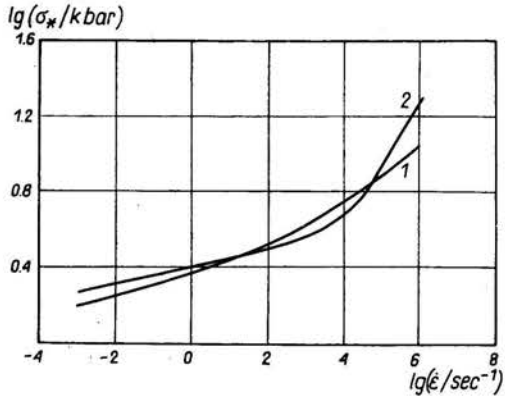

FIG. 2.

calculated values σ_* coincides satisfactorily with the experimental ones up to the highest values $\dot{\varepsilon}$. The character of the curves *10* and *11* differs significantly from the corresponding curves *8*, and this accounts for neglect of the influence of sample heating in Eq. (7).

It is of interest to clarify the influence of different parameters, characterizing the field of dislocations, on the value of the yield limit being calculated. The initial number of dislocations turned out to influnce the value insignificantly; the variation of N_0 in the range $10^5 \leqslant N_0 \leqslant 10^9$ changes σ_* only by 2–3%. σ_* and, on the contrary, it is very sensitive to the variation of the multiplication constant M. Evidence of this is given by the curve in Fig. 3 plotted for $\dot{\varepsilon} = 10^3$ 1/s at the rest constant parameters in the expression (7).

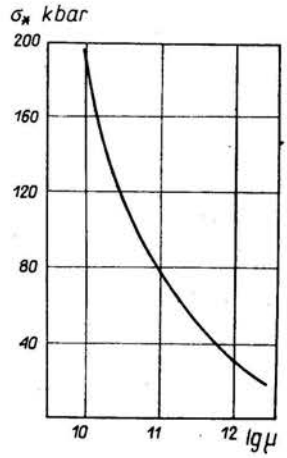

FIG. 3.

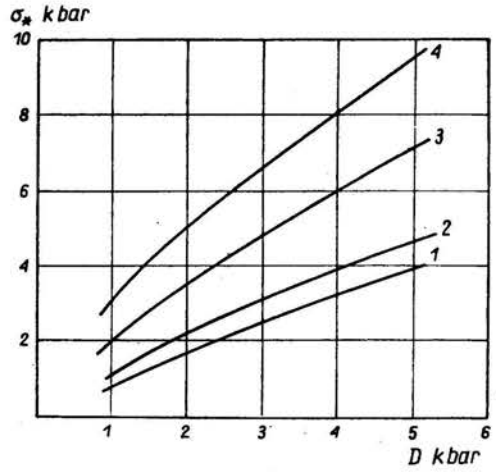

FIG. 4.

The dependence of σ_* on the characteristic stress of braking H_0 is shown in Fig. 4; the curves (*1–4*) correspond to $\dot{\varepsilon} = 10, 10^2, 10^4, 10^5$. Let us refer now to the calculation of unsteady processes within the framework of the formulated model. The method of disintegration

of discontinuities [14], proposed and widely used for the solution of gasdynamic problems, was taken as the basis of numerical calculation. The method was worked out with a one-dimensional variant of the model (2), however, its extension to the more general case makes no difficulties in principle [14]. According to this method the parameters are calculated at each time step in two stages. At the first stage corresponding to an intermediate step in time it is assumed that in each cell of the difference net the parameters describing the state of substance are equal. Disintegration of discontinuities occur at boundaries of cell, this stage ends in the calculation of mass, impulse, and energy fluxes through the given boundary. At the second stage, i.e. a new step in time, the parameters of medium in cells are calculated by means of integral conservation laws, using the obtained fluxes. Details of the method and examples of its application are given in the monography [14].

An accurate calculation of disintegration of discontinuities even in the case of gasdynamic problems requires significant expenditures of computer time. And still this is aggravated by the fact that even the system (2) does not allow automodel solutions, and the problem of discontinuity disintegration for the accepted model still requires and additional development. At the same time the experience of computer calculations shows that calculation of intermediate parameters, i.e. the first stage, can be made on the basic of linearized equations of an initial system. Such approximation gives good results even in the regions wherein the discontinuities of calculated values are present, apart from the regions where the process is described by smooth functions. Taking this into account, the system of equations obtained by the linearization of the system (2) was used to calculate the first stage:

$$
\begin{aligned}
&\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}-\mathscr{L}\frac{\partial \alpha}{\partial x}-\mathscr{R}\frac{\partial \beta}{\partial x}-\mathscr{K}\frac{\partial S}{\partial x}=0,\\
&\frac{\partial \alpha}{\partial t}+U\frac{\partial \alpha}{\partial x}-\frac{\partial u}{\partial x}=0,\\
&\frac{\partial \beta}{\partial t}+U\frac{\partial \beta}{\partial x}=0,\\
&\frac{\partial S}{\partial t}+U\frac{\partial S}{\partial x}=0,
\end{aligned}
\tag{9}
$$

