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The asymptotic motion of the concentrated defect

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THE ANALYSIS of asymptotic solutions of equations of a concentrated defect is presented. The equation of zero order approximation of the position of the defect and form of the adiabatic invariant of its motion are given.

Przedstawiono asymptotyczne rozwiązanie równań skoncentrowanego defektu. Podano równania zerowej aproksymacji dla położenia defektu i postać adiabatycznego niezmiennika jego ruchu.

Представлено асимптотическое решение уравнений сосредоточенного дефекта. Приведены уравнения нулевой аппроксимации для положения дефекта и форма адиабатического инварианта его движения.

Introduction

THE AIM of this paper is to investigate the asymptotics of solutions of equations of the concentrated defect in an unbounded linear elastic medium. The defect considered is of the type of the variable in the time jump of the normal component of the displacement on the surface of the sphere. The concentration of the defect indicates that the surface of the defect has a radius negligible in relation to the characteristic linear parameters of the external elastic field. The equation of motion of the concentrated defect was obtained in the paper [1] and has the form

$$\varepsilon \ddddot{\boldsymbol{\xi}} + \mathbf{Q}(t, \boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \ddot{\boldsymbol{\xi}}) = \sqrt{\varepsilon}\mathbf{F}(t, \boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \sqrt{\varepsilon}\,\dddot{\boldsymbol{\xi}}; \sqrt{\varepsilon}), \tag{1}$$

where $\boldsymbol{\xi} = \boldsymbol{\xi}(t) \in R^3$, $t \in R$ — the position of the centre of the surface of the defect (in which the defect is "concentrated"). $\varepsilon = Ct^2 \approx t_0^2$ — the small parameter designated by time to needed by the sound signal to go round the sphere bounded by the surface of the defect. $\mathbf{Q}$ — the vector-function, the form of which is determined by the external elastic field. $\mathbf{F}$ — the vector-function independent of the external field and disappearing if the defect is constant in time. $\dot{\boldsymbol{\xi}}, \ddot{\boldsymbol{\xi}}, \dddot{\boldsymbol{\xi}}, \ddddot{\boldsymbol{\xi}}$ — the derivatives with respect to time t.

This is a system of three ordinary differential equations of the fourth order with the unknown vector-function $\boldsymbol{\xi}(t)$. Introducing the auxiliary variables

$$\mathbf{x} = \ddot{\boldsymbol{\xi}}, \quad \mathbf{y} = \sqrt{\varepsilon}\,\dddot{\boldsymbol{\xi}}, \quad \mathbf{v} = \dot{\boldsymbol{\xi}}, \quad \boldsymbol{\mu} = (t, \boldsymbol{\xi}, \mathbf{v}), \quad \tau = t/\sqrt{\varepsilon},$$
$$\mathbf{x}, \mathbf{y}, \mathbf{v} \in R^3, \quad \boldsymbol{\mu} \in R^7, \quad \tau \in R,$$

we can write Eq. (1) in the form

$$\begin{aligned} &\frac{d\mathbf{x}}{d\tau} = \frac{\partial H}{\partial \mathbf{y}}, \quad \frac{d\mathbf{y}}{d\tau} = -\frac{\partial H}{\partial \mathbf{x}} + \sqrt{\varepsilon}\mathbf{F}(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}; \sqrt{\varepsilon}), \\ &\frac{d\boldsymbol{\mu}}{d\tau} = \sqrt{\varepsilon}\boldsymbol{\varphi}(\mathbf{x}, \boldsymbol{\mu}), \end{aligned} \tag{2}$$

where

$$H = H(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}) = \frac{1}{2} y^2 + \frac{1}{2a(t)} Q^2, \quad \mathbf{y} = \frac{d\mathbf{x}}{d\tau}, \quad y = ||\mathbf{y}||,$$

