Archives of Mechanics • Archiwum Mechaniki Stosowanej • 30, 4–5, pp. 461–475, Warszawa 1978

Dispersion of shock waves in liquid foams of high dryness fraction

J. S. de KRASIŃSKI, A. KHOSLA and V. RAMESH (CALGARY)

DRY FOAMS in which $\alpha > 0.5$ appear to have very high dispersive qualities, a trend opposite as predicted by present day theories. G. I. Taylor's statistical approach to shock wave diffusion in turbulent atmosphere [18] was used to predict this new trend. It appears that the scattering of the wave againts individual bubbles becomes a predominant mechanism when α increases. An expression has been derived for the standard deviation in time of the progressing shock wave.

Suche pianki, w których $\alpha > 0.5$ cechują się bardzo wysokimi efektami dyspersyjnymi, co jest sprzeczne z wynikami otrzymanymi według dotychczasowych teorii. Celem potwierdzenia tych efektów zastosowano statystyczne podejście G. I. Taylora dla dyfuzji fali uderzeniowej w turbulentnej atmosferze. Okazuje się, że rozproszenie fali wskutek odbijania się jej od pojedynczych pęcherzyków powietrza staje się wraz ze wzrostem α mechanizmem dominującym. Wyprowadzono wyrażenie dla standardowego odchylenia w czasie postępującej fali uderzeniowej.

Сухие пены, в которых $\alpha > 0,5$, характеризуются очень высокими дисперсионными эффектами, что находится в противоречии с результатами полученными согласно существующим до сих пор теориям. С целью подтверждения этих эффектов применен статистический подход Г. И. Тейлора для диффузии ударной волны в турбулентной атмосфере. Оказывается, что рассеяние волны, вследствие улетучивания единичных пузырьков воздуха, становится, совместно с ростом α, преобладающим механизмом. Выведено выражение для стандартного отклонения во времени распространяющейся ударной волны.

1. Introduction

THIS PAPER deals with the dispersion of shock waves in mixtures of gas with a small quantity of liquid forming bubbles. It appears that the propagation of sound and shock waves in frothy mixtures with large quantities of liquid and small gas content is reasonably well understood although there are still some puzzling discrepancies between the existing theories and experimental data.

The extremely low sound velocity high dispersion characteristics and almost isothermal behaviour of frothy mixtures have attracted the attention of many scientists and a large literature is available on the subject. MALLOCK [11], CAMPBELL and PITCHER [5], BATCHELOR [1], WIJNGAARDEN [19, 20, 21, 22] and NOORDZIJ [12] should be considered as pioneers in this field. It is questionable, however, if for comparatively dry foams in which the dryness fraction α_0 lies between 0.6 and 1.0 all the arguments are valid. The classical approach starts with one bubble oscillating in an infinite fluid and most of the detailed theories are based on such a model.

Recently, WIJNGAARDEN [22] has summarized the dispersion mechanism which appears in a general equation of a wave of finite amplitude progressing in one direction. This equation is similar to the one Korteweg-de Vries studied in recent years in connection with nonlinear wave propagation of various kinds [10]. The mechanisms can be divided into the following groups:

i) Frequency dispersion due to thermal conduction. The heat is conducted from the fluid to the bubbles and vice-versa, causing a difference of phase between the pressure in the bubble and the external pressure. This component depends upon the bubble diameter and wave length and appears to be dominant. It tends to flatten the wave.

ii) Acoustic dispersion due to acoustic radiation.This component is comparatively small.

iii) Amplitude dispersion due to the nonlinearity of the wave equations. It causes the wave to steepen.

iv) Viscous dissipation due to the friction of the individual bubbles during oscillations. It is related to the viscosity coefficient of the fluid.

WIJNGAARDEN advanced a theory [20] which is based on the idea that there is a balance between the tendency to steepen the wave through nonlinear effects and to spread it out by means of linear dispersion. Recently, NOORDZIJ [12] conducted test in a longer shock tube and found that the change in shock profile cannot be explained by Wijngaarden's theory. Moreover, he brought an argument of relaxation effects due to the time interval in which the viscous forces alter the velocity of individual bubbles relative to the fluid which add to the viscous dissipation mechanism. HAMILTON [6] conducted a series of tests in which out of 42 cases, 37 wave fronts broadened and only 5 steepened without any apparent reason.

