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Diffraction of acoustic wave at a thin wing

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ACOUSTIC waves fall on a foil moving at subsonic speed in an infinite volume of a perfect gas. The foil is made of a thin sheet with a small curvature and set at a low angle of attack. The solution is obtained in a closed form, in quadrature, when the wing moves at subsonic speed according to an arbitrarily given law, and when the acoustic wave falls on the foil at an arbitrary angle. The solution is expressed in the form of a recurrent formula which accounts for the effect of any number of diffraction waves which succesively arise at the foil edge.

Fale askustyczne padają na skrzydło poruszające się z prędkością poddźwiękową w nieskończonej objętości gazu doskonałego. Skrzydło wykonane jest z cienkiej blachy o małej krzywiźnie i ustawione pod małym kątem natarcia. Rozwiązanie otrzymano w postaci zamkniętej, w kwadraturach, gdy skrzydło porusza się z prędkością poddźwiękową według dowolnego prawa i gdy fala akustyczna pada na profil pod dowolnym kątem. Rozwiązanie przedstawiono w postaci wzoru rekurencyjnego, który uwzględnia wpływ dowolnej ilości fal załamanych pojawiających się kolejno na krawędzi skrzydła.

Акустическая волна падает на крыло, движущееся с дозвуковой скоростью в неограниченном объеме идеального газа. Крыло предполагается тонким, слабоизогнутым и наклоненным под малым углом атаки. Решение задачи получено в замкнутом виде, в квадратурах, когда крыло движется с дозвуковой скоростью по произвольно заданному закону и когда акустическая волна падает на крыло под произвольным углом. Решение представлено в виде рекуррентных формул, учитывающих влияние любого числа дифракционных волн, последовательно возникающих на кромках крыла.

Introduction

AN ACOUSTIC wave propagates in an unlimited volume of an ideally compressible medium. The wave is incident on a wing moving at subsonic velocity u. The wing is assumed to be thin, slightly curved and inclined at a small angle of attack. Besides, the wing may perform some small arbitrary additional motions in the course of which the wing surface may slightly deform. The wing chord equals d (Fig. 1).

We shall deal with plane-parallel irrotational flows of gas in the fixed coordinate axes Oxz. The velocity potential Φ for the gas motion is in the form $\Phi = \varphi_0+\varphi_1+\varphi$, where the potential φ_0 is the solution of the problem of the given wing flow-past (with regard for small additional motions) when this wing moves by itself, i.e. a waveless medium; $\varphi_1(x, z, t)$ is an arbitrary velocity potential of the incident wave, φ_1 defines the parameters of gas in the incident wave; $\varphi(x, z, t)$ is the unknown velocity potential depending on the backward and diffracted waves [1, 2].

The sum of the potentials $\varphi_1+\varphi$ defines the velocity field excited by the acoustic wave incident on a plane plate of width d, when the plate moves at velocity u at a zero angle of attack (Fig. 2).

Diffraction waves spreading at the velocity of sound c in the gas are successively generated at the foil edges. The fronts of these waves divide the plane (xz) into regions with different analytical solutions of the problem for the function φ and its derivatives.

A space defined by the variables x, z, t is introduced. The velocity potential φ is expressed as a double integral of some elementary solutions of the wave equation distributed

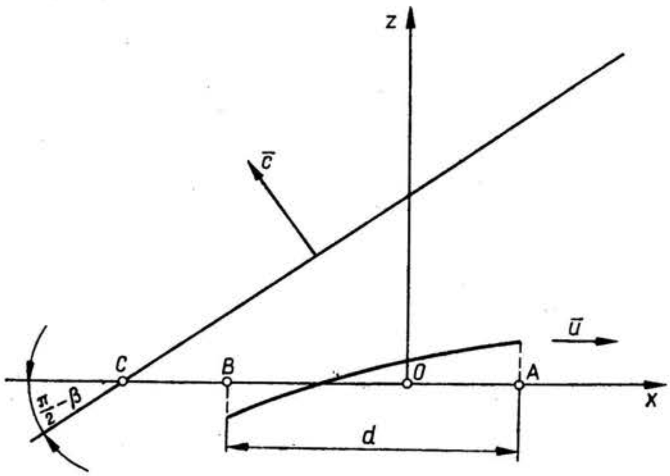

Fig. 1.

