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Preliminary Optimization of Pipe-Z Reconfiguration

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Abstract

Pipe-Z (PZ) is a parametric design system which comprised of a congruent modules (PZM) allows the creation of complex three-dimensional, single-branch structures which can be represented by mathematical knots. Once the geometrical parameters are set for the PZM, the shape of PZ is controlled solely by relative twists of the PZMs in a sequence. Therefore each PZM has one degree of freedom (1DOF). This paper presents the preliminary optimization of PZ reconfiguration from a "straight tube" to a half-torus. Here the displacement of PZMs transverse to the "bending direction" is to be minimized. In other words, it resembles "truing" of a wheel. In the considered case, the PZ is comprised of eight hexagonal PZMs. Thus every PZM can have six possible positions relative to the previous module. The initial (PZ_I) and target (PZ_T) configurations are given. Since the time-steps and relative twists are discrete, it is a discrete optimization and has combinatorial nature. The number of possible configurations grows astronomically with the assumed number of time-steps from one position to another and the number of PZMs. However, the optimization algorithm can be naturally parallelized. At first the concept of PZ is outlined, followed by the experiment. The results are illustrated and discussed.

Keywords: Extremely Modular System, Pipe-Z, Arm-Z, discrete optimization, dihedral rotation, "snakebot", reconfiguration.

1 Introduction

The idea of Extremely Modular System (EMS) has been introduced in 2016 [1] by the author. The purpose of EMS is to create free-form objects in a given environment (E) without obstacle-, and self-collisions. The concepts of two EMSs: Truss-Z (TZ) and Pipe-Z (PZ) have been presented by the author in Ref. [3], and Ref. [2], respectively.

EMS jointly meets the following three criteria:

1. EMS allows for creation of structurally sound free-form structure.
2. **The number of module types in EMS is minimal.**
3. EMS is not constrained by a regular tessellation of space.

Chronologically, TZ has been developed first. It is a skeletal system for creating free-form *pedestrian ramps* and *ramp networks* among any number of terminals in space. TZ structures are composed of four variations of a single basic module (TZM) subjected to affine transformations (mirror reflection, rotation and combination of both).

PZ is more "fundamental" and forms spatial single-branch *knot-like structures* by assembly of one type of module, called PZM. PZ has been proposed as deployable and temporary system, potentially suitable for extreme and outer-space environments [4]. In particular, PZ has been suggested for emergency connectors and habitats in so-called, "banana-split" configuration (orientation). In principle PZs can be composed of modules whose based on circles or regular polygons of arbitrary number of *n* sides, as shown in Fig.1.

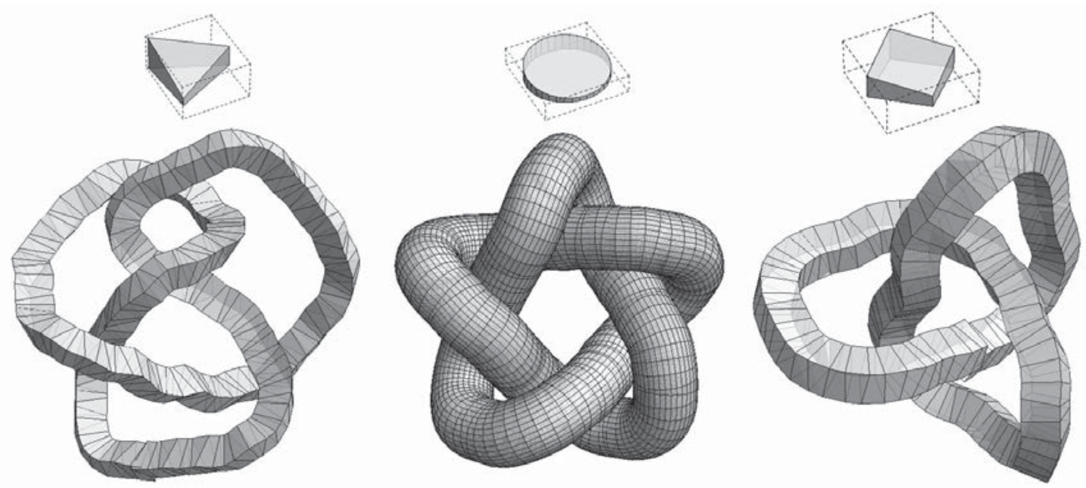

Figure 1: Various PZ knots assembled with different basic units. From the left: Figure-eight (4_1) @ the number of PZM sides: $n = 3$, *Cinquefoil* (5_1) @ $n = 36$, and *Trefoil* (3_1) @ $n = 4$.

