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Thermal exciting energy of atmospheric tides(*)

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A NUMERICAL approach is given to evaluate the contributions of different modes to the exciting thermal energy of atmospheric tides. The relative response of the different modes to the excitation is shown to be affected by their corresponding wavelengths; the latter are affected to a large extent the variation in temperature. The exciting energy and its upward propagation through layers of different transmissivity in the upper atmosphere are discussed.

Przedstawiono podejście numeryczne do określenia udziału poszczególnych postaci (modów) termicznej energii wzbudzenia pływów atmosferycznych. Pokazano, że zachowanie się poszczególnych modów zależy od odpowiedniej długości fali, która z kolei w silny sposób zależy od zmian temperatury. Zbadano energię wzbudzenia oraz jej przenoszenie w górę przez warstwy górnej atmosfery o różnej charakterystyce przewodzenia.

Представлен численный подход к оценке вклада различных мод в термическое возбуждение атмосферных приливных течений. Показывается, что на относительный ответ различных мод при возбуждении оказывают влияние их соответствующие длины волн обусловленные в значительной степени изменениями температуры. Обсуждается энергия возбуждения и ее распространение вверх сквозь верхние слои атмосферы с различной проницаемостью.

1. Introduction

ATMOSPHERIC tides may be looked upon as global internal gravity waves resulting from a particular excitation. By tides we mean oscillations in any field whose periods are integral fractions of either a lunar or solar day. The subject of internal waves in media, such as the atmosphere and oceans, is one of the oldest in fluid dynamics [1, 2] and, consequently, atmospheric tides have attracted the attention of many investigators for the last two centuries [3, 4, 5, 6, 7].

The continuous development of the theory of atmospheric tides [8, 9] was then to answer the question: why does the more strongly driven diurnal tide give rise to a diurnal surface pressure oscillation which is not very much greater than the semidiurnal? In explaining the relative smallness of the diurnal surface of the diurnal surface amplitude, recent investigations of the classical tidal theory showed that diurnal oscillations can have modes of a trapped nature [10, 11]. Although these theoretical predictions have successfully explained the major features, minor discrepancies appeared to be almost real when compared with the observations in the 30–60 km region [12].

The discrepancy, or part of it, would arise from one or more of the various simplified assumptions on which the theory is based. GRÖVES [13] has found that changes as large as 300% in the upward energy flux (of a particular mode) can result for various distributions

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of water vapour and temperature. It has been argued that a significant revision of the amplitudes and phases of the thermal excitation is required if observations are to be consistent with the classical theory [14]. When these assumptions are overcome, however, height and latitude dependences cannot be generally expressed in separable form and the analysis becomes considerably more complicated.

Therefore, the inverse problem of deriving sources of excitation, consistent with the observed values, seems to be more appropriate. More specifically, if the amplitudes and phases of the wind oscillations are known for a particular height interval and location, the problem is to investigate to what an extent the properties of the source can be described. This is the aim of the present work.

2. Theoretical and observed tidal wind fields

In the development of the tidal theory, the assumption is made such that the dependence of the tides on the height, z, co-latitude, θ, and longitude, ϕ, are separable. This leads to the decomposition of an oscillation S_l, of a given period, l^{-1} of a solar day, into wave types $S_{l,n}^s$, where n is the more number and s is the longitudinal wave number. The actual field will result from the summation over all n's [8]:

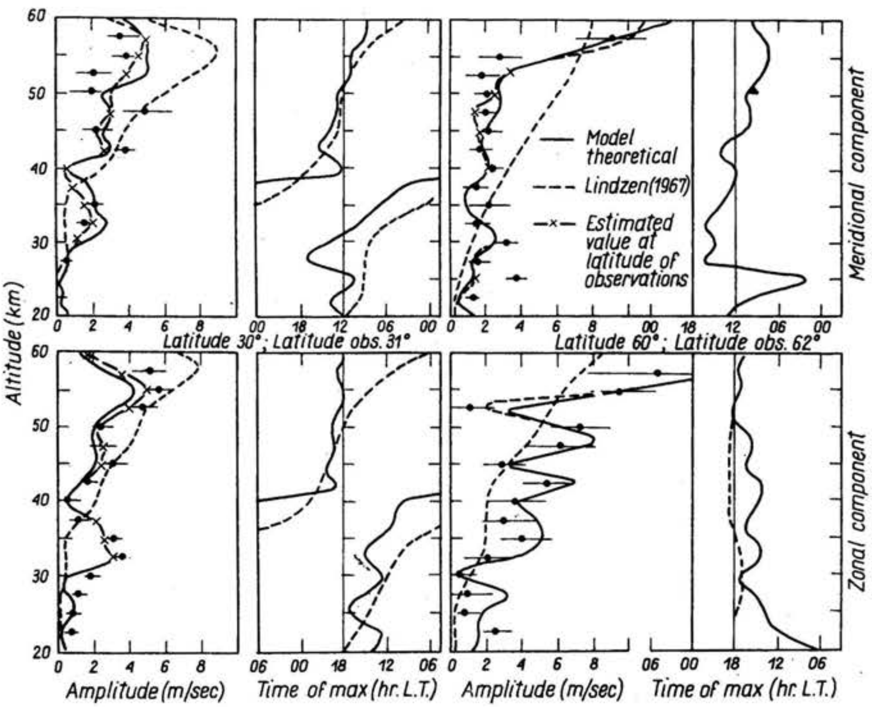

Fig. 1. Altitude distribution of the diurnal oscillation, after Makarious [14], of meridional and zonal wind components,

—•— computed amplitude, based on observations, at 2.5 km height interval, at 31° and 62° latitudes, × estimated values at 30° and 60° latitudes, ⌒ curve that joins the anticipated values, based on the superposition of 6 modes (Eq. 1), - - - theoretical curve [7], based on 5 modes, in an isothermal atmosphere.

$$Sv_l(z,\theta,t) = \sum_n Lv_{l,n}(\theta) Z_{l,n}(z) e^{i(\sigma t + s\phi)}, \tag{2.1}$$

where σ is the angular frequency of the oscillation. The latitude-dependences $Lv_{l,n}(\theta)$ of the horizontal wind components are expressed in terms of the Hough functions $\Theta_n(\theta)$, the latter being the solutions of Laplace's equation:

$$\frac{d}{d\mu}\left[\frac{(1-\mu^2)}{(f^2-\mu^2)}\frac{d\Theta_n}{d\mu}\right] - \frac{1}{(f^2-\mu^2)}\left[\frac{s}{f}\frac{(f^2+\mu^2)}{(f^2-\mu^2)} + \frac{s^2}{(1-\mu^2)}\right]\Theta_n + \frac{4a^2\omega^2}{gh_1}\Theta_n = 0 \tag{2.2}$$

whose eigenvalues are the corresponding equivalent depths h_n for the given mode, $f = \sigma/2\overline{\omega}$, $\overline{\omega}$ is the angular velocity of the earth's rotation and a is the radius of the earth. The definition of h_n arises from the theory of internal gravity waves on a rotating plane where it is a measure of the square of the wave's horizontal wavelength; h_n may be negative for diurnal oscillation.

Laplace's tidal equation has been recently re-investigated and the latitude-dependences of the tidal wind fields have been evaluated [15] for $l = 1$ ($s = 1 : n = -5, -3, -1, 1, 3, 5$) and $l = 2$ ($s = 0 : n = 2$ and $s = 2 : n = 2, 4, 6, 8, 10$), negative mode numbers are given to the modes of negative equivalent depths.

In observations, a detailed analysis was carried out [16] in order to remove the two major variations, namely irregular and seasonal, which tend to mask the tidal wind oscilla-