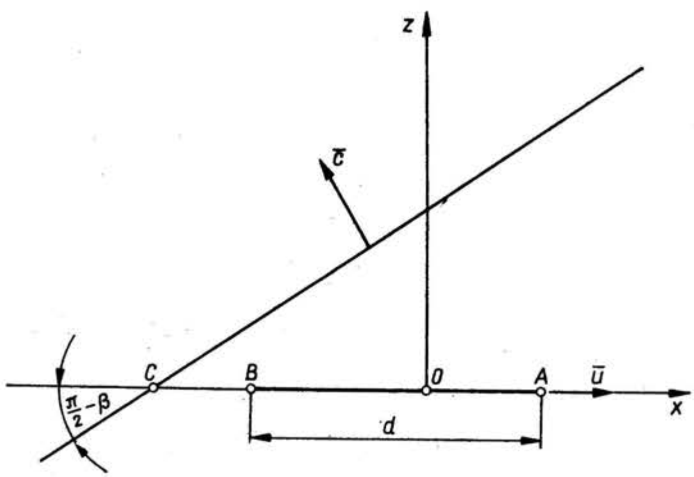

Fig. 2.

in the plane (xt). The domain of integration is a part of the plane (xt) inside the characteristic cone with its vertex at a point where there is a potential. The derivative φ_z is contained in the integrand. It is an unknown quantily outside the foil at any instant of time [3, 4].

The family of characteristic cones of the wave equation with their vertices at points in the plane (xt) with the coordinates x_i and t_i $(i = 0, 1, 2, 3, \dots)$ divide the space (xzt), in particular, the plane (xt), into regions with different analytical expressions for the solutions of the problem (Fig. 3). The points $x_0, x_1, x_2, x_3, \ldots$ on the axis Ox are the centers for the diffraction waves generated at the foil edges at the instants $t_0, t_1, t_2, t_3, \ldots$, respectively.

Using the boundary conditions of the problem we obtain a Volterra double integral equation for each characteristic region of the plane (xt) outside the foil. The unknown

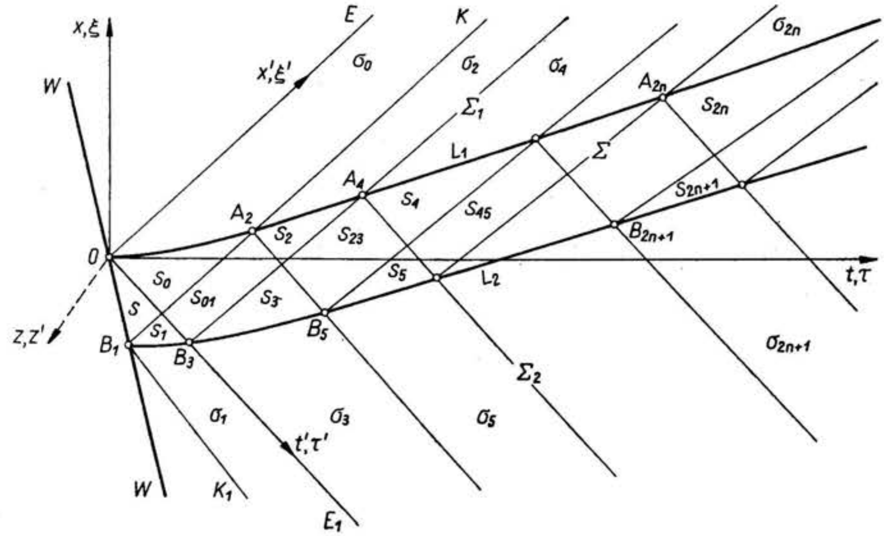

FIG. 3.

function φ_z satisfies these integral equations. The solution of each of these equations is found by moving in the direction of the time axis Ot and consecutively passing from one region into the other [5].

1. The initial-boundary value problem

The initial-boundary value problem which defines the function φ is as follows. In the half-plane $z \geqslant 0$ find a function $\varphi(x, z, t)$ satisfying the wave equation

$$c^2(\varphi_{xx}+\varphi_{zz})-\varphi_{tt} = 0 \tag{1.1}$$

and the conditions at the Ox-axis: before the plate

$$\varphi = 0, \tag{1.2}$$

behind the wave front at the plate

$$\varphi_z = -\varphi_{1z}(x, 0, t) = \mathscr{A}(x, t_1) \tag{1.3}$$

and behind the plate

$$\varphi_t = 0. \tag{1.4}$$

Besides, at each time the Chaplygin-Zhukovsky condition should be met at the plate boundary corresponding to the rear edge of the wing.

In the half-plate $z < 0$ the function φ is obtained through the condition

$$\varphi(x, -z, t) = -\varphi(x, z, t). \tag{1.5}$$

2. The solution of the problem

Consider a general case of the wing motion with subsonic velocity. Assume that the wing (plate) motion law is given in the form

$$x = F(t),$$

where F is an arbitrary continuous time function, $F'(t) = u$.