$$Q = Q(\mathbf{x}, \boldsymbol{\mu}) = ||\mathbf{Q}(\mathbf{x}, \boldsymbol{\mu})||, \quad \mathbf{Q}(\mathbf{x}, \boldsymbol{\mu}) = a(t)[\mathbf{x} - \mathbf{f}(\boldsymbol{\mu})] \in R^3,$$

$$\mathbf{f}(\boldsymbol{\mu}) = M(t)^{-1}[\mathbf{P}(\boldsymbol{\mu}) - \dot{M}(t)\mathbf{v}] \in R^3,$$

$$\mathbf{F}(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}; \sqrt{\varepsilon}) = -\sqrt{\varepsilon}[F_1(t)\mathbf{v} + F_2(t)\mathbf{x}] - F^3(t)\mathbf{y} \in R^3,$$

$$\boldsymbol{\varphi}(\mathbf{x}, \boldsymbol{\mu}) = (1, \mathbf{v}, \mathbf{x}) \in R^7;$$

$P(\boldsymbol{\mu})$ is the force with which the external field reacts on the defect concentrated at the point $\boldsymbol{\xi}$; $F_i(t)$, $M(t)$, $a(t)$ are functions of time independent of the external field but dependent on the defect.

The zero-approximation of Eq. (2) has the Hamiltonian form

$$\frac{d\mathbf{x}}{d\tau} = \frac{\partial H}{\partial \mathbf{y}}, \quad \frac{d\mathbf{y}}{d\tau} = -\frac{\partial H}{\partial \mathbf{x}}, \tag{3}$$

where the function H is dependent on the parameter

$$\boldsymbol{\mu} = \text{const}.$$

In order to find out when Eq. (3) can be interpreted as describing the motion of a material point, let us consider the quantity $M(t)$:

$$M(t) = \alpha U(t)^2 + m(t),$$

where $\alpha < 0$ — the constant and $(U(t), m(t))$ — a pair of functions defining the defect: $U(t)$ — magnitude of the defect, $m(t)$ — quantity measuring mass.

It has been assumed in the paper that

$$m(t) = m = \text{const}.$$

As sgn $H =$ sgn M, then Eq. (3) is the equation of the motion of the material point (possessing mass equal 1) if

$$\bigwedge_t M(t) > 0. \tag{*}$$

The condition (*) limits the magnitude of the defect:

(*) iff $$\bigwedge_t U(t) < \sqrt{-m/\alpha}, \quad m > 0.$$

We shall consider further Eqs. (1)–(3) satisfying the condition (*). Equation (3) will be considered together with the initial condition of the form

$$\begin{aligned} \mathbf{x}(0) &= \boldsymbol{\alpha}_0 \neq \mathbf{f}(\boldsymbol{\mu}), \\ \mathbf{y}(0) &= \dot{\mathbf{x}}(0) = \boldsymbol{\beta}_0. \end{aligned} \tag{3'}$$

Analysis of the properties of the solution of Eq. (3)

The solution of the initial problem (3) and (3′) has the form

$$\begin{aligned} \mathbf{x}(\tau) &= \frac{1}{a}\sqrt{1+P^2}Q[\cos\sqrt{a}(\tau-\tau_0)\mathbf{M} + \cos\sqrt{a}(\tau+\tau_0)\mathbf{N}] + \mathbf{f}(\boldsymbol{\mu}), \\ \mathbf{y}(\tau) &= -\frac{1}{\sqrt{a}}\sqrt{1+P^2}Q[\sin\sqrt{a}(\tau-\tau_0)\mathbf{M} + \sin\sqrt{a}(\tau+\tau_0)\mathbf{N}], \end{aligned} \tag{4}$$

where for $\boldsymbol{\mu} = (t, \boldsymbol{\xi}, \mathbf{v}) = \text{const.}$:

$$a = a(t) = \gamma M(t) U(t)^{-2} > 0, \quad \gamma \in R,$$

$$Q = Q(\boldsymbol{\alpha}_0, \boldsymbol{\mu}) = ||\mathbf{Q}(\boldsymbol{\alpha}_0, \boldsymbol{\mu})||,$$

$$P = P(\boldsymbol{\alpha}_0, \boldsymbol{\beta}_0, \boldsymbol{\mu}) = Q(\boldsymbol{\alpha}_0, \boldsymbol{\mu})^{-1}\sqrt{a(t)\beta_0}, \quad \beta_0 = ||\boldsymbol{\beta}_0||,$$

$$\mathbf{M} = \mathbf{M}(\boldsymbol{\alpha}_0, \boldsymbol{\beta}_0, \boldsymbol{\mu}) = \frac{1}{2}[\mathbf{m}(\boldsymbol{\alpha}_0, \boldsymbol{\mu}) + \mathbf{n}(\boldsymbol{\beta}_0)]\varkappa,$$

$$\mathbf{N} = \mathbf{N}(\boldsymbol{\alpha}_0, \boldsymbol{\beta}_0, \boldsymbol{\mu}) = \frac{1}{2}[\mathbf{m}(\boldsymbol{\alpha}_0, \boldsymbol{\mu}) - \mathbf{n}(\boldsymbol{\beta}_0)]\varkappa,$$

$$\mathbf{m}(\boldsymbol{\alpha}_0, \boldsymbol{\mu}) = Q(\boldsymbol{\alpha}_0, \boldsymbol{\mu})^{-1}\mathbf{Q}(\boldsymbol{\alpha}_0, \boldsymbol{\mu}), \quad \mathbf{n}(\beta_0) = \beta_0^{-1}\boldsymbol{\beta}_0,$$

$$\tau_0 = \frac{1}{\sqrt{a}} \arccos \sqrt{\frac{1}{1+P^2}}, \quad \varkappa = \operatorname{sgn}(\sqrt{a}\tau_0).$$

This solution describes a periodic motion with the period

$$T(\boldsymbol{\mu}) = \frac{2\pi}{\sqrt{a(t)}}$$

and depends on the multi-dimensional parameter

$$\mathbf{Y}_0 = (\boldsymbol{\alpha}_0, \boldsymbol{\beta}_0, \boldsymbol{\mu}) \in R^{13}.$$

Equation (3) has locally the full system of the first integrals, i.e. that there is a neighbourhood G so that $\mathbf{Y}_0 \in G$ and the system of the function $\mathbf{H}$:

$$\mathbf{H} = (H_1, \ldots, H_5): G \to R^5, \quad H_i \in C^2(G)$$

which are first integrals Eq. (3) on G.

Let us designate $\mathbf{h} = \mathbf{H}(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu})$, $\mathbf{r} = (\mathbf{h}, \boldsymbol{\mu})$ and let us introduce the mappings

$$p: G \to R^{12}, \quad \pi: G \to R^6$$

by the rules

$$p(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}) = (\mathbf{h}, \boldsymbol{\mu}) \quad \text{for} \quad \mathbf{h} = \mathbf{H}(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}),$$

$$\pi(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}) = (\mathbf{x}, \mathbf{y}).$$

Let us denote by $G_p = p(G)$ and $G_\pi = \pi(G)$ the images of the set G using the functions p and π. On the basis of the paper [2] we can prove the existence of the neighbourhood G possesing the additional properties as follows:

A. Each trajectory of Eq. (3) possesses a neighbourhood G_π composed of disjoint trajectories of this equation, that is:

$$G_\pi = \bigcup_{\mathbf{r} \in G_p} M_{\mathbf{r}}, \quad M_{\mathbf{r}_1} \cap M_{\mathbf{r}_2} = \phi \quad \text{for} \quad \mathbf{r}_1 \neq \mathbf{r}_2,$$

where $M_{\mathbf{r}}$ is compact and connected one-dimensional manifold so that $p^{-1}(\mathbf{r}) = M_{\mathbf{r}} \times \{\boldsymbol{\mu}\}$ for $\mathbf{r} = (\mathbf{h}, \boldsymbol{\mu})$.