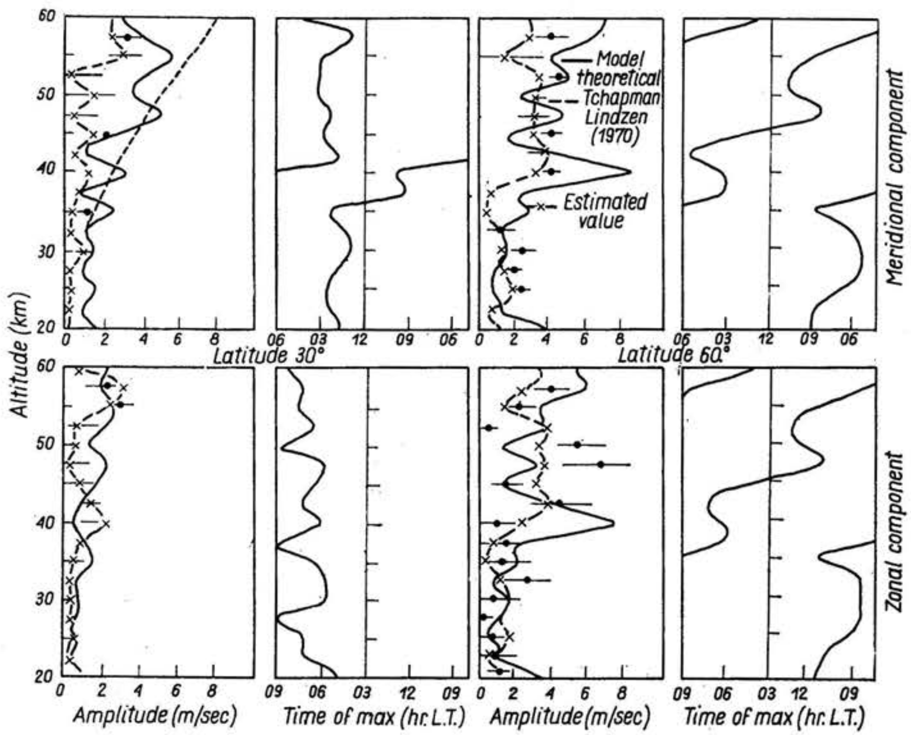

FIG. 2. Altitude distribution of the semidiurnal oscillation, after MAKARIOUS [14], similar to Fig. 1, except that the theoretical curve is available only for the meridional wind amplitude at 30° N.

tions. The analysis was carried out at five latitude ranges (8°S, 20°N, 30°N, 37°N and 61°N) for diurnal and semidiurnal oscillations of S-N and W-E wind components, at heights 2.5, 5.0, ..., 60 km, using Meteorological Rocket Network soundings conducted in the year 1959–1965. The deviations of the hourly means from the daily mean are approximated as

$$Sv(z,\theta,t) = \sum_{l=1}^{2} [av_l(z,\theta)\sin lt + bv_l(z,\theta)\cos lt]. \tag{2.3}$$

Let the height-dependence terms $Z_n(z)$ in (2.1) be expressed in the form

$$Z_{l,n}(z) = X_{l,n}(z) + iY_{l,n}(z). \tag{2.4}$$

For each oscillation the least-squares criterion is satisfied by combining the 10 equations of condition (for two wind components at five latitudes) into 6 normal equations (for 6 modes). A return to (2.1) has then been made to obtain another set for the amplitudes and phases [14] (Figs. 1 and 2).

For diurnal wind oscillations (Figs. 1), it is shown that the most important deviations of the anticipated model from theoretical results [7] are within the range of 35 and 55 km: the lower level deviation is due to the isothermal assumption in the theory and the upper deviation is due to the inclusion of the $(1, -5)$ mode in the present analysis.

The results for semidiurnal oscillations are presented in Fig. 2. In spite of the fact that complete theoretical results of this oscillation are available for comparison, the main features are revealed, namely the clockwise rotation of the wind vectoı as viewed from above and the rapid phase shift of 6 hr around 40 km. The latter feature is the sole characteristic of the semidiurnal oscillation principal modes [17, 18], and at a level which is higher than theoretically predicted [19]. This conflict has been attributed to the response of the oscillation to the changing distribution of mean zonal winds in the stratosphere [20].

3. Numerical approach to the vertical structure equation

The height-dependence terms $Z_n(z)$ in the tidal wind fields are expressed in the form

$$Z_n(z) = \left(\frac{dy_n}{dx} - \frac{y_1}{z}\right) e^{-x/2}, \tag{3.1}$$

where

$$y_n(x)e^{x/2} = \chi_n(z) - \frac{kI_n(z)}{gH(z)},$$

$$\chi = \nabla \cdot V = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}(v\sin\theta) + \frac{1}{a\sin\theta}\frac{\partial u}{\partial\phi} + \frac{\partial w}{\partial z},$$

$$\chi = \sum_n \chi(z)\Theta^s_{l,n}(\theta)e^{i(\sigma t + s\phi)}, \tag{3.2}$$

$$x = \int_0^z \frac{dz'}{H(z')}, \quad H = \frac{RT_0}{g}.$$

H is the scale height, T_0 is the static temperature, J_n is the thermotidal heating rate per unit mass, $k = (\gamma-1)$, $\gamma = 2/7$, $\gamma = C_p/C_v = 1.4$, and y_n is the solution of the equation formulated in [8] as

$$\frac{d^2y_n}{dx^2} - \frac{1}{4}\left[1 - \frac{4}{h_n}\left(kH + \frac{dH}{dx}\right)\right] y_n = \frac{kJ_n}{\gamma g h_n} e^{-x/2}. \tag{3.3}$$