Introduce a space of variables x, z and t. In the plate (xt) define the domains Σ, Σ_1 and Σ_2, for which the conditions (1.3), (1.2) and (1.4), respectively, are specified. The domain Σ is limited by the curves L_1, L_2 and W. The curves L_1 and L_2 describe the motion laws for the points A and B — the plate's boundaries. The curve W describes the motion law for the point C, the point of intersection of the incident wave and the Ox-axis. The points O and B_1 in Fig. 3 correspond to the boundaries A and B of the plate at the times of the wave front propagating through them. The straight lines OE, OE_1 and B_1K, B_1K_1 are the lines of intersection of the plane (xt) and the characteristic cones of Eq. (1.1). The curve L_1 and the straight line OE are the boundaries for the domain Σ_1, while L_2 and B_1K_1 are those for the domain Σ_2 (Fig. 3).

The straight lines OE_1 and B_1K are repeatedly reflected from the curves L_1 and L_2. Along with the coordinates of the points O and B_1 note the coordinates of the points of reflection: $O(x_0 = 0, t_0 = 0)$, $A_2(x_2, t_2)$, $A_4(x_4, t_4)$, ..., $A_{2n}(x_{2n}, t_{2n})$, ..., $B_1(x_1, t_1)$, $B_3(x_3, t_3)$, $B_5(x_5, t_5)$, ..., $B_{2n+1}(x_{2n+1}, t_{2n+1})$, ...

The points of the Ox-axis with the coordinates $x_0, x_2, x_4, \ldots, x_{2n}, \ldots$ are the centres of tube diffraction waves arising at the boundary A of the plate at the times $t_0, t_2, t_4, \ldots, t_{2n}, \ldots$, respectively, and propagating in the gas at the sonic velocity c.

The points of the Ox-axis with the coordinates $x_1, x_3, x_5, \ldots, x_{2n+1}, \ldots$ are the centres of similar waves arising at the boundary B of the plate at the times $t_1, t_3, t_5, \ldots, t_{2n+1}, \ldots$, respectively.

The characteristic cones with vertices located at the points $O, A_2, A_4, \ldots, A_{2n}, \ldots$, $B_1, B_3, B_5, \ldots, B_{2n+1}, \ldots$ divide the space (xzt) into domains with different analytical solutions of the problem. In particular, the plane domain Σ_1 is divided into the domains $\sigma_0, \sigma_2, \sigma_4, \ldots \sigma_{2n}, \ldots$, the domain Σ_2 into $\sigma_1, \sigma_3, \sigma_5, \ldots \sigma_{2n+1}, \ldots$ and the domain Σ into $s, s_0, s_1, s_{01}, s_2, s_3, s_{23}, \ldots, s_{2n}, s_{2n+1}, \ldots$.

Let us take the solution of Eq. (1.1) in the form of a formula linking the function φ in an arbitrary point of the plate (xz) with a derivative φ_z at the Ox-axis for any time

$$\varphi^*(x', z', t') = -\frac{1}{2\pi} \iint \frac{\varphi^*_{z'}(\xi', 0, \tau')}{\sqrt{(x'-\xi')(t'-\tau')-z'^2}}\, d\xi' d\tau'.$$

The characteristic coordinates x', t' and z' are linked with the coordinates x, t and z by the relations $c(x'-t') = 2x$, $x'+t' = 2t$, $cz' = z$. The functions φ and $c\varphi_z$ in the new variables are denoted as φ^* and $\varphi^*_{z'}$.

Using the boundary condition (1.3) present the potential φ for $z \geqslant 0$ in the form

$$\varphi^*(x', z', t') = -\frac{1}{2\pi}\iint\limits_{\sigma} \mathscr{A}^*(\xi', \tau')\frac{d\xi' d\tau'}{r} - \frac{1}{2\pi}\iint\limits_{\sigma_1'} \varphi^*_{z'}(\xi', 0, \tau')\frac{d\xi' d\tau'}{r} - \frac{1}{2\pi}\iint\limits_{\sigma_2'} \varphi^*_{z'}(\xi', 0, \tau')\frac{d\xi' d\tau'}{r},$$

$$r = \sqrt{(x'-\xi')(t'-\tau')-z'^2}. \tag{2.1}$$

The domains σ, σ_1' and σ_2' are parts of the domains Σ, Σ_1 and Σ_2, respectively, cut off by the characteristic cone with a vertex in the point with the coordinates x', z' and t' directed towards decreasing time values. The function $c\mathscr{A}$ in the new variables is denoted as $\mathscr{A}^*$.