B. There is a smooth function

$$\mathbf{\Lambda} = (\boldsymbol{\alpha}, \boldsymbol{\beta}): G_p \to G_\pi$$

so that if $\mathbf{\Lambda}(\mathbf{r}_1), \mathbf{\Lambda}(\mathbf{r}_2) \in M_\mathbf{r}$ then $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$.

This is due to the properties A and B that each solution $\mathbf{X}(\tau) = (\mathbf{x}(\tau), \mathbf{y}(\tau))$ of Eq. (3) contained in G_π has the form

$$\mathbf{X}(\tau) = \mathbf{X}^0(\tau; \mathbf{h}, \boldsymbol{\mu}) = (\mathbf{x}^0(\tau; \mathbf{h}, \boldsymbol{\mu}), \mathbf{y}^0(\tau; \mathbf{h}, \boldsymbol{\mu})), \tag{5}$$

where the functions $\mathbf{x}^0$ and $\mathbf{y}^0$ are determined by the solution (4) of Eq. (3) with the initial condition of the form

$$(\boldsymbol{\alpha}_0, \boldsymbol{\beta}_0) = (\boldsymbol{\alpha}(\mathbf{h}, \boldsymbol{\mu}), \boldsymbol{\beta}(\mathbf{h}, \boldsymbol{\mu})) = \mathbf{\Lambda}(\mathbf{h}, \boldsymbol{\mu}).$$

The function $\mathbf{X}^0$ depends on the parameters $\mathbf{h}$ and $\boldsymbol{\mu}$ in a one-to-one and smooth manner.

Equations of the asymptotic quantities

Let $M_\mathbf{r} \subset G_\pi$, $\mathbf{r} = (\mathbf{h}, \boldsymbol{\mu}) \in G_p$ be the trajectory of Eq. (3) and $\mathbf{X}^0$ — the parametrization (5) of $M_\mathbf{r}$.

Let φ: $G \to R$ be a continuous function. Let us denote

$$\bar{\varphi}(\mathbf{r}) = \frac{1}{T(\boldsymbol{\mu})} \int_0^{T(\boldsymbol{\mu})} \varphi(\mathbf{X}^0(\tau; \mathbf{h}, \boldsymbol{\mu}), \boldsymbol{\mu}) d\tau.$$

The function

$$\bar{\varphi}: \mathbf{r} \in G_p \to \bar{\varphi}(\mathbf{r}) \in R$$

will be called the averaged function. The function $\bar{\varphi}$ is smooth if the function φ is smooth ([2]).

Let us denote $\mathbf{X} = (\mathbf{x}, \mathbf{y}) \in R^6$, $\mathbf{Y} = (\mathbf{X}, \boldsymbol{\mu}) \in R^{13}$ and let us consider the equation of the motion of the defect in the form (2) with the initial condition

$$\begin{gathered} \mathbf{Y}(0) = \mathbf{Y}_0 = (\mathbf{X}_0, \boldsymbol{\mu}_0), \\ \mathbf{X}_0 = (\boldsymbol{\alpha}_0, \boldsymbol{\beta}_0), \quad \boldsymbol{\mu}_0 = (0, \boldsymbol{\xi}_0, \mathbf{v}_0). \end{gathered} \tag{2'}$$

Let

$$\mathbf{Y}(\tau; \sqrt{\varepsilon}) = (\mathbf{X}(\tau; \sqrt{\varepsilon}), \boldsymbol{\mu}(\tau; \sqrt{\varepsilon})),$$

$$\mathbf{X}(\tau; \sqrt{\varepsilon}) = (\mathbf{x}(\tau; \sqrt{\varepsilon}), \mathbf{y}(\tau; \sqrt{\varepsilon}))$$

be the solution of the initial problem (2) and (2') determined on the interval $I = \langle 0, b/\sqrt{\varepsilon_0} \rangle$ so that

$$\bigwedge_{\tau \in I} \mathbf{Y}(\tau; \sqrt{\varepsilon}) \in G, 0 \leqslant \varepsilon \leqslant \varepsilon_0.$$