Equation (3.3) is an inhomogeneous equation which, given two boundary conditions, has a unique solution for the vertical structure of a given mode. By analogy with the propagation of electromagnetic waves in a medium having a variable refractive index λ, the equivalent refractive index is

$$\lambda_n^2 = \left[4\left(kH + \frac{dH}{dx}\right)/h_n - 1\right]/4. \tag{3.4}$$

Upward propagation of the energy, if mainly put into the atmosphere by tidal or thermal causes in the lower layers, is effectively blocked if $\lambda_n^2 < 0$. The conditions favouring $\lambda_n^2 < 0$ are that H should be small and dH/dx should be either small positive or negative; in addition, it is possible for h_n to be negative. The number of barriers to energy flow depends on the number of such regions of upward decreasing temperature, but they alone are not sufficient to give a barrier unless the value of h_n is appropriate, which in turn depends on the mode and period of oscillation considered.

To examine how the basic temperature distribution (H) affects the vertical structure of the tide, a temperature profile corresponding to 45°N is used and the results for the wavelengths of the propagating modes ($2\pi H/\lambda_n$) and exponential scales of trapped modes ($H/|\lambda_n|$) are shown in Fig. 3 [16]. It is clear that the vertical wavelengths of the propagating

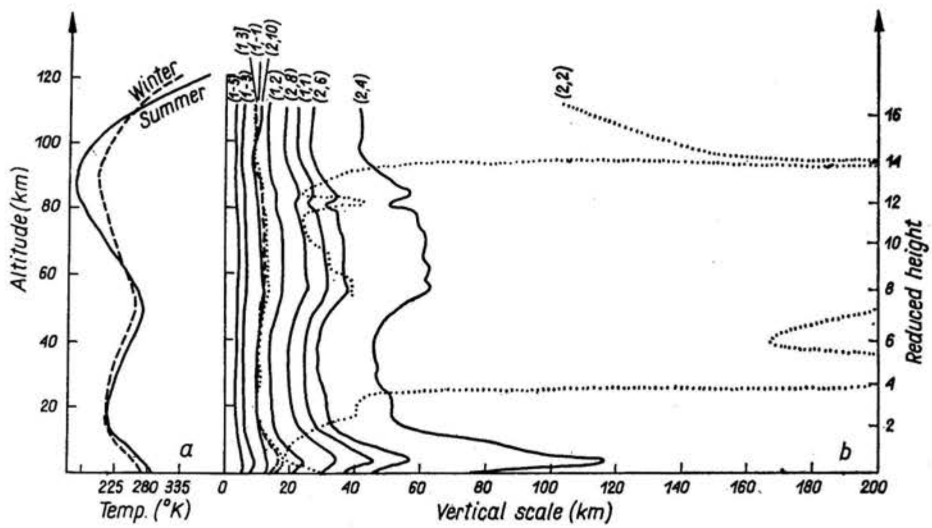

FIG. 3. Vertical scales of diurnal and semidiurnal migrating solar wave types in a model atmosphere, after MAKARIOUS [16]

Fig. 3a. Represents the temperature structure in the atmosphere at 45° N. Fig. 3b. Wavelengths (in km) of the propagating modes and exponential scales of trapped modes.

modes are much more affected by the variation of the temperature in the vertical than the decay scales of the trapped modes. It is also interesting to note the rapid increase in the wavelength of the (2,2) mode in the regions of strong inversion of the atmosphere ($\sim$ 30 and 90 km).