Find the derivative $\varphi^*_{z'}$ in the domains Σ_1 and Σ_2 through integral equations. Denote the derivative $\varphi^*_{z'}$ in the domains $\sigma_{2n} \subset \Sigma_1$ and $\sigma_{2n+1} \subset \Sigma_2$ as θ_{2n} and ϑ_{2n+1} $(n = 0, 1, 2, 3, \ldots)$, respectively.

Assuming the coordinate $z' = 0$ in Eq. (2.1) and using the condition (1.2) obtain an integral equation for the function $\theta_0(\xi', \tau') = \varphi^*_{z'}(\xi', 0, \tau')$ in the domain σ_0:

$$\int\limits_0^{\tau'}\int\limits_{\mathscr{F}_1(\tau'')}^{\xi'} \theta_0(\xi'', \tau'')\frac{d\xi'' d\tau''}{\sqrt{(\xi'-\xi'')(\tau'-\tau'')}} + \int\limits_0^{\tau'}\int\limits_{\omega(\tau'')}^{\mathscr{F}_1(\tau'')} \mathscr{A}^*(\xi'', \tau'')\frac{d\xi' d\tau''}{\sqrt{(\xi'-\xi'')(\tau'-\tau'')}} = 0,$$

where the function $\xi'' = \mathscr{F}_1(\tau'')$ is the equation for the curve L_1 and the function $\xi'' = \omega(\tau'')$, that for the curve W. Let us present the equation in the form

$$\int\limits_0^{\tau'}\frac{1}{\sqrt{\tau'-\tau''}}\left\{\int\limits_{\mathscr{F}_1(\tau'')}^{\xi'} \theta_0(\xi'', \tau'')\frac{d\xi''}{\sqrt{\xi'-\xi''}} + \int\limits_{\omega(\tau'')}^{\mathscr{F}_1(\tau'')} \mathscr{A}(\xi'', \tau'')\frac{d\xi''}{\sqrt{\xi'-\xi''}}\right\}d\tau'' = 0.$$

This is an Abel integral equation with the right-hand part identically equal to zero. Therefore

$$\int\limits_{\mathscr{F}_1(\tau')}^{\xi'} \theta_0(\xi'', \tau')\frac{d\xi''}{\sqrt{\xi'-\xi''}} = f_0(\xi', \tau'), \tag{2.2}$$

where

$$f_0(\xi', \tau') = -\int\limits_{\omega(\tau')}^{\mathscr{F}_1(\tau')} \mathscr{A}^*(\xi'', \tau')\frac{d\xi''}{\sqrt{\xi'-\xi''}}. \tag{2.3}$$

In a similar way obtain integral equations with the functions $\theta_2, \theta_4, \ldots, \theta_{2n}, \ldots$ satisfying them

$$\int\limits_{\mathscr{F}_1(\tau')}^{\xi'} \theta_{2n}(\xi'', \tau')\frac{d\xi''}{\sqrt{\xi'-\xi''}} = f_{2n}(\xi', \tau'), \tag{2.4}$$

where for $n \geqslant 1$

$$(2.5)\quad f_{2n}(\xi', \tau') = -\int\limits_{\mathscr{F}_2(\tau')}^{\mathscr{F}_1(\tau')} \mathscr{A}^*(\xi'', \tau')\frac{d\xi''}{\sqrt{\xi'-\xi''}} - \sum_{l=0}^{n-2}\int\limits_{x'_{2l+1}}^{x'_{2l+3}} \vartheta_{2l+1}(\xi'', \tau')\frac{d\xi''}{\sqrt{\xi'-\xi''}} - \int\limits_{x'_{2n+1}}^{\mathscr{F}_2(\tau')} \vartheta_{2n-1}(\xi'', \tau')\frac{d\xi''}{\sqrt{\xi'-\xi''}}.$$

The function $\xi' = \mathscr{F}_2(\tau')$ is an equation for the curve L_2. The limits of integration x'_1, $x'_3, \dots, x'_{2n-1}$ are the coordinates of the points $B_1, B_3, \dots, B_{2n-1}$ in terms of the new variables.