Arm-Z (AZ) is a concept of a kinematic system derived from PZ presented in Ref.[6]. It is closely related to the idea of modular robots, in particular, so-called "snakebots", that is biomorphic hyper-redundant robots that resemble biological snakes. AZ is composed of congruent and rigid modules and it is capable of free movements (translation, extension and flexure), as demonstrated in [6]. Both in PZ and AZ, each module has only one degree of freedom (1DOF), which makes them extremely *simple* and *robust*, but also rather *unintuitive* to work with. In principle, the problem of PZ reconfiguration is broader than AZ manipulation, as it also allows for folding of PZMs (studied in Ref.[4]). However, in this paper the PZMs are also rigid, therefore this PZ reconfiguration is geometrically equivalent to the AZ manipulation.

2 Pipe-Z

Although PZ can be composed of modules based on circle or any regular polygon, in this paper the considered PZM is hexagonal, as shown in Fig.2.

2.1 The Pipe-Z module (PZM)

Pipe-Z module (PZM) is a geometrical object analog to a sector of circular torus described in [5]. It is defined by parameters: r: $(0, \infty)$, d: $(0, \infty)$ and ζ: $(0,\infty)$, which denote: radius, corresponding radius and central angle, respectively; r, d, $\zeta \in \mathbb{R}$. PZMs are terminated by two faces T and B, corresponding to the top and the bottom of a unit. Although in principle they do not have to be congruent, for practicality, however, it is desirable that PZM is symmetrical about the plane perpendicular to its axis, as shown on the left in Fig. 2.

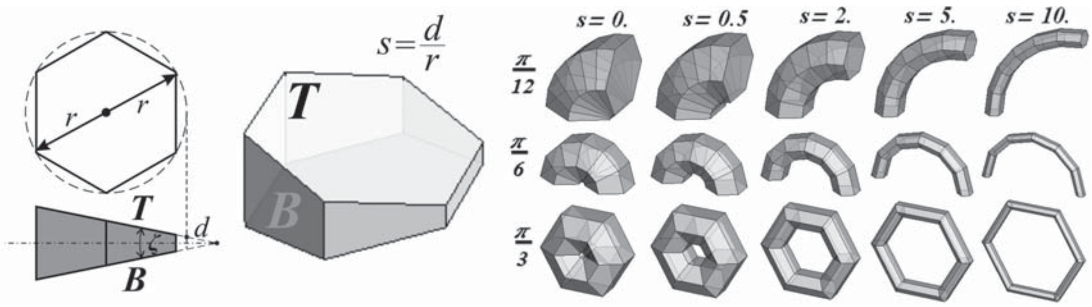

Figure 2: On the left: a visualization of PZM which is defined by parameters: r, d, ζ and n. Since n = 6, the top (T) and bottom (B) faces are hexagons. On the right: variety of PZs constructed with the same sequence of six units with relative twists $k_1 = k_2 = ... = k_5 = 0$ at increasing values of s and ζ along columns from the left and rows from the top, respectively. The value of parameter $r = 1$, however, the images are zoomed-to-fit.

Such a condition implicates that T and B are congruent. Their relative position is controlled by r, d and ζ. The faces of T and B can have shapes of circles or regular polygons of arbitrary number of n sides. Polygonal faces seem easier to fabricate and assemble than circular ones. In such a case the number of sides n: $n \in \mathbb{N}$ becomes an additional parameter, here set to 6. Moreover, it is convenient to introduce a new parameter $s = \frac{d}{r}$; $s :(0,\infty)$. Thus r is the "absolute" parameter controlling the size of

PZM in relation to the environment E, and s is the ”relative“ parameter defining the ”slenderness“ of PZMs.

PZ structures are assembled by a sequence of PZMs, so that top face (T) of the previous unit becomes the base (B) for the next unit. The successive PZM i is rotated by the relative twist angle κ_i, which can have real or discrete values. In the latter case such rotations are denoted by k_i. In the following examples PZMs are based on hexagon (6-gon), thus any subsequent unit can be added at six dihedral (rational) angles, so the facets of adjacent units are aligned.