where U is the main solution for velocity,

$$\mathscr{L}=E_{\alpha\alpha}-E_{\alpha},\quad \mathscr{R}=E_{\alpha\beta}-E_{\alpha},\quad \mathscr{K}=E_{\alpha S};$$

the subscript denotes the differentiation with respect to the corresponding variable. The system (9) was obtained with the assumption that the medium possessed completely elastic properties. This assumption is justified if disintegrations are considered to occur at times which are shorter than the relaxation time. The system has four families of characteristics, i.e. the two-fold characteristic

$$\frac{dx}{dt}=U,$$

with the conditions $\beta=\beta_0$, $S=S_0$ (trajectories with β and S remaining constant therealong), and two characteristics

$$\frac{dx}{dt}=U\pm\sqrt{\mathscr{L}},$$

with the conditions

$$u \mp \sqrt{\mathscr{L}}\alpha \mp \frac{\mathscr{R}}{\sqrt{\mathscr{L}}}\beta \mp \frac{\mathscr{K}}{\sqrt{\mathscr{L}}} S = \text{const} \quad (u_0, \alpha_0, \beta_0, S_0),$$

respectively. The substript "0" of all values corresponds to initial values of these parameters. The calculation of fluxes according to the characteristic form of the system (9) was made in accordance with the procedure described in [14]. To accomplish the second stage of calculation, rewrite the system (2) in the form of integral conservation laws:

$$\begin{aligned}
\oint \varrho(dx - udt) &= 0, \\
\oint [\varrho u dx - (\varrho u^2 - \sigma_1) dt] &= 0, \\
\oint \{[\varrho(E + u^2/2)] dx - [\varrho u(E + u^2/2) - \sigma_1 u] dt\} &= 0, \\
\oint \varrho(\beta dx - u\beta dt) &= \int\int \varrho \frac{\ln \varrho_0/\varrho - 3\beta}{3\tau}\, dx\, dt.
\end{aligned} \tag{10}$$

The calculation is made in a movable difference net, and in this case the algorithm allows the calculation region to be nonuniformly divided into cells. Let the index i possess a number of values from 1 to n where n is the number of calculational points. We shall refer the values with an integer index to the boundaries of cells, and indices which differ from i by $\pm 1/2$ to the values of parameters inside calculational cells. The line over the value means that it pertains to the new instant of time. Taking account of the difference net, the system (10) rewritten in the difference form is

$$\begin{aligned}
\bar{\varrho}_{i+1/2} &= \frac{1}{\bar{x}_{i+1} - \bar{x}_i} [\varrho_{i+1/2}(x_{i+1} - x_i) - [(\pi\mathscr{S})_{i+1} - (\pi\mathscr{S}_i)] \cdot \delta \\
&\qquad + \pi_{i+1}(\bar{x}_{i+1} - x_{i+1}) - \pi_i(\bar{x}_i - x_i), \\
\bar{u}_{i+1/2} &= \frac{1}{\bar{\varrho}_{i+1/2}(\bar{x}_{i+1} - \bar{x}_i)} [(\varrho u)_{i+1/2}(x_{i+1} - x_i) - [(\pi\mathscr{S}^2 - \Sigma_1)_{i+1} \\
&\qquad - (\pi\mathscr{S}^2 - \Sigma_1)_1] \cdot \delta + (\pi\mathscr{S})_{i+1}(\bar{x}_{i+1} - x_{i+1}) - (\pi\mathscr{S})_i(\bar{x}_i - x_i)], \\
\bar{E}_{i+1} &= \frac{1}{\bar{\varrho}_{i+1/2}(\bar{x}_{i+1} - \bar{x}_i)} \{[\varrho(E + u^2/2)]_{i+1/2}(\bar{x}_i - x_i) \\
&\qquad - \{[\pi\mathscr{S}(\ni + {}^1/_2\mathscr{S}^2) - \Sigma_1\mathscr{S}]_{i+1} - [\pi\mathscr{S}(\ni + {}^1/_2\mathscr{S}^2) - \Sigma_1\mathscr{S}]_1\} \cdot \delta \\
&\qquad + [\pi(\ni + {}^1/_2\mathscr{S}^2]_{i+1}(\bar{x}_{i+1} - x_{i+1}) - [\pi(\ni + {}^1/_2\mathscr{S}^2)]_i(\bar{x}_i - x_i)\} - {}^1/_2 u_{i+2}^2, \\
\beta_{i+1/2} &= \left\{(\varrho\beta)_{i+1/2}(\bar{x}_{i+1} - x_i) - [(\pi\mathscr{S}\mathscr{B})_{i+1} - (\pi\mathscr{S}\mathscr{B})_i] \cdot \delta \right. \\
&\qquad + (\pi\mathscr{B})_{i+1}(\bar{x}_{i+1} - x_{i+1}) - (\pi\mathscr{B})_i(\bar{x}_i - x_i) + \frac{\varrho_{i+1/2}}{3\tau_{i+1/2}} \ln \frac{\varrho_0}{\varrho_{i+1/2}} \\
&\qquad \left. \frac{(\bar{x}_{i+1} - \bar{x}_i) + (x_{i+1} - x_i)}{2} \cdot \delta \right\} \Big/ \left\{ \varrho_{i+1/2} \left[(\bar{x}_{i+1} - \bar{x}_i) + 3 \frac{(\bar{x}_{i+1} - \bar{x}_i) + (x_{i+1} - x_i)}{2\tau_{i+1/2}} \delta \right] \right\},
\end{aligned}$$