Assuming that in Eq. (2.1) the coordinate z' equals zero and using the condition (1.4) which takes the form $\varphi^*_{x'}+\varphi^*_{t'} = 0$ for the characteristic variables, obtain the equation of the function $\vartheta_1(\xi', \tau') = \varphi^*_{z'}(\xi', 0, \tau')$ in the domain σ_1:

$$(2.6)\quad \frac{\partial}{\partial \xi'}\int\limits_{x'_1}^{\xi'}\int\limits_{\mathscr{F}_2^0(\xi'')}^{\tau'} \vartheta_1(\xi'', \tau'')\frac{d\tau''d\xi''}{\sqrt{(\xi'-\xi'')(\tau'-\tau'')}} + \frac{\partial}{\partial \tau'}\int\limits_{x'_1}^{\xi'}\int\limits_{\mathscr{F}_2^0(\xi'')}^{\tau'} \vartheta_1(\xi'', \tau'')\frac{d\tau''d\xi''}{\sqrt{(\xi'-\xi'')(\tau'-\tau'')}}$$
$$+ \frac{\partial}{\partial \xi'}\int\limits_{x'_1}^{\xi'}\int\limits_{\omega^0(\xi'')}^{\mathscr{F}_2^0(\xi'')} \mathscr{A}^*(\xi'', \tau'')\frac{d\tau''d\xi''}{\sqrt{(\xi'-\xi'')(\tau'-\tau'')}} + \frac{\partial}{\partial \tau'}\int\limits_{x'_1}^{\xi'}\int\limits_{\omega^0(\xi'')}^{\mathscr{F}_2^0(\xi'')} \mathscr{A}^*(\xi'', \tau'')\frac{d\tau''d\xi''}{\sqrt{(\xi'-\xi'')(\tau'-\tau'')}} = 0,$$

where the functions $\mathscr{F}_2^0$ and ω^0 are inversions of the functions $\mathscr{F}_2$ and ω.

In the first and the third terms of Eq. (2.6) let us perform integration by parts over the variable ξ'', then perform the differentiation with respect to the parameter ξ' and present equation (2.6) in the form

$$\int\limits_{x'_1}^{\xi'}\frac{1}{\sqrt{\xi'-\xi''}}\Bigg\{\frac{\partial}{\partial \xi''}\int\limits_{\mathscr{F}_2^0(\xi'')}^{\tau'} \vartheta_1(\xi'', \tau')\frac{d\tau''}{\sqrt{\tau'-\tau''}} + \frac{\partial}{\partial \tau'}\int\limits_{\mathscr{F}_2^0(\xi'')}^{\tau'} \vartheta_1(\xi'', \tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}} + \frac{\partial}{\partial \xi''}\int\limits_{\omega^0(\xi'')}^{\mathscr{F}_2^0(\xi'')} \mathscr{A}^*(\xi'', \tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}$$
$$+ \frac{\partial}{\partial \tau'}\int\limits_{\omega^0(\xi'')}^{\mathscr{F}_2^0(\xi'')} \mathscr{A}^*(\xi'', \tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}\Bigg\}d\xi'' = 0.$$

This is an Abel integral equation with the right-hand part identically equal to zero. Therefore,

$$(2.7)\quad \frac{\partial}{\partial \xi'}\int\limits_{\mathscr{F}_2^0(\xi')}^{\tau'} \vartheta_1(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}+\frac{\partial}{\partial \tau'}\int\limits_{\mathscr{F}_2^0(\xi')}^{\tau'} \vartheta_1(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}$$
$$+\frac{\partial}{\partial \xi'}\int\limits_{\omega^0(\xi')}^{\mathscr{F}_2^0(\xi')} \mathscr{A}^*(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}+\frac{\partial}{\partial \tau'}\int\limits_{\omega^0(\xi')}^{\mathscr{F}_2^0(\xi')} \mathscr{A}^*(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}=0.$$

Differentiating with respect to the parameters ξ' and τ' and using the constraint on the curve L_2,

$$(2.8)\qquad \vartheta_{2n+1}[\xi',\mathscr{F}_2^0(\xi')]=\mathscr{A}^*[\xi',\mathscr{F}_2^0(\xi')],\qquad n=0,1,2,3,\ldots,$$

which follows from the Chaplygin-Zhukovsky condition, Eq. (2.7) is reduced to an Abel integral equation for the function $\vartheta_{1\xi'}+\vartheta_{1\tau'}$

$$(2.9)\qquad \int\limits_{\mathscr{F}_2^0(\xi')}^{\tau'}[\vartheta_{1\xi'}(\xi',\tau'')+\vartheta_{1\tau'}(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}=f_1(\xi',\tau'),$$

where

$$(2.10)\quad f_1(\xi',\tau')$$
$$=-\frac{\mathscr{A}^*[\xi',\omega^0(\xi')]}{\sqrt{\tau'-\omega^0(\xi')}}\left\{1-\frac{d\omega^0(\xi')}{d\xi'}\right\}-\int\limits_{\omega^0(\xi')}^{\mathscr{F}_2^0(\xi')}[\mathscr{A}^*_{\xi'}(\xi',\tau'')+\mathscr{A}^*_{\tau''}(\xi',\tau'')]\frac{d\tau''}{\sqrt{\tau'-\tau''}}.$$