Entire PZs are encoded as: $PZ = \{\{n, r, s, \zeta\}, V_s, L\}$, where n, r, s, and ζ are the PZM parameters, V_s is the initial vector which positions the first unit in space, and L is the sequence of dihedral twists k_i, where i is the index of the i$^{\text{th}}$ unit. Fig. 2 on the right shows six PZMs assembled at constant $k_i = 0$ and $r = 1$, and various values of s and ζ.

3 The reconfiguration of Pipe-Z

Ref.[6] demonstrates that a modular system based on PZ, whose units have only one degree of freedom (1DOF) is capable of practically any 3D motion. The fundamental movements of: *extension*, *translation*, and *flexure* have been executed rather satisfactorily. *Flexure* is used in this paper as an example of reconfiguration from a “straight tube” to a (half or full) torus. Fig.3 shows a possible solution presented in [6], which has not been subjected to optimization. This paper introduces a minimization criterion, and the new problem is formulated as follows:

1. The change of relative position of an i$^{\text{th}}$ PZM is constrained to maximum of one dihedral rotation (k_i) of $\frac{2\pi}{6} = \frac{\pi}{3}$ at a time-step, i.e.: $k_i \in \{-\frac{\pi}{3}, 0, \frac{\pi}{3}\}$.
2. The deviation in transverse direction is to be minimal.

Fig.3 shows axonometric views of PZ at each of 34 time-steps presented in [6], together with projections on horizontal and vertical-transverse planes. These projections indicate that at certain time-steps the deviation of PZ from the vertical-longitudinal plane is quite visible. Since the intention here, is to keep all PZMs in the same XZ plane, this deviation is called here the *buckling error* (ε_Y), and is expressed as follows:

$$\varepsilon_Y = \sum_{1}^{U} |\delta_Y(i)| \tag{1}$$

where $\delta_Y(i)$ is the difference between the Y-coordinate of the centroid of the initial module (PZM_0) and the Y-coordinate of the i$^{\text{th}}$ module (PZM_i);
U is the number of PZMs.

Since the position and orientation of the initial module (PZM_0) are given, it is excluded from the computation.