A strong dispersion of the shock front observed by the authors in comparatively dry foams [7, 13] seemed to contradict the existing theories. Even if Noordzij's argument were to account for the required dispersion mechanisms, the postulated individual motion of the bubbles could occur in liquid but not in dry foams. The alternatively broadening or steepening of the shock front reported by HAMILTON [6] would indicate that there may be some other dispersive mechanism at stake. HAMILTON [7] and KHOSLA [7] also investigated the possible effects of the bulk modulus of the mixture. SHAPIRO [16] has demonstrated that if $\frac{\partial^2 p}{\partial v^2} > 0$, the compression waves would steepen and the expansion waves broaden, but if $\frac{\partial^2 p}{\partial v^2} < 0$ the opposite argument would hold. Neither HAMILTON [6] using frothy mixtures nor KHOSLA [7] could observe such a behaviour. If the shock wave broadened along its trajectory, the expansion wave did so much more.

Because of the non-satisfactory explanation by the present theories of the shock wave dispersion for dry foam ($0.6 < \alpha_0 < 1$) observed by the authors, a new mechanism is proposed which is based on the scattering of the shock front by individual bubbles and which should be added to the list of the existing ones. It appears that its effect is more prominent when the foam becomes dryer.

2. Experimental details

2.1. The shock tube

A shock tube was constructed specially for the purpose of investigating comparatively dry foams. Its details are shown in Fig. 1. The pressures were measured by means of Kistler 603A pressure transducers and amplified by Kistler charge amplifiers. The output was

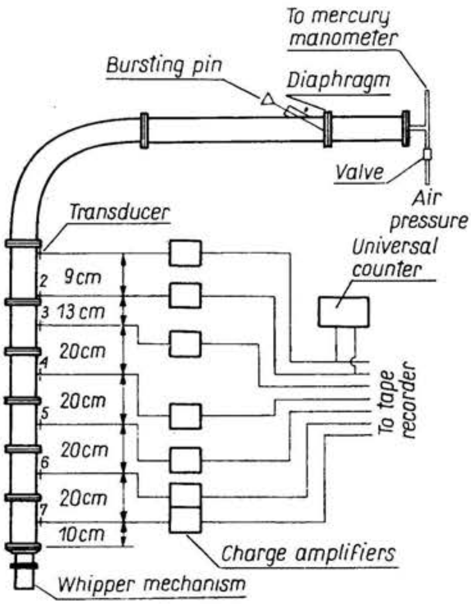

FIG. 1. The shock tube lay-out.

recorded on a 14 channel Hewlett Packard 3955 magnetic tape recorder and data reproduced on a Honeywell Visicorder. The standard shock tube technique was used to measure the shock strength and velocity. One may note at the end of the tube that a whipper mechanism was used when the dryness fraction α had to be varied systematically. In most cases the foam was at an atmospheric pressure of about 0.88 bars and moderate strength shock waves were produced by bursting a cellophane diaphgram using 2.2 to 2.3 bars in the driver. Transducers 1 and 2 (see Fig. 1) recorded the shock strength in the air and the remaining ones were submerged in the foam. Some experiments were also carried out in the shock tube described in [7].

2.2. The generation of foam

Five mechanism of foam generation were tried.

i) The wire gauze. The air was blown into the foaming mixture through a wire gauze of varying mesh size. The bubbles were very uneven and tended to grow in size by the time they reached to top of the container.

ii) The discrete hole. A method similar to the one above, except that the wire gauze was removed. The results were better than with the wire gauze but here also the foam was not homogeneous.

iii) The whipper. A whipping rotating blade was activated by an electrical motor at 15000 rpm. The advantage of this system was the exact control of the void fraction by measuring the height of the foam column and the original height of the foaming mixture. Foams were very stable.

iv) Ready made mixtures. These are available on the market and their content was obtained from the manufactures. These foams were also very stable and had a comparatively even distribution of bubble size.

v) The liquified gas. A liquified gas of low critical pressure was forced in a cold bath into the foaming mixture. After returning to atmospheric conditions very fine stable and even bubbles were formed.

Rug shampoo, distilled water and methanol were used to obtain the foaming mixture. In some experiments 75% of Glycerol and 15% of water and rug shampoo were used to increase the viscosity coefficient by 3 orders of magnitude. For more details see [13].

3. The geometrical structure of the foam

Some of the discrepancies observed by various experimentators can be attributed to different foam structures produced under various laboratory conditions. For each fraction a foam possesses a spectrum of bubble diameters which is usually skewed. An example of it is given in Fig. 2 as obtained by KHOSLA [7]. Although finer analysis would require a more precise knowledge of such a spectrum, there are still almost unsurmountable experimental difficulties in obtaining it for each test; average values have to be taken usu-