where $\pi, \mathscr{S}, \ni, \mathscr{B}, \Sigma_1$ are the fluxes of density, velocity, energy, β and σ_1, respectively; δ is the step in time. It should be noted that to provide stability in calculation, the value β

in the right part of the last equation of these system is taken on a new time step. The stability condition puts restriction on the step in time, which is similar to that for gasdynamic equations:

$$\delta \leqslant \min_i \frac{\bar{x}_{i+1}-\bar{x}_i}{\max_i (u_i+c, u_i-c)},$$

c is the velocity of propagation of longitudinal sound disturbances in a given material. To complete the calculation fully at the next step in time, the following equation should be used:

$$\bar{\alpha}_{i+1/2} = \ln (\varrho_0/\bar{\varrho}_{i+1/2}) - 2\bar{\beta}_{i+1/2}.$$

In the algorithm used the value of velocity of boundaries was given as boundary conditions in the form of time functions.

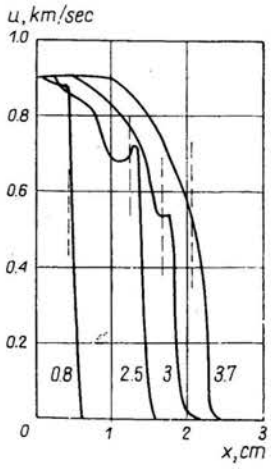

FIG. 5.

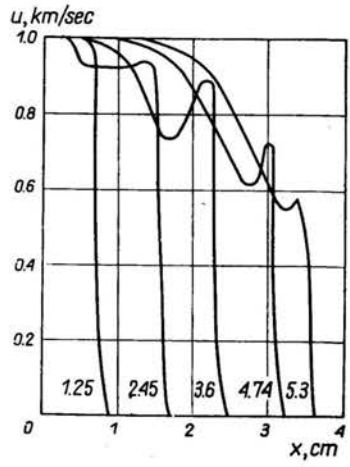

FIG. 6.

Figures 5 and 6 give the results of calculation of a collision process of two similar plates. The velocity of collision v = 1.8 km/s and 2 km/s, respectively. The diagrams show the dependence of mass velocity on the x coordinate at instants denoted by numerals near each curve (time in mcsec.). Mass velocities behind the front of waves propagating into plates are precisely equal to $v/2$. The values of the remaining parameters, i.e. density, stresses and temperature behind fronts, coincide practically with the values calculated using the known Hugoniot of material. The process of appearance and decay of elastic precursor amplitude is qualitatively reflected by calculations; the velocity of precursor propagation corresponds to the observed one during experiment. Shown by a dashed line in figures is the position of a shock wave front calculated at corresponding instants, using a Hugoniot. Practically at all instants it corresponds to the beginning of a plastic wave. The plastic wave overtakes the elastic precursor which had appeared before, and starting with some instant a two-wave configuration transfers into that with one wave. Figure 6 shows the dependencies of the distance between the beginning of the plastic wave (mini-

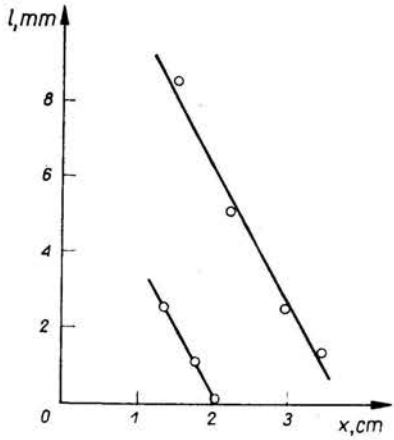

FIG. 7.

mum on the wave profile) and the front of the elastic precursor. This distance turned out to decrease linearly with the propagation of wave, and this indicates constancy of the difference of velocities of waves themselves. On the whole the calculations showed the applicability of the relation (7) and the proposed method of thorough calculation to calculate unsteady shock-wave processes in metals, taking account of the peculiarities of their course.

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INSTITUTE OF HYDRODYNAMICS,
SIBERIAN BRANCH OF USSSR ACADEMY OF SCIENCES, NOVOSIBIRSK, USSR.

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