In accordance with the definition of the parameter $\mathbf{h}$ we have the identity $\mathbf{h} = \mathbf{H}(\mathbf{X}^0(\tau; \mathbf{h}, \boldsymbol{\mu}), \boldsymbol{\mu})$. Let us introduce the mapping $\mathbf{h}$

$$\mathbf{h}: I \times \langle 0, \sqrt{\varepsilon_0} \rangle \to R^5$$

making use of the rule

$$\mathbf{h}(\tau; \sqrt{\varepsilon}) = \mathbf{H}(\mathbf{x}(\tau; \sqrt{\varepsilon}), \boldsymbol{\mu}(\tau; \sqrt{\varepsilon}))$$

and let us designate

$$\mathbf{H}(\mathbf{Y}) = \mathbf{H}(\mathbf{X}, \boldsymbol{\mu}) = \mathbf{H}(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}) \in R^5, \quad \boldsymbol{\varphi}(\mathbf{Y}) = \boldsymbol{\varphi}(\mathbf{X}, \boldsymbol{\mu}) = \boldsymbol{\varphi}(\mathbf{x}, \boldsymbol{\mu}) \in R^7,$$

$$\mathbf{F}(\mathbf{Y}; \sqrt{\varepsilon}) = \mathbf{F}(\mathbf{X}, \boldsymbol{\mu}; \sqrt{\varepsilon}) = \mathbf{F}(\mathbf{x}, \mathbf{y}, \boldsymbol{\mu}; \sqrt{\varepsilon}) \in R^3,$$

$$R(\mathbf{Y}; \sqrt{\varepsilon}) = (\mathbf{0}, \mathbf{F}(\mathbf{Y}; \sqrt{\varepsilon}), \boldsymbol{\varphi}(\mathbf{Y})) \in R^{13}, \quad \mathbf{R}_0(\mathbf{Y}) = \mathbf{R}(\mathbf{Y}; 0).$$

Since

$$\frac{d\mathbf{h}}{d\tau}(\tau; \sqrt{\varepsilon}) = \sqrt{\varepsilon}(\nabla\mathbf{H} \cdot \mathbf{R})(\mathbf{Y}(\tau; \sqrt{\varepsilon})),$$

$$\frac{d\boldsymbol{\mu}}{d\tau}(\tau; \sqrt{\varepsilon}) = \sqrt{\varepsilon}\boldsymbol{\varphi}(\mathbf{Y}(\tau; \sqrt{\varepsilon})),$$

the equations of the zero-approximation for $\mathbf{h}$ and $\boldsymbol{\mu}$ have the form of the "averaged equations" ([2]):

$$\frac{d\bar{\mathbf{h}}}{d\tau} = \sqrt{\varepsilon}\overline{(\nabla\mathbf{H} \cdot \mathbf{R}_0)}(\bar{\mathbf{r}}), \tag{6}$$

$$\frac{d\bar{\boldsymbol{\mu}}}{d\tau} = \sqrt{\varepsilon}\bar{\boldsymbol{\varphi}}(\bar{\mathbf{r}}),$$

where $^- = (\bar{\mathbf{h}}, \bar{\boldsymbol{\mu}})$, $\tau \in \langle 0, b/\sqrt{\varepsilon_0} \rangle$, $\nabla = \left(\frac{\partial}{\partial Y_1}, \ldots, \frac{\partial}{\partial Y_{13}}\right)$. We accept