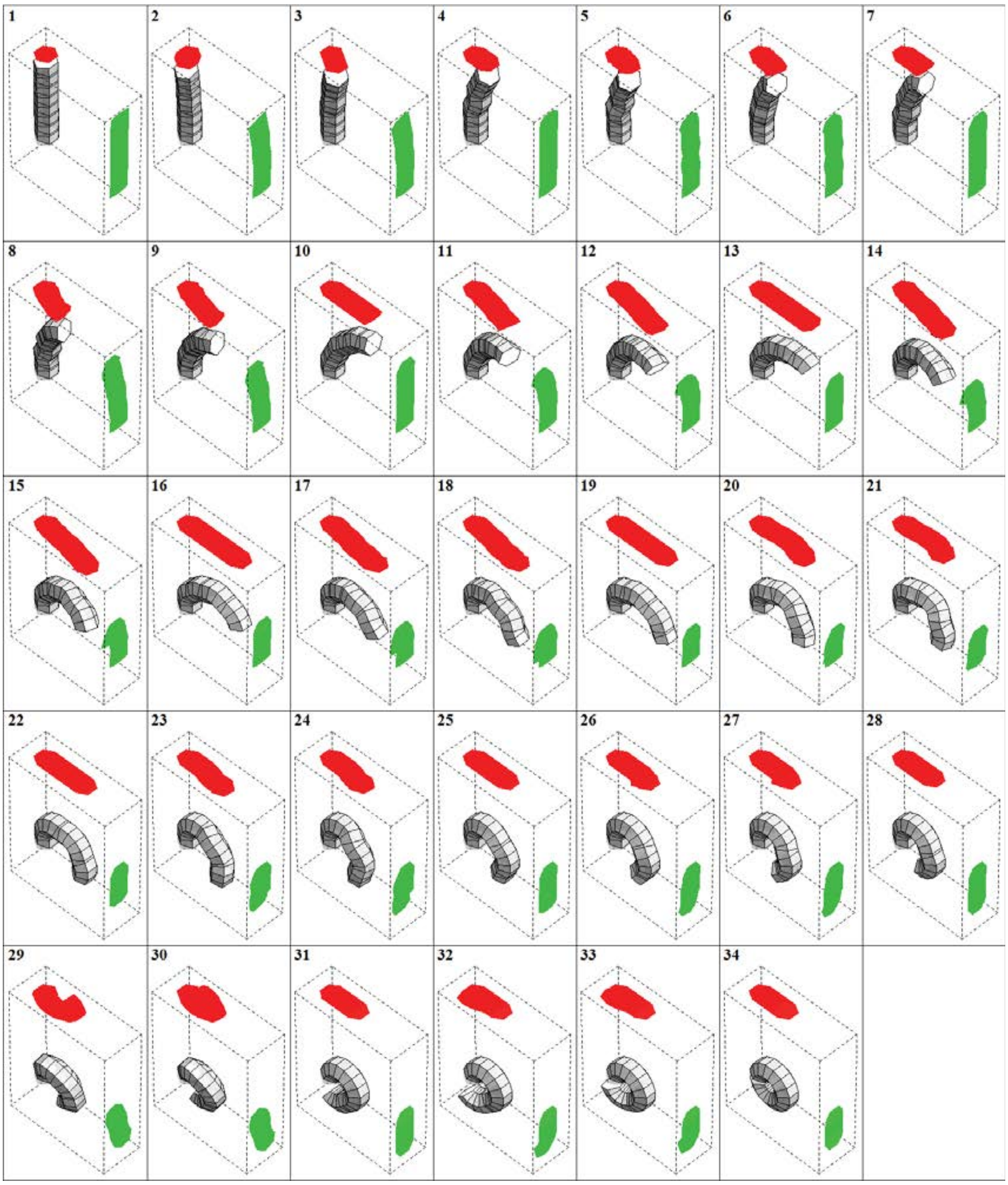

Figure 3: A possible transition from a “straight tube” to a full torus in 34 time-steps. The projections of PZ on horizontal and vertical-transverse planes are shown in red and green, respectively. The number of PZMs $U = 16$. The initial module (PZM_0) is shown as transparent.

4 Exhaustive search

The problem described here is discrete due to: modularity of the PZ system and dihedral relative twists of the modules. Therefore the optimization has combinatorial nature. In order to find the *ideal* solution(s) exhaustive search is performed. In the considered example, the conditions are given as follows:

- The position and orientation of the initial module (PZM_0) are set arbitrarily.
- Since each PZ module is based on hexagon ($n = 6$), every PZM_i for $i : [1, 2, \ldots, U]$ can be placed at six twist angles k_n relative to the previous PZM, that is: $k_n \in \{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\}$.
- The initial and target configurations are given by corresponding lists of relative twists: $\mathrm{PZ_I}$ and $\mathrm{PZ_T}$. Their length (L) is equal to the number of modules (U).

$$\mathrm{PZ_I} = \{\pi_1, \pi_2, \ldots, \pi_U\};\ \mathrm{PZ_T} = \{0_1, 0_2, \ldots, 0_U\} \tag{2}$$

$$L[\mathrm{PZ_I}] = L[\mathrm{PZ_T}] = U \tag{3}$$

- At each time-step the position of each PZM can be altered by maximum one dihedral twist of $\frac{\pi}{3}$, that is: $\{-\frac{\pi}{3}, 0, +\frac{\pi}{3}\}$.

4.1 The straightforward transition (*S*-*T*) from $\mathrm{PZ_I}$ to $\mathrm{PZ_T}$

There are two alternative simplest transitions from $\mathrm{PZ_I}$ to $\mathrm{PZ_T}$ which meet all the aforementioned conditions. These are the straightforward (*S*-*T*). The completion requires only three time-steps ($t = 3$), and each PZM twists in either order: $\{\pi \to \frac{2\pi}{3} \to \frac{\pi}{3} \to 0\}$, or $\{\pi \to \frac{4\pi}{3} \to \frac{5\pi}{3} \to 2\pi (= 0)\}$. Fig.4 illustrates the former sequence.