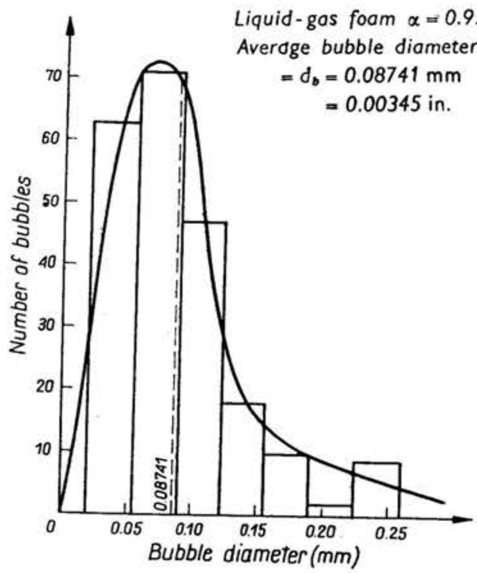

FIG. 2. The foam spectrum.

ally from practical considerations. In this case the shape of the bubbles cannot be considered and a tacit assumption is made that the bubbles are on the whole spherical. For a vessel containing N bubbles of an average diameter d_0 and average thickness t_0 of the liquid film separating the bubbles, the volumetric void fraction α_0 represents the (volume of gas)/(total volume). It is sometimes useful to consider linear relations as though the foam was observed through a narrow slit, than the linear α corresponds to the (length of gas intervals)/ (total length). The relation between the two definitions is $\alpha = \sqrt[3]{\alpha_0}$. Similarly, if there are N_0 bubbles in the vessel $\frac{N_0}{\text{Volume}} = n_0$ and the amount of bubbles/unit length $n = \sqrt[3]{n_0}$. It follows from geometrical considerations that the average gas gap g, the film thickness t and the bubble diameter d_0 are given by

$$g = \frac{\alpha}{n}, \quad t = \frac{1-\alpha}{n}, \quad d_0 = \sqrt{\frac{6\alpha_0}{\pi n_0}}. \tag{3.1}$$

These equations relate three variables, a minimum essential for the description of the structure of the foam. A knowledge of α_0 and n_0 is required to determine the diameter d_0. The void fraction α and n are in principle independent variables which are usually related by the way how the foam is produced. Thus, for example, bubbling of gas through the liquid may produce large bubbles at low values of α, and a whipping mechanism will produce small bubbles at large values of α. On the other hand, a ready made foam of high gas content and with small bubbles will, under expansion, produce large bubbles at very high void fractions. The same foam subjected to compression will possess very small bubbles at low void fractions until all the bubbles disappear in the liquid. Larger bubbles tend to be broken by shock waves [6] and their growth is limited at high void fractions by their stability. The quotation of the void fraction α only, is obviously not sufficient, $\boldsymbol{\alpha}$ and **N** or α and g are the minimum requirements for the repeatability of the tests. Even this may be questioned if large differences occur in the bubble spectra (see Sect. 4).

It may be pointed out that an important parameter related to the heat transfer between the liquid and the gas is the interfacial area of the bubbles per unit mass of the liquid requires both α_0 and n_0 for its description.

In the study of the dispersion phenomena the dispersive function proposed by WIJNGAARDEN [19] depends upon a non-dimensional ratio of the interbubble distance b_0 to the bubble diameter d_0 and this can be easily shown to depend only upon α_0. Thus

$$\frac{b_0}{d_0} = 1 + \left[\frac{1-\alpha_0}{\alpha_0}\right]^{1/3}. \tag{3.2}$$

4. The nature of dispersive losses

In a shock tube of a very large diameter filled with foam the impulse due to the shock wave

$$I = \int_0^T p\,dt = \text{const}, \tag{4.1}$$

where T is the positive time duration of the wave.

If dispersive phenomena do take place, the wave will flatten and the positive time duration T will increase by ΔT along the trajectory when measured from the foot to the tail of the wave. A triangular wave shape with the transmitted pressure P_1 at the gas-foam interface and positive time duration T_1 will reduce its maximum pressure to P_2 and increase its positive time duration to $T_2 = T_1+\Delta T$. If

$$\delta = \frac{P_1-P_2}{P_1} \tag{4.2}$$

is the pressure defficiency parameter, it can be shown that for a constant impulse ΔT and δ are related through

$$\Delta T = T\frac{\delta}{1-\delta}, \tag{4.3}$$

$$\delta = \frac{\Delta T}{T+\Delta T}. \tag{4.4}$$

The same argument can be extended to any wave shape, in this case instead of P_1 and P_2 the mean values

$$\overline{P}_1 = \frac{\int_0^{T_1} pdt}{T_1}, \tag{4.5}$$

$$\overline{P}_2 = \frac{\int_0^{T_1+\Delta T} pdt}{T_1+\Delta T} \tag{4.6}$$

should be used. In the shock tube experiments it is not easy to obtain such integrals because the reflected wave interferes with the tail of the trasmitted wave and time $T+\Delta T$ cannot often be read accurately from the record.

If the dispersive phenomena due to frequency and the internal viscous dissipation were to be equilibrated by the nonlinear effects as postulated by WIJNGAARDEN [20], the pressure deficiency parameter δ would reach a limit smaller than unity.