Similarly, obtain the equations with the functions $\vartheta_{3\xi'}+\vartheta_{3\tau'}$, $\vartheta_{5\xi'}+\vartheta_{5\tau'}$ $\ldots$, $\vartheta_{(2n+1)'\xi}+$ $+\vartheta_{(2n+1)\tau'}$, ... satisfying them:

$$(2.11)\qquad \int\limits_{\mathscr{F}_2^0(\xi')}^{\tau'}[\vartheta_{(2n+1)\xi'}(\xi',\tau'')+\vartheta_{(2n+1)\tau''}(\xi',\tau'')]\frac{d\tau''}{\sqrt{\tau'-\tau''}}=f_{2n+1}(\xi',\tau'),$$

where, for $n\geqslant 1$,

$$(2.12)\quad f_{2n+1}(\xi',\tau')$$
$$=-\int\limits_{\mathscr{F}_1^0(\xi')}^{\mathscr{F}_2^0(\xi')}[\mathscr{A}^*_{\xi'}(\xi',\tau'')+\mathscr{A}^*_{\tau''}(\xi',\tau'')]\frac{d\tau''}{\sqrt{\tau'-\tau''}}-\frac{\mathscr{A}^*[\xi',\mathscr{F}_1^0(\xi')]}{\sqrt{\tau'-\mathscr{F}_1^0(\xi')}}\left\{1-\frac{d\mathscr{F}_1^0(\xi')}{d\xi'}\right\}$$
$$-\sum_{i=0}^{n-2}\frac{\partial}{\partial\xi'}\int\limits_{t'_{2i}}^{t'_{2i+2}}\theta_{2i}(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}-\frac{\partial}{\partial\xi'}\int\limits_{t'_{2i+2}}^{\mathscr{F}_1^0(\xi')}\theta_{2n-2}(\xi',\tau')\frac{d\tau''}{\sqrt{\tau'-\tau''}}$$
$$-\sum_{i=0}^{n-2}\frac{\partial}{\partial\tau'}\int\limits_{t'_{2i}}^{t'_{2i+2}}\theta_{2i}(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}-\frac{\partial}{\partial\tau'}\int\limits_{t'_{2n-2}}^{\mathscr{F}_1^0(\xi')}\theta_{2n-2}(\xi',\tau'')\frac{d\tau''}{\sqrt{\tau'-\tau''}}.$$

The function $\mathscr{F}_1^0$ is the inversion of the function $\mathscr{F}_1$. The limits of integration $t_0, t_2, t_4, \ldots,$ t_{2n-2} are the coordinates of the points $O, A_2, A_4, \ldots, A_{2n}$.

In the expressions (2.5) and (2.12) the sum is found for $n\geqslant 2$.

Using the inversion formula of the Abel integral equation obtain the solutions of Eqs. (2.4) and (2.11) for the functions θ_{2n} and $\vartheta_{(2n+1)\xi'}+\vartheta_{(2n+1)\tau'}$ in the form [6, 7]

$$(2.13)\qquad \theta_{2n}(\xi',\tau')=\frac{1}{\pi}\frac{f_{2n}[\mathscr{F}_1(\tau'),\tau']}{\sqrt{\xi'-\mathscr{F}_1(\tau')}}+\frac{1}{\pi}\int\limits_{\mathscr{F}_1(\tau')}^{\xi'}\frac{\partial}{\partial\xi'}[f_{2n}(\xi'',\tau')]\frac{d\xi''}{\sqrt{\xi'-\xi''}}$$

and

$$(2.14)\qquad \vartheta_{(2n+1)\xi'}(\xi',\tau')+\vartheta_{(2n+1)\tau'}(\xi',\tau'')$$
$$=\frac{1}{\pi}\frac{f_{2n+1}[\xi',\mathscr{F}_2^0(\xi')]}{\sqrt{\tau'-\mathscr{F}_0^2(\xi')}}+\frac{1}{\pi}\int\limits_{\mathscr{F}_2^0(\xi')}^{\tau'}\frac{\partial}{\partial\tau''}[f_{2n+1}(\xi',\tau'')]\frac{d\tau''}{\sqrt{\tau'-\tau''}}.$$