$$\bar{\mathbf{h}}(0) = \mathbf{h}_0 = \mathbf{h}(0) = \mathbf{H}(\mathbf{X}_0, \boldsymbol{\mu}_0), \tag{6'}$$

$$\bar{\boldsymbol{\mu}}(0) = \boldsymbol{\mu}_0 = \boldsymbol{\mu}(0) = (0, \boldsymbol{\xi}_0, \mathbf{v}_0)$$

as the initial condition for the solution of Eq. (6). It may easily be shown that

$$\bar{\boldsymbol{\varphi}}(\mathbf{r}) = (1, \mathbf{v}, \mathbf{f}(\boldsymbol{\mu}))$$

for $\mathbf{r} = (\mathbf{h}, \boldsymbol{\mu})$, $\boldsymbol{\mu} = (t, \boldsymbol{\xi}, \mathbf{v})$. Then the second equation in the set (6) is independent of the choice of the system of the first integrals $\mathbf{H}$ and it is possible to separate it in the form of the equation

$$\frac{d^2\bar{\boldsymbol{\xi}}}{dt^2} = \mathbf{f}(t, \bar{\boldsymbol{\xi}}, \dot{\bar{\boldsymbol{\xi}}}), \quad t \in \langle 0, b \rangle, \tag{7}$$

$$\bar{\boldsymbol{\xi}}(0) = \boldsymbol{\xi}_0, \quad \dot{\bar{\boldsymbol{\xi}}}(0) = \mathbf{v}_0.$$

Let $\bar{\boldsymbol{\xi}}(t) = \bar{\boldsymbol{\xi}}(t; \boldsymbol{\xi}_0, \mathbf{v}_0)$ be the solution of the initial problem (7) and let $\boldsymbol{\xi}(t; \sqrt{\varepsilon}) = \boldsymbol{\xi}(t; \mathbf{X}_0, \boldsymbol{\mu}_0, \sqrt{\varepsilon})$ be the solution of the equation of the concentrated defect in the form (1) satysfying the initial conditions

$$(1') \qquad \begin{aligned} &\boldsymbol{\xi}(0; \sqrt{\varepsilon}) = \boldsymbol{\xi}_0, \qquad \dot{\boldsymbol{\xi}}(0; \sqrt{\varepsilon}) = \mathbf{v}_0, \\ &\ddot{\boldsymbol{\xi}}(0; \sqrt{\varepsilon}) = \boldsymbol{\alpha}_0, \quad \sqrt{\varepsilon}\,\dddot{\boldsymbol{\xi}}(0; \sqrt{\varepsilon}) = \boldsymbol{\beta}_0. \end{aligned}$$

With these notations we have $\boldsymbol{\mu}(t; \sqrt{\varepsilon}) = (t, \boldsymbol{\xi}(t; \sqrt{\varepsilon}), \dot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon}))$, $\bar{\boldsymbol{\mu}}(t) = (t, \bar{\boldsymbol{\xi}}(t), \dot{\bar{\boldsymbol{\xi}}}(t))$ and $\boldsymbol{\mu}(0; \sqrt{\varepsilon}) = \bar{\boldsymbol{\mu}}(0)$. Using the conclusions of the paper [2] (and on the basis of the formula (4)), we can formulate the following theorem about the asymptotics of Eq. (1):

THEOREM

$$(*) \qquad \lim_{\varepsilon\to 0} ||\boldsymbol{\mu}(t; \sqrt{\varepsilon}) - \bar{\boldsymbol{\mu}}(t)|| = 0$$

tends uniformly towards $t \in \langle 0, b\rangle$ *and* $(\mathbf{X}_0, \boldsymbol{\mu}_0) \in G$.