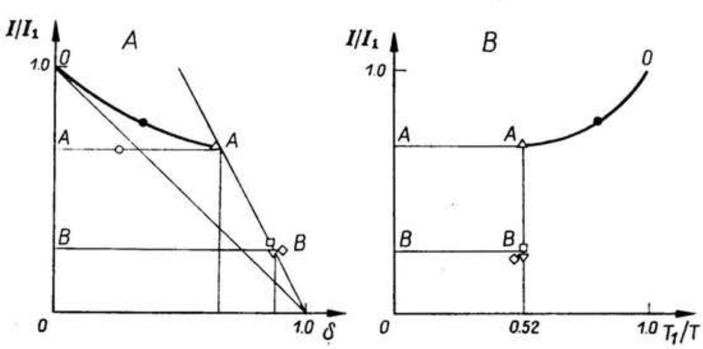

FIG. 3. The path of the shock wave in the two-coordinate systems.

For a shock tube of a finite diameter d and filled with foam some work is also done at the boundaries. If there were no dispersive phenomena, the pressure deficiency parameter δ would increase at $T = \text{const}$. In reality both phenomena take place simultaneously and it is not easy to discern each contribution separately. If the impulse integral can be measured, the simplest coordinate system for this purpose is to plot the ratio I/I_1 against the pressure deficiency parameter δ, or I/I_1 against $\frac{T_1}{T_1+\Delta T}$. Such two plots are shown in Fig. 3a, b for an experiment in which the work due to friction was artificially increased by submerging a thin-walled honeycomb in the foam [7]. One observes in both figures that as the wave progresses through the foam, the impulse slowly decreases from 0 to A because of the external work. This process modifies the broadening of the shock front. As the wave crosses the honeycomb the impulse suffers a sudden decrease at constant T from A to B as would be expected, and the pressures deficiency parameter δ also increases. A more refined method to discern and estimate the frictional losses at the boundaries was elaborated by KHOSLA [7] and presented previously by the authors [8]. It follows that δ would, in reality, be larger than when computed by means of Eq. (4.4) on the principle of constant impulse because of the friction against the walls of the shock tube. It appears from this short discussion that the broadening of the shock front and dispersion processes in the foam will depend not only on the foam structure liquid viscosity and shock strength as postulated by NOORDZIJ [12] but also on the characteristic of the shock tube, in particular on the ratio between its diameter and the distance travelled by the wave.

These considerations shed additional light on the reasons for the discrepancies observed when comparing experimental data. Also the nature of the shock tube measurements with the associated scatter of results and the difficulties to obtain uniform foams complicate further the issue.

5. Experimental results and their analysis

5.1. The velocity of propagation of shock waves in foam

By varying systematically the void fraction α the velocity of shock propagation conformed well with the classical simplified expressions based on the isothermal bulk modulus and sound velocity at low frequencies as reported, for example, by HAMILTON [6], namely,

$$U = \left[\left(\frac{P_1}{P_0}\right)\frac{P_0}{[\alpha_0(\alpha_0\varrho_g+(1-\alpha_0)\varrho_l}\right]^{1/2}, \tag{5.1}$$

where P_1/P_0 is the shock strength Z and ϱ_g and ϱ_l are the densities of the gas and the liquid, respectively. It was observed that for higher void fractions $0.9 < \alpha_0 < 0.99$, the velocity of the shock wave approached that of adiabatic thermal equilibrium [14], i.e. the results obtained from Eq. (5.1) should be multiplied by $\sqrt{\gamma}$, with $\gamma = 1.4$. Thus the effect was small. Typical data collected during these tests are shown in Fig. 4 for the shock strength 1.82 measured at the foam interface.

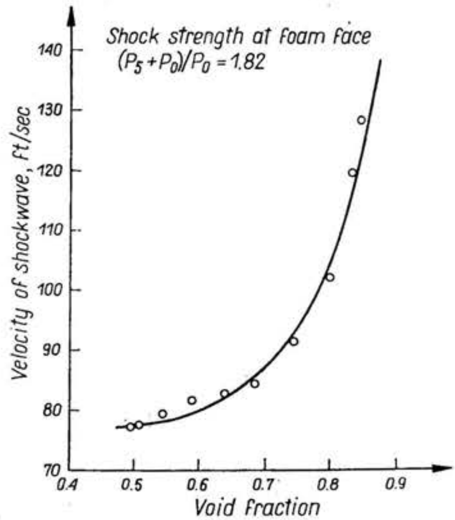

FIG. 4. Measured velocity of shock-wave as function of α.