Integrating Eq. (2.14) along the time axis Ot find the derivative $\varphi^*_{z'}=\vartheta_{2n+1}(\xi',\tau')$ in the domain σ_{2n+1} ($n=0,1,2,3,\ldots$). In that part of this domain where the inequality $\xi'\geqslant\tau'-t_1'+x_1'$ is true,

$$(2.15)\quad \vartheta_{2n+1}(\xi',\tau')=\mathscr{A}^*(x^0,t^0)+\int\limits_{t^0}^{\tau'}[\vartheta_{(2n+1)\xi}(\xi,\tau)+\vartheta_{(2n+1)\tau}(\xi,\tau)]_{\xi=\tau+\xi'-\tau'}\,d\tau,$$

where $\mathscr{A}^*(x^0,t^0)$ is the specified value of the derivative $\varphi^*_{z'}$ on the curve L_2. The coordinates x^0 and t^0 are defined from the system of equations

$$x^0-\mathscr{F}_2^0(x^0)=0,\qquad x^0-\xi'+\tau'-t^0=0.$$

In the part of the domain σ_{2n+1}, where the inequality $\xi'<\tau'-t_1'+x_1'$ is true,

$$(2.16)\qquad \vartheta_{2n+1}(\xi',\tau')=\int\limits_{t_1'}^{\tau'}[\vartheta_{(2n+1)\xi}(\xi,\tau)+\vartheta_{(2n+1)\tau}(\xi,\tau)]_{\xi=\tau+\xi'-\tau'}\,d\tau.$$

The functions f_{2n} and f_{2n+1} depend on the functions ϑ_{2k+1} and θ_{2k} for the indices $k\leqslant n-1$. The functions f_{2n} and f_{2n+1} with $n=0$ are obtained from Eqs. (2.3) and (2.10). If the functions θ_{2k} and ϑ_{2k+1} are already obtained for all the indices $k\leqslant n-1$, the right-hand parts of Eqs. (2.4) and (2.11) for the index n are known and one may compute the functions θ_{2n} and ϑ_{2n+1} through the formulae (2.13)–(2.16).

The solutions of Eqs. (2.13) and (2.14) with $n=0$ are of the form [5]

$$(2.17)\qquad \theta_0(\xi',\tau')=-\frac{1}{\pi}\frac{1}{\sqrt{\xi'-\mathscr{F}_1(\tau')}}\int\limits_{\omega(\tau')}^{\mathscr{F}_1(\tau')}\mathscr{A}^*(\xi'',\tau')\frac{\sqrt{\mathscr{F}_1(\xi')-\xi''}}{\xi'-\xi''}\,d\xi'',$$

$$(2.18)\qquad \vartheta_{1\xi'}(\xi',\tau')+\vartheta_{1\tau'}(\xi',\tau')$$
$$=-\frac{1}{\pi}\frac{1}{\sqrt{\tau'-\mathscr{F}_1^0(\xi')}}\int\limits_{\omega^0(\xi')}^{\mathscr{F}_2^0(\xi')}[\mathscr{A}^*_{\xi'}(\xi',\tau'')+\mathscr{A}^*_{\tau''}(\xi',\tau'')]\frac{\sqrt{\mathscr{F}_2^0(\xi')-\tau''}}{\tau'-\tau''}\,d\tau''$$
$$-\frac{1}{\pi}\mathscr{A}^*[\xi',\omega^0(\xi')]\frac{\sqrt{\mathscr{F}_2^0(\xi')-\omega^0(\xi')}}{\tau'-\omega^0(\xi')}\left\{1-\frac{d\omega^0(\xi')}{d\xi'}\right\}.$$

Thus to obtain the solutions for θ_0 and ϑ_1 the functions θ_2 and ϑ_3, θ_4 and $\vartheta_5,\ldots,\theta_{2n}$ and ϑ_{2n+1} are found successively for any value of n.

In our case of the wave front motions at constant velocity equal to the sonic velocity c, the curve W is a straight line described by the equation

$$\xi'=\omega(\tau')=-\tau'\mathrm{tg}^2\left(\frac{\beta}{2}\right).$$

By virtue of the formulae (2.17) and (2.18) the function θ_0 is defined when the angle β satisfies the inequality $0 \leqslant \beta < \pi$ and the function ϑ_1 is defined when $0 < \beta \leqslant \pi$.