When H is constant, the homogeneous form of Eq. (3.3) has solutions which are either exponential or sinusoidal. For more complex problems, i.e. to account for the vertical temperature structure, the solutions should be approached numerically and this will be as follows. Let us divide our x-domain into a number of discrete levels x_m ($m = 0, 1, 2, \ldots$ $\ldots, M$), $x_0 = 0$ and x_m corresponds to 60 km. At x_m, the derivatives of the function $y(x)$ may be approximated as

$$\text{(3.5)} \qquad \begin{aligned} \frac{dy_m}{dx} &\simeq \frac{y_{n+1}-y_{n-1}}{2\delta x}, \\ \frac{d^2y_m}{dx^2} &\simeq \frac{y_{n+1}-2y_n+y_{n-1}}{(\delta x)^2}. \end{aligned}$$

Our procedure in solving Eq. (3.3) is to let

$$\text{(3.6)} \qquad \begin{aligned} y_m &= \alpha_m y_{n+1}+\beta_m, \\ y_{n-1} &= \alpha_{m-1} y_n+\beta_{n-1}. \end{aligned}$$

Substituting for y_{m-1} in (3.3) and comparing with the expression for y_m, we obtain

$$\text{(3.7)} \qquad \begin{aligned} \alpha_m &= \frac{-1}{\alpha_{m-1}+B_m}, \\ \beta_m &= \frac{D_m-\beta_{m-1}}{\alpha_{m-1}+B_m}, \end{aligned}$$

where

$$\text{(3.8)} \qquad B_m = -2+(\delta x)^2\lambda_m^2, \quad D_m = (\delta x)^2 \frac{kJ_m}{\gamma g h_n} e^{-x/2},$$

α_0 and β_0 are obtained from the assumption of a smooth spherical earth implying that the vertical component of wind velocity vanishes at $x = 0$ (as lower boundary condition). Then,

$$\text{(3.9)} \qquad \frac{dy_n}{dx} + \left(\frac{H}{h_n} - \frac{1}{2}\right) y_n \bigg|_{x=0} = 0.$$

On using Eqs. (3.5) and comparing with Eqs. (3.6), we get

$$\text{(3.10)} \qquad \alpha_0 = \frac{1}{1-\delta x\left(\dfrac{H_0}{h_n} - \dfrac{1}{2}\right)}, \quad \beta_0 = 0.$$

In order to evaluate α_m and β_m at all levels, J_m should be given to evaluate D_m in Eq. (3.8). However, the real difficulty in evaluating the theoretical tides lies in specifying the thermal drive with sufficient accuracy from our knowledge of the radiative processes and temperature changes. On the other hand, if J is known, we now merely have to know y at some high level, and Eqs. (3.6) will give its values at all lower levels, and thus the integration is completed. For this condition it is generally required that the kinetic energy density $\varrho_0(z) V^2/2$ remains bounded as $z \to \infty$. This is in turn the requirement that $y(x)$ remains bounded as $x \to \infty$ [21]. LINDZEN [7], in his theoretical treatment, assumed that J is zero above some level ($\sim$ 110 km), and his results for isothermal atmosphere are shown in Fig. 1 as dotted curves. It has been concluded, however, that the present theory fails to predict the observed decay of amplitude above 105 km [9].

Therefore, to overcome the above-mentioned difficulties, α_m was evaluated by substituting for $Z_n(z)$, as defined by Eq. (3.1), at all levels, and provided that $y(0)$ is known. In order to evaluate $y(0)$, the pressure oscillations at the ground [22, 23] are used since they are the best developed data associated with atmospheric tides. The surface pressure oscillation is given in terms of $y(0)$ as follows:

$$\delta P_n(0) = \frac{i\gamma}{\sigma} p_0(0) y_n(0). \tag{3.11}$$

On using Eqs. (3.6), the values for β_m are obtained and J_m is thus evaluated in a complex form. The results obtained for the rates of the thermotidal heating per unit mass of the atmosphere are shown in Figs. 4 and 5, for diurnal and semidiurnal oscillations, respectively.