5.2. The shock wave dispersion

Preliminary experiments performed on ready made foams with a void fraction $\alpha_0 = 0.94$ and by systematically increasing the void fraction α from 0.5 to about 0.84 [7, 14] have shown a small increase of dispersive characteristics of the foam with an increase of the void fraction α_0. At what α_0 a maximum was achieved was not quite ascertained. Approaching $\alpha_0 \to 1.0$ causes extreme thinness of the bubble walls and their collapse, hence one would expect that in the limit the pressure deficiency parameter would reach again a very small

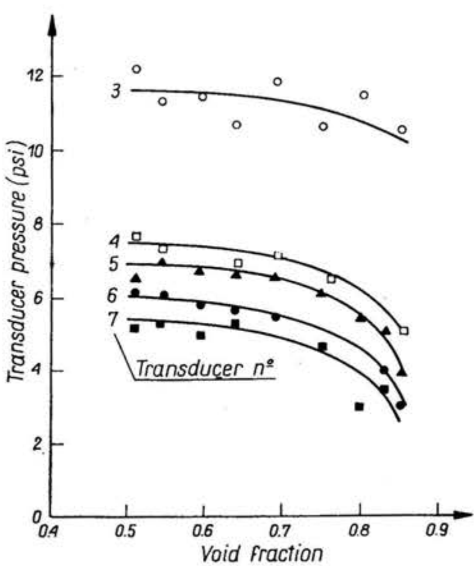

FIG. 5. Pressure records of transducer 3-7 as function of α_0,

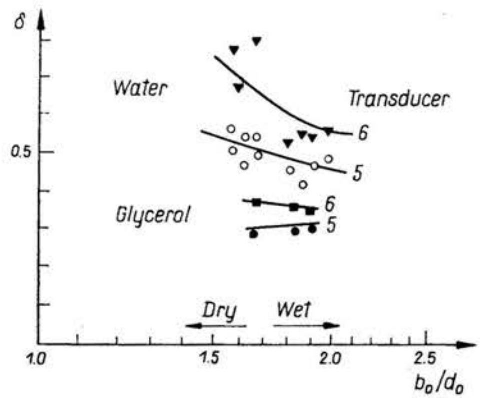

FIG. 6. Measured pressure deficiency parameter δ against b_0/d_0.

value. The mean bubble diameter in those experiments was about 0.08 mm. The pressures recorded at the tranducers 3 to 7 plotted against α_0 are shown in Fig. 5 and indicate an increase in attenuation with increasing α_0. The same results replotted for the transducers 5 and 6 in terms of the pressure deficency parameter δ as a function of b_0/d_0 are shown in Fig. 6 and give the same trend.

WIJNGAARDEN [11] has shown that the frequency dispersion depends upon a dispersion parameter σ which is a function of $b_0/d_0 = \frac{\text{interbubble distance}}{\text{diameter of the bubble}}$. This ratio was given in Eq. (3.2). The parameter σ is plotted against b_0/d_0 in Fig. 7 and reaches a minimum when

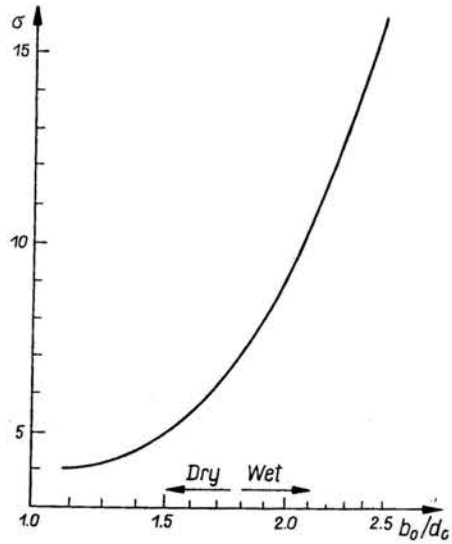

FIG. 7. Dispersion parameter σ [19] against b_0/d_0.

α_0 approaches unity. It appears therefore that the criteria for frequency dispersion for wet foams do not apply to dry ones. WIJNGAARDEN has also shown [19] that viscous pressure losses decrease exponentially with the viscosity according to the factor exp. ϕ, where

$$\phi = \frac{-2\nu X}{UR^2}. \tag{5.2}$$

To find the effects of increased viscosity, 75% of Glycerol was used in the foaming mixture increasing the viscosity by a factor of about 1000. The pressure deficiency parameter δ for this new set of tests is also given in Fig. 6. A slight decrease in the values of δ could be attributed to a small difference in the size of the bubbles, but no increase in δ is observable.

As mentioned before, any irregularities in the bulk modulus giving $\frac{\partial^2 p}{\partial v^2} < 0$ would mean a broadening of the compression steepening of the expansion waves. This was never observed neither in this laboratory [7] nor by HAMILTON [6]. The expansion waves broadened always much more than the compression ones.