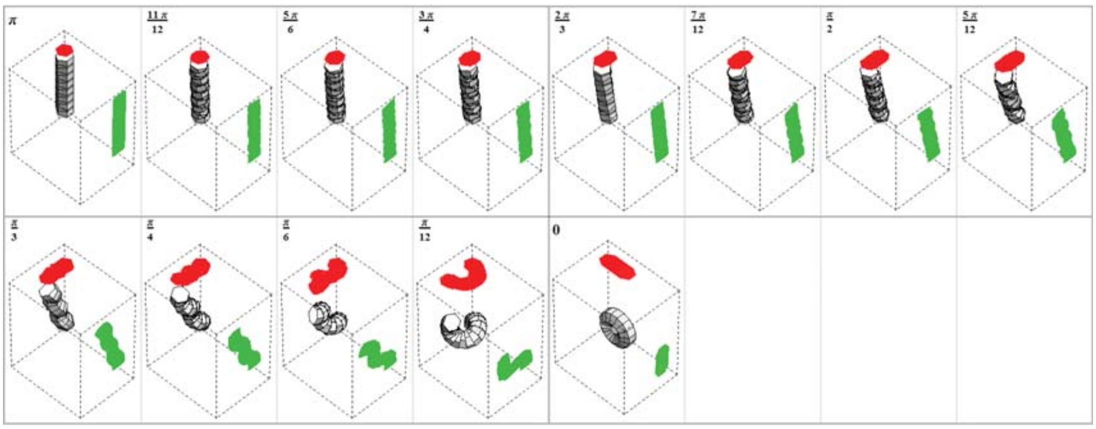

Figure 4: *S*-*T*: Every module changes its relative position as follows: $\{\pi \to \frac{2\pi}{3} \to \frac{\pi}{3} \to 0\}$. Additionally, three intermediate time-steps at fractional twists are shown.

As Fig.4 indicates, in *S-T*, *buckling* at positions $\{\frac{2\pi}{3}, 0\}$ is noticeable, but at $\frac{\pi}{3}$ is severe. It becomes even more dramatic at the three intermediate time-steps, which are not considered in the optimization, but are shown here for illustrative purpose only.

4.2 Perfect ternary trees

As shown in Fig.4, *S-T* is very quick, but the *buckling* is very strong which is unacceptable. In this subsection the application of exhaustive search for finding the transition with minimal *buckling* is described. As mentioned above, each but initial PZM can be rotated relative to the subsequent module by the following angles: $\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\}$. For simplicity, further in text, these six angles will be represented by the following corresponding six positions: $\{0,1,2,3,4,5\}$. Fig.5 visualizes the entire search space for three time-steps in *S-T* for each module (except the initial one).

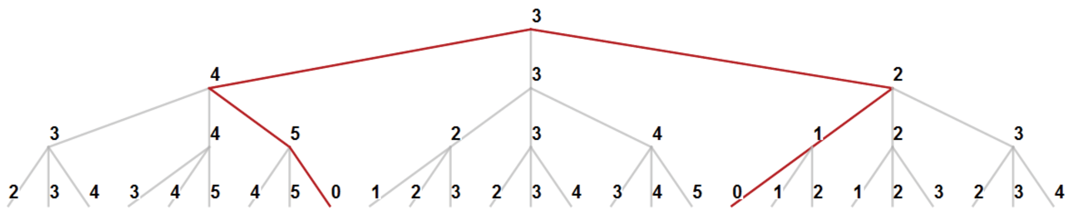

Figure 5: Perfect ternary tree for each PZM in *S-T*: there are two alternative paths from position 3 to 0; $t = 3$.

As Fig.5 indicates, in the three-time-step procedure there are two “symmetrical” paths, namely:$\{3,4,5,0\}$, and $\{3,2,1,0\}$. This means that in such a case, a PZ structure comprised of U-number of units can be reconfigured in 2^U unique sequences. The formula for the total number of sequences for transition from PZ_I to PZ_T can be generalized as follows:

$$3^t \tag{4}$$

where 3 corresponds to the number of possible relative twists at each time-step $\{-\frac{\pi}{3}, 0, +\frac{\pi}{3}\}$, and t is the number of time-steps.

The calculation of the number of all possible paths for a given number of time-steps (t) is straightforward, which is not the case for calculation of all *potential paths*, *PP* for short. In this paper the number of *PP*s are calculated by depth-first algorithm. Fig.6 shows the log-plot for the number of: all paths, and *PP*s as a function of the number of time-steps (t). As Fig.6 indicates the rate of the exponential growth is approximately the same for both numbers.