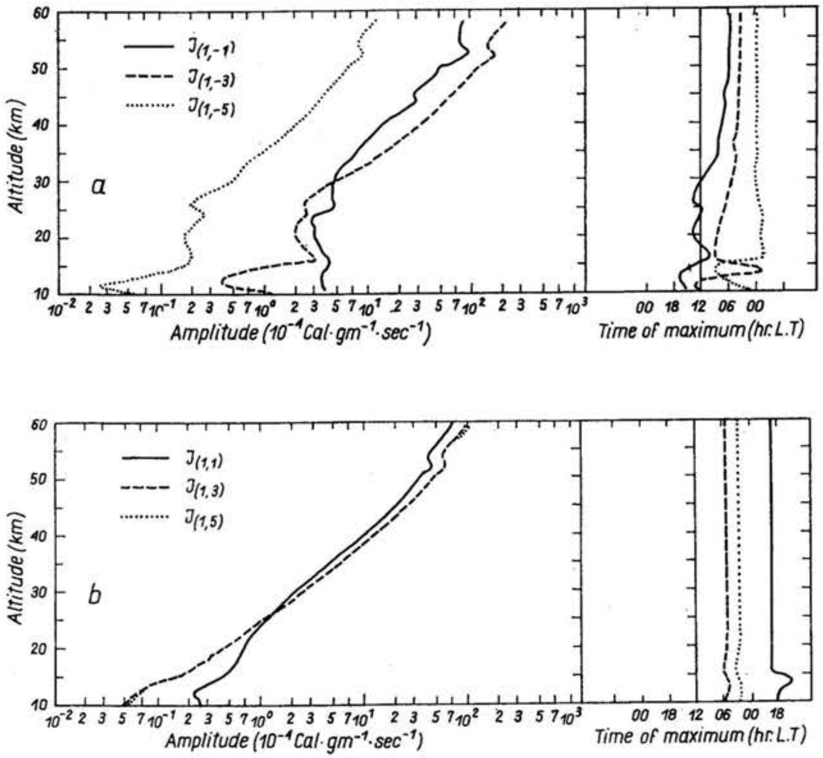

FIG. 4. Rates of thermotidal heating per unit mass of atmosphere. Diurnal oscillations amplitudes and time of maximum amplitude occurrence.

FIG. 4a. Negative modes, (1, −1), (1, −3), (1, −5), Fig. 4b. Positive modes, (1, 1), (1, 3), (1, 5).

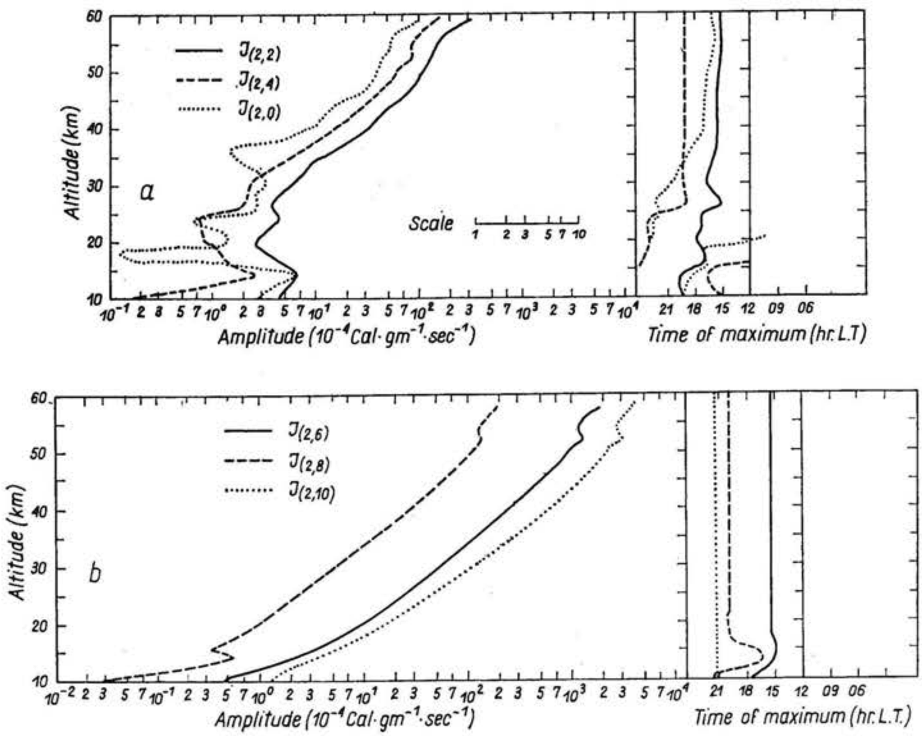

FIG. 5. Rates of thermotidal heating per unit mass of atmosphere. Semidiurnal oscillations amplitudes and time of maximum amplitude occurrence.

Fig. 5a. (2, 2), (2, 4), (0, 2) modes, Fig. 5b. (2, 6), (2, 8), (2, 10) modes.