It appears from this discussion that another mechanism of dispersion is at work for dry foams, althrough one would suspect that it also exists in frothy mixtures.

6. The scattering of the shock front from the bubble surfaces

6.1. Preliminary observations

The process of scattering of the shock front on loosely distributed solid bodies has been analysed in [9]. Dry foams represent a gas disturbed by small quantities of liquid. The shock front at each bubble interface meets a high acoustic impedance of the liquid. The motion of a shock front amongst solid spheres, like golf balls, would be somewhat similar to the motion amongst tightly packed gas bubbles in which the dispersive characteristics based on their radial motion namely $\partial R/\partial t$ and $\partial^2 R/\partial t^2$ would be practically nil.

It may be recalled in this context [2, 3, 15] that if a shock front meets a medium of lower acoustic impedance, it will reflect as an expansion wave and progress into the medium with the strength at which it was reflected; if, on the other hand, high acoustic impedance is encountered at the interface, the shock will reflect back as a compression wave and the correspondingly stronger wave will be transmitted into the other medium. One may ask what would be the situation at the interface of a thin-walled shell.

6.2. Auxiliary experiments

A thin-walled sphere was selected whose thickness of the wall t and diameter d determined $\alpha_0 = 0.952$, the acoustic impedance of the wall was similar to that of the water, and its inertia was close to a gas bubble covered with water of the same wall thickness. Unfortunately, the elastic properties could not be simulated as the walls were comparatively too rigid. The sphere was loosely set on a stand and subjected to a shock wave in atmospheric conditons of 917 mb and Mach number 1.27 and a high speed cine camera was used. Figure 8a shows the shock front just crossing the sphere, Fig. 8b, after 167 μ sec, the shock is about 4 diameters behind the sphere, a reflected wave is clearly visible

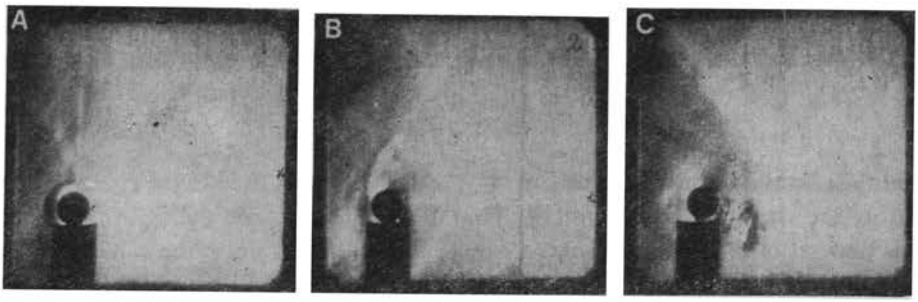

FIG. 8. High speed cine recordings of a thin-walled sphere crossed by a shock wave.

and is moving in the opposite direction than the main wave and the sphere is motionless. In Fig. 8c, only after 668 μ sec the sphere begins to move and both the main and reflected waves are not in the field of vision anymore. This experiment illustrates well the inertia and scattering effects due to one disturbance of this type.

If a similar situation were to occur in the foam, then one would expect that the particle velocity in the foam would also suffer a time delay behind the shock front. For this purpose the motion of the foam behind the shock was observed with a high speed cine and the result of the analysis is shown in Fig. 9. One observes that the foam requires about 550 μ sec to reach its limiting velocity. There appears to be a time lag between the crossing of the wave front and the moment when the particle velocity reaches its full strength. This observation

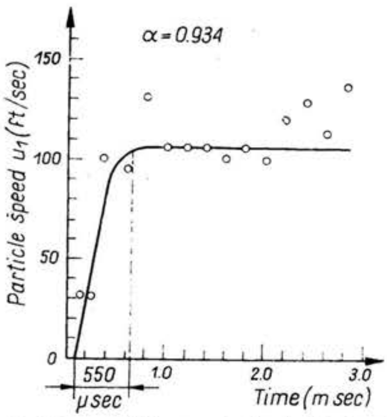

FIG. 9. Measured foam velocity behind a shock wave as f(time).

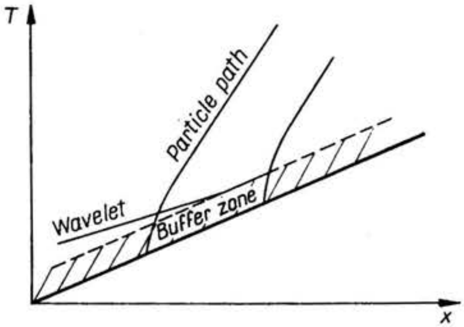

FIG. 10. A diagram of the process of overtaking of a shock front by a wavelet.

casts some doubts if one can treat the foam using classical concepts without considering some type of inertial relaxation time for the foam as a whole. The time lag in the particle velocity would indicate on the t-x diagram a curved path as shown in Fig 10. The nonlinear effects of steepening of the wave in isothermal conditions can be physically seen as being caused by the additional velocity in the field behind the wave, helping the disturbances to catch up with the wave front. It is illustrated in Fig. 10 that a wavelet following the shock front may never reach it as there appears to exist a buffer zone between the shock front and the rest of the field.