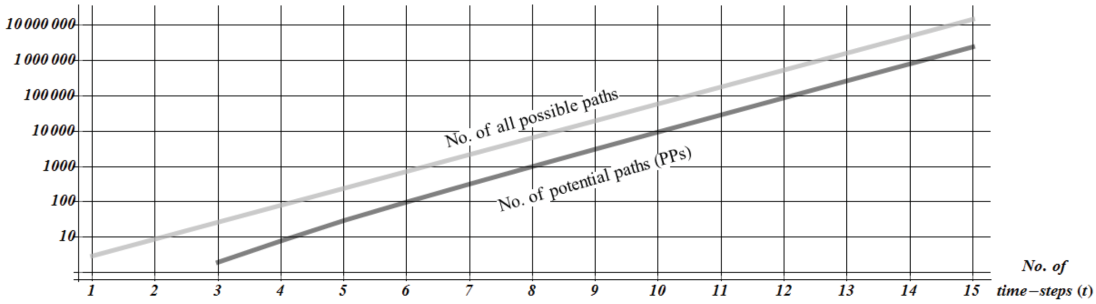

Figure 6: The log-plot showing the exponential growth of the numbers of: all possible paths and all *PP*s from PZ_I (3) to PZ_T (0) for a given number of time-steps (t).

Since the quickest transition is burden by strong buckling, it is natural to increase the number of time-steps. As a consequence, the number of possible paths also increases, as illustrated in Fig.7.

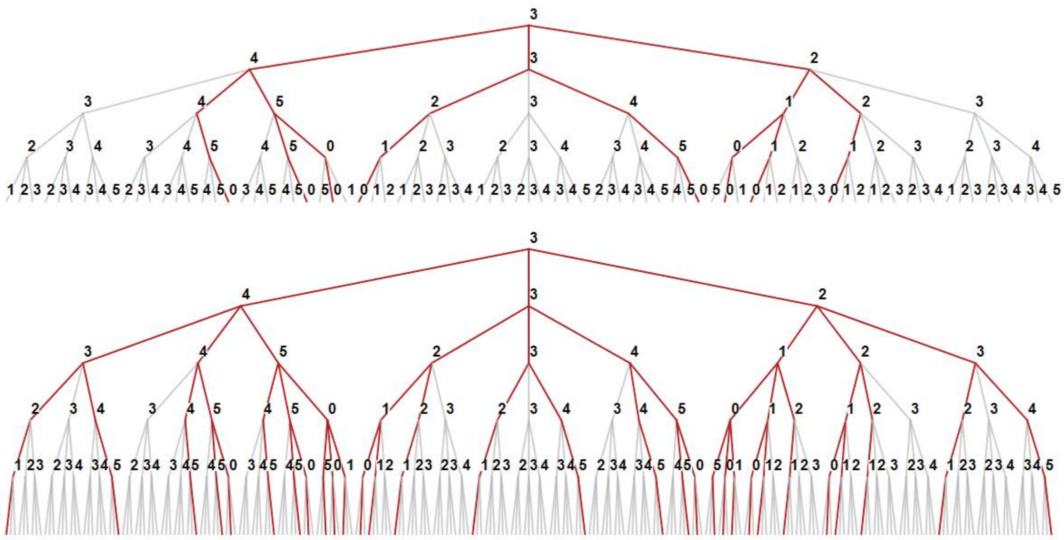

Figure 7: The ternary trees for each PZM at increased number of time-steps (t). On the top: at $t = 4$, there are 8 *PP*s. On the bottom: at $t = 5$, the number of *PP*s is 30.

Therefore, the number of all possible transition sequences from PZ_I to PZ_T for a PZ comprised of U-number of units can be expressed as follows:

$$p^U \tag{5}$$

where p is the number of *potential paths* (*PP*) that is paths in the ternary tree connecting the *initial position* with the *target position* for each module; U is the number of modules of the PZ.

Therefore, the growth of the number of all possible transition sequences from PZ_I to PZ_T is *double exponential*, as visualized in Fig.8.