Additionally we have:

a. The function $\boldsymbol{\mu}(t; \sqrt{\varepsilon})$ differs from the function $\bar{\boldsymbol{\mu}}(t)$ by the term of the order of the small parameter $\sqrt{\varepsilon}$, i.e.:

$$\bigvee_{\varepsilon_0 > 0} \bigwedge_{0 < \varepsilon \leqslant \varepsilon_0} ||\boldsymbol{\mu}(t; \sqrt{\varepsilon}) - \bar{\boldsymbol{\mu}}(t)|| = O(\sqrt{\varepsilon}).$$

b. The function $\ddot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon})$ is the function oscillating around the points $\hat{\mathbf{x}}(t) = \mathbf{f}(t, \bar{\boldsymbol{\xi}}(t), \dot{\bar{\boldsymbol{\xi}}}(t))$, but on the whole

$$\lim_{\varepsilon\to 0} ||\ddot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon}) - \ddot{\bar{\boldsymbol{\xi}}}(t)|| \neq 0.$$

c. The function $\dddot{\boldsymbol{\xi}}(\mathrm{t}; \sqrt{\varepsilon})$ is a function oscillating around the point $\hat{\mathbf{y}} = \mathbf{0}$, but on the whole,

$$\lim_{\varepsilon\to 0} ||\dddot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon})|| = \infty.$$

We see that the solution $\bar{\boldsymbol{\xi}}(t)$ of Eq. (7) and the function $\dot{\bar{\boldsymbol{\xi}}}(t)$ are the zero-approximation of the functions $\boldsymbol{\xi}(t; \sqrt{\varepsilon})$ and $\dot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon})$, but the functions $\ddot{\bar{\boldsymbol{\xi}}}(t)$ and $\dddot{\bar{\boldsymbol{\xi}}}(t)$ — are not the same approximation of the functions $\ddot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon})$ and $\dddot{\boldsymbol{\xi}}(t; \sqrt{\varepsilon})$.

The first equation in (6) cannot be solved on the whole without knowledge of the forms of all the first integrals of Eq. (3). If, however, we limit ourselves to the case in which

$$\boldsymbol{\beta}_0 = \dddot{\boldsymbol{\xi}}(0; \sqrt{\varepsilon}) = \mathbf{0},$$

then, taking $H_1 = H$, $h_1 = h$ and $\bar{h} = E$, we get the equation of the zero-approximation of the energy H:

$$(8) \qquad \begin{aligned} &\frac{dE}{dt} = \lambda(t)E + \theta(t)||\mathbf{f}(t, \bar{\boldsymbol{\xi}}(t), \dot{\bar{\boldsymbol{\xi}}}(t)||^2, \\ &E(0) = H(\boldsymbol{\alpha}_0, \mathbf{0}, \boldsymbol{\mu}_0), \end{aligned}$$

where

$$\lambda(t) = -4(\dot{U}U^{-1})(t) + \frac{1}{2}\Delta^1 \dot{a}(t) + (\dot{M}M^{-1})(t),$$

$$\theta(t) = -(\dot{M}M^{-1}a)(t), \quad \Delta^1 \text{ — a constant ([1]).}$$

About the functions $E(t)$ and $h(t;\sqrt{\varepsilon})$, we can make an analogous assertion as in the case of the functions $\overline{\mu}(t)$ and $\mu(t;\sqrt{\varepsilon})$. (Theorem: (*) and a). If the defect is constant (i.e. $m(t) = =$ const., $U(t) =$ const.) then Eq. (8) is reduced to the formula

$$\frac{dE}{dt} = 0.$$

This formula means that for a constant defect the function $h(t;\sqrt{\varepsilon}) = H(\ddot{\xi}(t;\sqrt{\varepsilon}), \sqrt{\varepsilon}\,\dot{\ddot{\xi}}(t;\sqrt{\varepsilon}), \mu(t;\sqrt{\varepsilon}))$ is an adiabatic invariant of the motion $\xi(t;\sqrt{\varepsilon})$ of the defect:

$$h(t;\sqrt{\varepsilon}) = H(\alpha_0, 0, \mu_0) + O(\sqrt{\varepsilon}).$$

References

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