These considerations lead to the conclusion that in some respects the discontinuities in the acoustic impedance encountered by the shock front as it progresses through the foam bear similarities to a wave moving amongst solid objects. The shock front is asso-

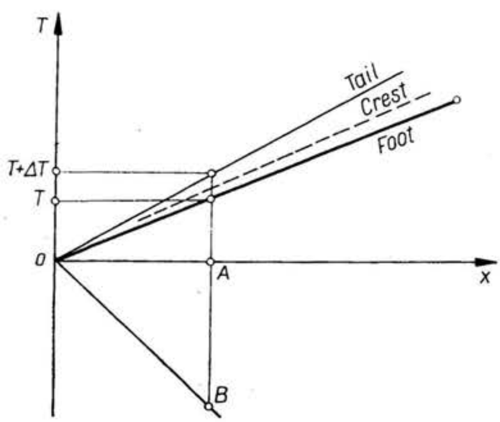

FIG. 11. A diagram of the process of spreading of a shock wave in foam.

ciated with a weak family of right and left waves. This scattering effect becomes particularly pronounced in dryer foams and is believed to be another important mechanism of dispersion. The process may also be illustrated on a t-x diagram (Fig. 11). At time t the foot of the wave has reached the distance OA. At time $T+\Delta T$ the remaining waves moving with the same speed have reached on different paths a distance OB marked on the inclined axis, but in the X direction they have only progressed the distance OA. Hence the wave broadens similarly to an expansion wave. In this case one can distinguish the foot, the crest and the tail of the wave. The thicknes of the shock would be considered as the distance between the foot and the crest. If all the dissipative mechanisms are stronger than the nonlinear "amplitude dispersion" anticipated by Wijngaarden which tends to steepen the wave, then the concept of the "shock thickness" has as little meaning as that of the thickness of an expansion wave in classical gas dynamics.

7. The application of G. I. Taylor's concept of diffussion

7.1. Introductory remarks

Taylor's famous concept of diffusion by continuous movements [17] was applied by him much later to study the propagation of blast waves over the ground in a turbulent atmosphere [18]. In this rather little known paper he considers the probability of encountering the shock front which is scatteed by the atmospheric eddies. With certain simplifications related to the correlation coefficient, he obtains the standard deviation to which the shock front is subjected in the form (standard deviation) = (deviation due to one event) × $\sqrt{\text{amount of events}}$

$$\sigma_x \sim \sigma_1 \sqrt{N}. \tag{7.1}$$

After estimating the standard deviation in space σ_x, the application of the probability integral indicates that the wave flattens at constant impulse and the pressure deficiency parameter δ increases along the trajectory. Calculations of this kind have been carried out by KHOSLA [7] assuming a certain ratio between the standard deviation and the distance of the positive time duration. This principle has been further extended to estimate the scattering of the wave as it progresses amongst loosely arranged rigid bodies [9]. From the previous discussion it appears that the same principle can be invoked as an additional mechanism of shock dispersion in foams besides those listed in Sect. 1 of this paper. If correctly applied, one would expect that the spreading of the wave in time should depend upon the structure of the wave, namely α, n and the distance of propagation. Also one would expect the pressure deficiency parameter δ to rise slightly with the increase of the void fraction and finally fall when it reaches unity. These effects would be of course blurred by the other mechanisms of dispersion discussed before and friction at the boundaries in the case of a shock tube.

7.2. The standard deviation of a scattered wave in foam

Considering all the simplifications with regard to the correlation coefficient [18] and using Eq. (7.1) one may, in the case of foam, assume that the deviation due to one event is proportional to the gap diameter g and the number of events $N \sim nX$, where n is the amount of bubbles per unit length and X is the distance travelled by the wave; thus, for a wave element

$$\sigma_x \sim g\sqrt{nX} \tag{7.2}$$

combining this relation with Eqs. $(3.1)_{1,2}$

$$\sigma_x \sim \sqrt{g\alpha X} \sim \sqrt{\frac{X}{n}}, \tag{7.3}$$

$$\sigma_x \sim \alpha\sqrt{\frac{Xt}{1-\alpha}}. \tag{7.4}$$

One may note that the standard deviation depends both upon the size of the gaps of gas g that the wave encounters and the dryness fraction α. In all cases the spread of the wave is proportional to $X^{1/2}$.