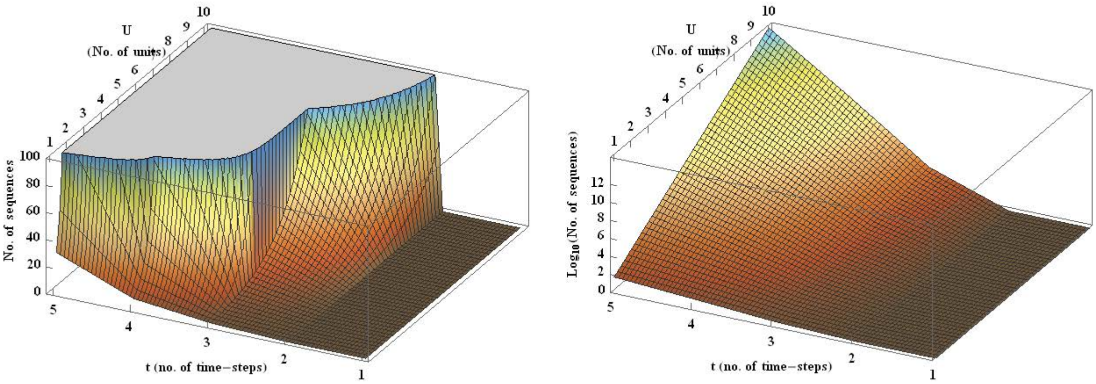

Figure 8: The visualization of the double exponential growth of the number of all possible transition sequences from $PZ_I = 3$ to $PZ_T = 0$. Linear and logarithmic scales are shown on the left and right, respectively.

5 The experiment: exhaustive search

As mentioned above, the number of possible solutions and therefore the search space grow astronomically with the numbers of: time-steps and PZMs. In such cases, for problems of practical size, proposed solutions are usually produced by meta-heuristic methods and the quality of solutions is not *ideal*, but "just good". Nevertheless, for development of a proper meta-heuristic algorithm, it is desirable to know certain "benchmark solutions". A relatively small-size experiment has been performed in serial and in parallel, as described in the following subsections.

5.1 7 PZMs @ 4 time-steps

As shown on the top of Fig.7, there are eight *potential paths* (*PP*s) from positions 3 to 0. Since the initial PZM is fixed, the remaining 6 are being optimized. Therefore the total number of possible solutions is: $8^6 = 262144$.

In order to make the iteration process as simple and as little resource-consuming as possible, and also easily "parallelizable", each allowable solution has been indexed, as follows:

- There are six PZMs considered and eight *potential paths*:
  $PP_8 = \{\{3,2,1,0,0\}, \{3,2,1,1,0\}, \{3,2,2,1,0\}, \{3,3,2,1,0\}, \{3,3,4,5,0\}, \{3,4,4,5,0\}, \{3,4,5,5,0\}, \{3,4,5,0,0\}\}$.

- Therefore 6-element tuples of 8-element sets are to be considered. There are $8^6 = 262144$ such tuples.

- An octal (base-8) number system is used to index every i$^{\text{th}}$ tuple. E.g. 000077_8 represents a decimal 64$^{\text{th}}$ tuple, i.e. a list of six *PP*s: $\{PP_8^{(0+1)th}, PP_8^{(0+1)th}, PP_8^{(0+1)th}, PP_8^{(0+1)th}, PP_8^{(7+1)th}, PP_8^{(7+1)th}\}$, that is: $\{\{3,2,1,0,0\}, \{3,2,1,0,0\}, \{3,2,1,0,0\}, \{3,2,1,0,0\}, \{3,4,5,0,0\}, \{3,4,5,0,0\}\}$. In other words the first four PZMs will change position as follows: $\{3 \to 2 \to 1 \to 0 \to 0\}$, and the last two PZMs wil change the position as follows: $\{3 \to 4 \to 5 \to 0 \to 0\}$.

Due to symmetry of the search tree, there are two equivalent ideal solutions found at the 52521$^{\text{th}}$, and 209624$^{\text{th}}$ iterations, respectively:

$$\begin{bmatrix} 3 & 2 & 1 & 1 & 0 \\ 3 & 3 & 4 & 5 & 0 \\ 3 & 4 & 5 & 5 & 0 \\ 3 & 3 & 4 & 5 & 0 \\ 3 & 4 & 4 & 5 & 0 \\ 3 & 2 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 3 & 4 & 5 & 5 & 0 \\ 3 & 3 & 2 & 1 & 0 \\ 3 & 2 & 1 & 1 & 0 \\ 3 & 3 & 2 & 1 & 0 \\ 3 & 2 & 2 & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 \end{bmatrix} \tag{6}$$

The rows in the above matrices contain the sequences of relative positions of the individual PZMs. The first row from the top corresponds to PZ_1, the second one to PZ_2, and so forth. The solution shown on the left is illustrated in Fig.9 below.