4. Thermal exciting energy of atmospheric tides

4.1. Diurnal oscillations

For diurnal oscillations of negative h_n(Fig. 4a), or small positive $h_n(1, 1)$, λ_n^2 does not change sign anywhere in the atmosphere, thus attenuation and reflection associated with the transition between regions of propagation and evanescene are not to be expected. This trapping becomes increasingly great for the modes $(1, -3)$ and $(1, -5)$. Thus we are not surprised that the contributions to the modes with negative h_n from water vapour (near the ground) are larger than those from ozone (far above the ground). In the stratosphere, trapping and interference between different modes of diurnal oscillations do not preclude an effective response at the levels of excitation, and modes which propagate vertically are not trapped below the mesopause (Fig. 4b).

The effectiveness of the large stratospheric contribution in exciting the different modes of oscillation is shown to be severely affected by their corresponding wavelengths, the latter are affected by the variation in temperature. For the (1, 1) mode the wavelength is $\sim$ 25 km in the stratosphere and 43 km in the troposphere (Fig. 3). For this mode the region of water vapour excitation is not sufficiently thick for the process to be of great importance. This, however, is no longer true for the modes (1, 3) and (1, 5). On the other hand, the ozone excitation is distributied over a considerable depth of the atmosphere ($\sim$ 40 km), thus waves excited at one level can destructively interfere with waves excited at another level. The excitation of the mode (1, 3) by ozone heating is therefore subject to considerable selfcancellation due to the great depth of the region of heating in comparison with the 12 km wavelength of this mode, irrespective of the detailed representation of basic atmospheric scale-height.

4.2. Semidiurnal oscillation

For the main semidiurnal mode (2, 2) (Fig. 5a), λ_n^2 is almost zero through most of the atmosphere, i.e. extremely long vertical wavelength (150 km). Thus, not only does this mode receive the bulk of semidiurnal oscillation, but it must also respond to the excitation with particular efficiency. Below the mesopause H is small and dH/dx is negative; hence λ_n^2 is negative for this mode and energy is trapped.

The identification of higher-order semidiurnal modes in stratospheric wind oscillations leads to the question of their source of generation. Not all such modes would, however, be significant to the wind oscillations as tidal effectiveness of a mode depends on the correspondence of its vertical profile with that of the heat source, and modes of wind oscillations would still not be observed at latitudes near the zeros of corresponding wind functions Lv_n (Eq. 2.1).

In the classical treatment [9] a rapid phase reversal is obtained just below 30 km between adjacent regions of almost constant phase extending up to the mesopause and down to the surface. In the recent calculations [20], with a more realistic atmosphere, the chief modification is to the level at which the reversal occurs moving up closer to 50 km at middle and high latitudes in summer. This phase shift above 30 km is a recognized feature in the results of the principal semidiurnal modes (2, 2), (2, 4) and (0, 2).

5. Conclusion

In the present paper an attempt has been made to estimate the thermal exciting energy of the atmospheric tides based on the observations, through a numerical solution of the vertical structure (second-order differential) equation. The merit in this approach is that it takes into consideration the basic structure of the atmosphere which definitely governs the propagation of the energy through its effects on the atmospheric refractive index for the different modes.

For stratospherically excited modes, the assumption of longitudinal asymmetry should be justified if heating is simply dependent on solar declination and related photochemistry. Nevertheless, no account has been taken of the longitudinal dependence at present as most of the data used relate to $115\pm45°$ W. The analysis could be further improved should more latitude ranges, if available, be included in order to account for the rapid variation in the latitude-dependences $Lv_n(\theta)$, especially for higher order modes.

With respect to the accuracy of the empirical data used, the probable errors of the computed amplitudes of wind oscillations are shown in Figs. 1 and 2. These errors have been taken into consideration in evaluating the height-dependence terms $Z_n(z)$: the relative errors in the latter are found to be of the order of 10%. In Figs. 1 and 2 the close agreements between the anticipated and observed wind fields reveal that each oscillation can be fairly represented as superposition of the above-mentioned modes. To conclude, the accuracy in the determination of the solution of Eq. (3.3) ($y_n(x)$) can be further achieved by reducing the step size (1 km is used in the present study). However, this is rendered difficult at present since the observed tidal wind fields used are available every 2.5 km height and interpolation at very short interval could be misleading. Nevertheless, an empirical formula, if available, will be of practical use.

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