The relation between the spread in space and spread in time is

$$\sigma_T = \frac{\sigma_x}{U}, \tag{7.5}$$

where U is the velocity of propagation of the wave, which again is a function of the void fraction α_0. Also $\sigma_T \sim \Delta T$ discussed in Sect. 4 and again ΔT is related to the pressure deficiency parameter δ, through Eq. (4.3), what is correct for no friction at the shock tube walls. Taking Eq. (5.1) for the average velocity of the shock wave one readily obtains from Eq. (7.2)

$$\Delta T \sim \sigma_T \sim \left[\frac{gX\alpha_0^{4/3}[\alpha_0\varrho_g+(1-\alpha_0)\varrho_l]}{P_0 Z}\right]^{1/2} \tag{7.6}$$

as $g/d_0 = \text{const}$

$$\Delta T \sim \sigma_T \sim \left[\frac{d_0X\alpha_0^{4/3}[\alpha_0\varrho_g+(1-\alpha_0)\varrho_l]}{P_0 Z}\right]^{1/2}.$$

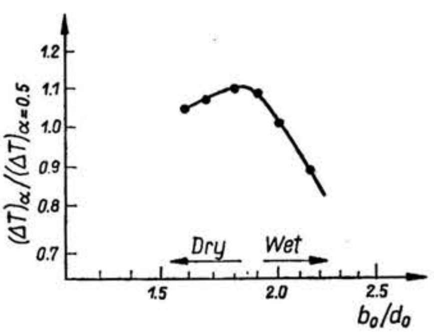

FIG. 12. The relative spread in time of a shock wave in foam due to scatter as function of b_0/d_0.

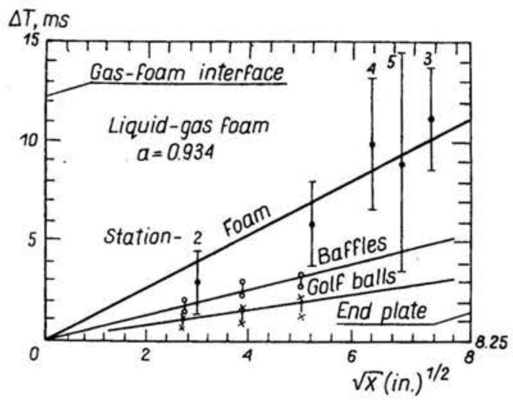

FIG. 13. Measured spread in time of shock waves in foam and through randomly distributed solid bodies.

This expressions stresses the importace of the foam structure upon the process of shock wave scattering (in time) and also indicates that for a given shock $\Delta T \sim X^{1/2}$.

In Fig. 12 the ratio $\frac{(\Delta T)\alpha}{(\Delta T)\alpha = 0.5}$ has been plotted against b_0/d_0. One observes a steady rise of the curve from $\alpha_0 = 0$ until it reaches a plateau at about $\alpha_0 = 0.65$; a trend different to the frequency dispersion postulated by Wijngaarden (Fig. 7). A similar trend is observable if the pressure deficiency parameter δ is plotted against b_0/d_0 by means of Eqs. (4.2) and (4.4). In real shock tube situations the growth of the pressure deficiency parameter δ is due to both the increase of ΔT and external frictions. There is some experimental evidence that as the void fraction α_0 grows beyond 0.7, the spread in time is somewhat smaller like indicated in Fig. 12; moreover, external friction effects become more important. Until a more precise control of the bubble diameter is achieved such a conclusions is premature.

If the scattering processes dominate dispersion at higher values of α_0, then ΔT for foams and solid should plot as a straight line agaist $X^{1/2}$. Figure 13 shows such results from numerous tests performed by KHOSLA [7] for foam $\alpha_0 = 0.93$, $d_0 = 0.08$ mm, golf balls and baffles. The trend is identical and shows the high dispersive quality of dry foams.

8. Conclusions

Shock tube experiments performed in comparatively dry foams ($\alpha > 0.5$) have shown an increase of dispersion with the incrase of α_0 contrary to present day theories which were developed for $\alpha \ll 1.0$. Advantage has been made of G. I. TAYLOR's statistical approach [17, 18] drawing attention to another mechanism of dispersion, namely the shock wave scattering against individual bubbles. To verify this new approach more tests should be done on dry foams with an exact control of bubble diameter.

Acknowledgements

The authors gratefully acknowledge the support of the Canadian Government Granting Agencies for this research as well as Mr. W. ANSON for his help in the Shock Tube Laboratory and with the high speed cine photography.

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DEPARTMENT OF MECHANICAL ENGINEERING,
THE UNIVERSITY OF CALGARY, ALBERTA, CANADA.

Received October 6, 1977.