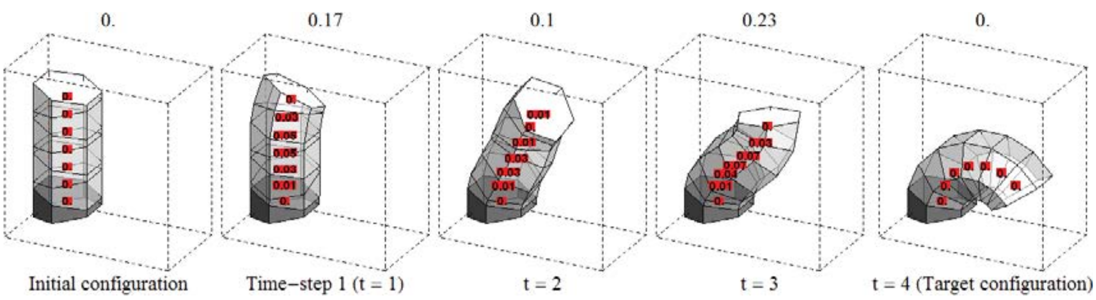

Figure 9: One of two ideal solutions for transition of a 7-unit hexagonal PZM from "straight tube" to (an almost) half-torus. The *buckling error* (ε_Y) at each time-step is shown on the top. Total error for this transition is 0.50308. The values of δ_Y are shown for individual PZMs on red background.

5.2 Serial vs. parallel computation on 2 & 4 kernels

The experiment has been performed in serial (on a single kernel), and parallelized on two and four kernels available on an Intel® Core™ i7-4790 CPU @ 3.60 GHz with 4 cores and 8 GB RAM.

Since the indexes of all 262144 potential solutions are explicitly known, they have been split in equal parts among two and four kernels. In the first experiment all 262144 PZM configurations were examined, in the second experiment kernels: *#1* and *#2* performed searches for indexes: $i = \{1,...,131072\}$ and $i = \{131073,...,262144\}$, respectively. In the last experiment kernels: *#1*, *#2*, *#3*, and *#4* performed searches for indexes: $i = \{1,...,65536\}$, $i = \{65537,...,131072\}$, $i = \{131073,...,196608\}$, and $i = \{196609,...,262144\}$, respectively. All setups returned the same ideal solutions. Strong scaling of this parallelization is illustrated in Fig.10.

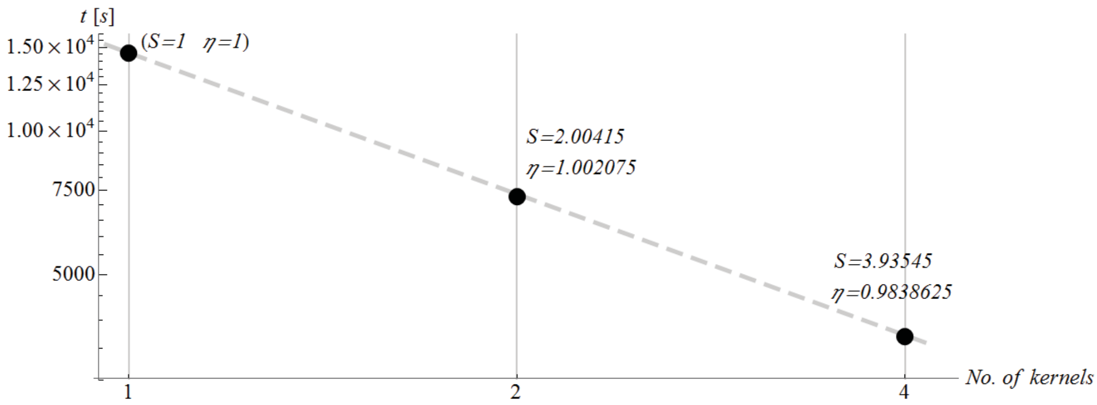

Figure 10: The speedup (S) and efficiency (η) for the experiment ran in parallel.

As Fig.10 indicates, the efficiency of parallelization is practically ideal.

6 Conclusion

In the considered optimization problem of Pipe-Z (PZ) reconfiguration, the growth of search space is *double exponential*. Therefore finding the ideal transitions by exhaustive search is extremely time-consuming or, in fact, the cases of larger PZMs – simply unrealistic. Nevertheless, by the indexing method described in Subsection 5.1, the domain can be directly split among any number of kernels, which makes it particularly suitable for parallel computation. In general, due to enormous size of solution space, population-based meta-heuristic methods seem the most suitable. However, smaller-size examples are well-suited for grid computation as the parallelization in straightforward and the scaling is strong.

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