In the extreme cases with $\beta = 0$ and $\beta = \pi$ this line coincides with OE_1 and OE, respectively. In the first case the domains Σ_1 and Σ_2 are divided into the domains σ_{4m}

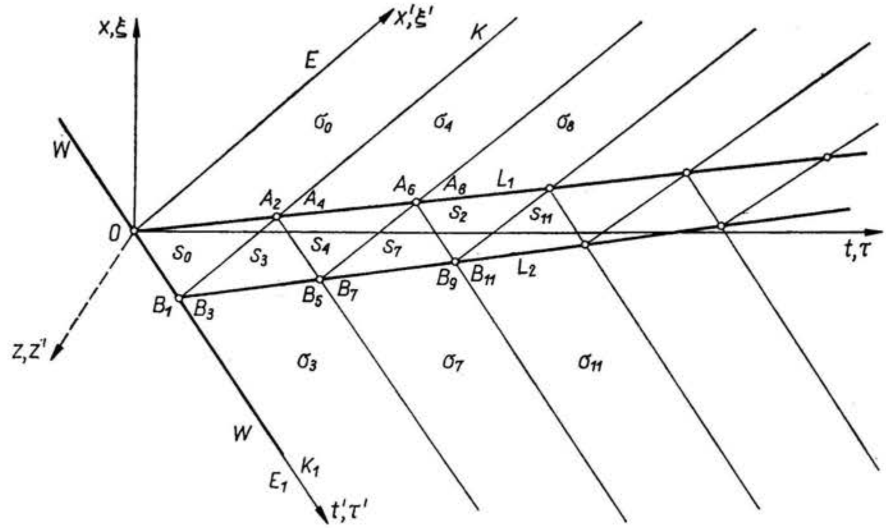

FIG. 4.

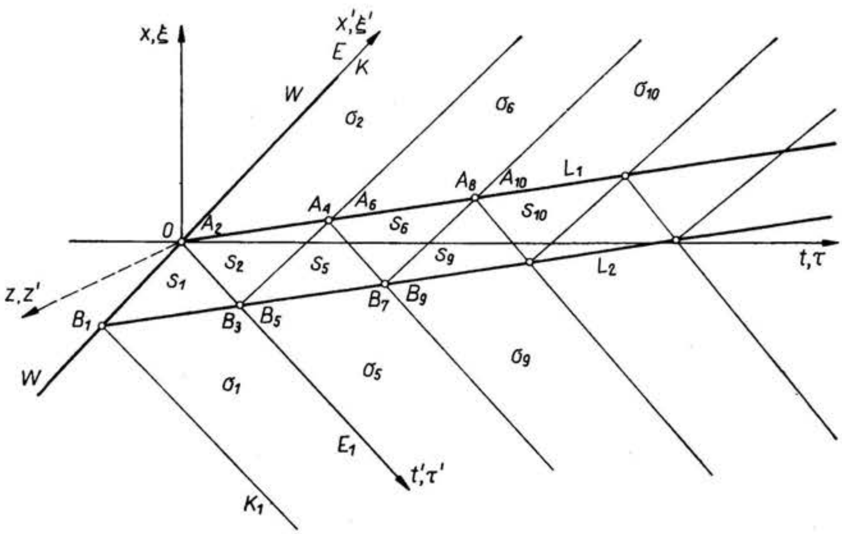

FIG. 5.

and σ_{4m+3} $(m = 0, 1, 2, 3, \ldots)$. Using the solution θ_0 the functions $\vartheta_3, \theta_4, \vartheta_7, \theta_{11}, \ldots, \theta_{4m}, \vartheta_{4m+3}, \ldots$ are obtained succesively (Fig. 4). In the second case the domains Σ_1 and Σ_2 are divided into σ_{4m+2} and σ_{4m+1}. Using the solution ϑ_1 the functions $\theta_2, \vartheta_5, \theta_6, \vartheta_9, \ldots, \theta_{4m+2}, \vartheta_{4m+1}$ are successively obtained for any value of m (Fig. 5).

With the derivatives φ_z known the velocity potential may be computed using the formula (2.1). The gas pressure field excited by the incident acoustic wave may be defined with the use of the Lagrange integral.

3. Concluding remarks

Depending on the velocity of the plate motion and on the angle between the plate and the incident wave, a certain version of the initial-boundary-value problem for the function φ corresponds to the diffraction problem. The plate may alternatively move at subsonic or supersonic velocity, occasionally cease motion, etc. Various versions of the initial-boundary-value problem differ by the form of the domains Σ, Σ_1 and Σ_2 and by their location with regard for the characteristic plates of the wave equations.

To solve any version, integral equations of the type (2.4) or (2.11) are constructed for each of the typical domains with a different analytical type of the derivative φ_z using the boundary conditions of the problem (1.2) or (1.4), respectively. Successively moving from one domain to another along the time axis Ot, the derivatives φ_z are found from the integral equations.

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