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Application of methods of characteristics and perturbation in solving the wave problems of viscoplasticity(*)

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THE AIM of the paper is to construct the mathematical procedure based on the method of perturbation along characteristics, describing the instantaneous state of stress and strain on the waves of strong discontinuity. Such a procedure is presented for the problem of a spherical hole in the elastic-viscoplastic medium subject to the impact loading on the surface $r = r_0$. The existence of the yield limit $\varkappa_1$ is assumed. The viscoplastic deformations P appear above this limit. To account for the changes of viscoplastic properties of the material with the change of the deformation rate, the appropriate evolution equations are delivered for the inelastic deformation P. In the viscoplasticity, we face two ranges of $\dot{P}$ (II and IV), in which two different physical mechanisms are responsible for the permanent deformations. Bearing in mind the evolution equations for each region, the nonlinear ordinary differential equations are derived to describe the stress change on the face of the shock wave. The perturbation method of solution is applied to these equations. The stress on the wave follows in the form of the power series with respect to the small parameter. The excess function Φ introduced in the paper makes possible the utilization of this method to the equations for P in the region II, while the assumption on the viscosity function λ to depend on the small parameter yields the utilization of this method in the region IV.

Celem pracy jest opracowanie procedury matematycznej polegającej na zastosowaniu metody perturbacyjnej wzdłuż charakterystyk do określenia aktualnego stanu naprężenia i odkształcenia na falach silnych nieciągłości. Procedurę tę opracowano dla przykładu zagadnienia pustki kulistej w ośrodku sprężysto-lepkoplastycznym poddanej na powierzchni $r = r_0$ nagłemu obciążeniu. Założono istnienie granicy plastyczności $\varkappa_1$, powyżej której pojawiają się deformacje lepkoplastyczne P. Zmieniające się wraz ze zmianą prędkości własności lepkoplastyczne materiału zostały uwzględnione przez podanie odpowiednich równań ewolucji na deformację niesprężystą P. W lepkoplastyczności mamy do czynienia z dwoma zakresami zmienności $\dot{P}$ (II i IV), w których występują różne fizyczne mechanizmy odpowiedzialne za trwałe deformacje. Wykorzystując równania ewolucji otrzymano dla każdego obszaru nieliniowe równania różniczkowe zwyczajne, opisujące zmianę naprężenia na czole fali uderzeniowej. Do rozwiązania tych równań zastosowano metodę perturbacyjną, naprężenie na fali otrzymano w postaci szeregu potęgowego względem małego parametru. Zastosowanie takiej metody było możliwe, ponieważ do równań na P w obszarze II wprowadzoną funkcję nadwyżki Φ, a w obszarze IV funkcję lepkości λ przyjęto jako funkcję małego parametru. Parametr ten można dobrać na podstawie wyników eksperymentalnych.

Целью работы является разработка математической процедуры, заключающейся в применении пертурбационного метода вдоль характеристик, для определения напряженного и деформационного состояний на волнах сильного разрыва. Эта процедура разработана для случая задачи сферической пустоты в упруго-вязкопластической среде, подвергнутой на поверхности $r = r_0$ внезапной нагрузке. Предположено существование предела пластичности $\varkappa_1$, свыше которого появляются вязкопластические деформации P. Изменяющиеся совместно с изменением скорости вязкопластические свойства материала учтены путем приведения соответствующих уравнений эволюции для неупругой деформации P. В вязкопластичности имеем дело с двумя интервалами изменения $\dot{P}$ (II и IV), в которых выступают разные физические механизмы отвечающие за остаточные деформации. Используя уравнения эволюции, получены, для каждой области, нелинейные обыкновенные дифференциальные уравнения, описывающие изменение напряжения на фронте ударной волны. Для решения этих уравнений применен пертурбационный метод; напря-

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жение на волне получено в виде степенного ряда по отношению к малому параметру. Применение такого метода было возможно, т. к. в уравнениях для P в области II введенная функция превышения Φ, а в области IV, функция вязкости λ, приняты как функции малого параметра. Этот параметр можно подобрать на основе экспериментальных результатов.

1. Introduction

IN THE PAPER, the problem of a spherical hole in the infinite medium is solved for the impact loading of the boundary. The rate of deformation depends on the value of this loading: the high rates correspond to the large pressure. Since the medium under considerations is elastic-viscoplastic then its properties depend on the deformation rates. For this reason the viscoplastic material cannot be described by a single constitutive equation for the whole range of rates (or, equivalently, for an arbitrary loading). Such problems have been treated in many papers for a limited rate (e.g. for aluminium up to 10^3 sec^{-1}). Hence these considerations were limited to the so-called region II, in which the dissipation effects are due to the thermal activation mechanism (SEEGER 1955 [11], PERZYNA 1966 [10]). The experimental results of the recent years (DHARAN, HAUSER [1], FERGUSON, KUMAR, DORN [2], KUMAR, KUMBLE [4]) allow to take into considerations also the higher rates, a so-called region IV. In this region, the constitutive equation (PERZYNA 1974 [7]), DHARAN, HAUSER [1]) takes into account the influence of two mechanisms on the plastic flow. These are the mechanisms of the motion of dislocations, namely the mechanism of phonon viscosity and the mechanism of phonon scattering.

For small deformation, this equation is of the form of the sum $\dot{e} = \dot{e}^e + \dot{P}$, where $\dot{e}$ — the total rate of deformation, $\dot{e}^e$ — the rate of elastic deformation, $\dot{P}$ — the rate of inelastic deformation. Simultaneously, we assume that the properties of material are mainly influenced by the rate of inelastic deformation $\dot{P}$. To determine the constitutive relations, the evolution equation should be, first of all, formulated for the rate of inelastic deformation $\dot{P}$ in the regions II and IV.

For such a formulation of the problem, bearing in mind all ranges of the rate of deformation $\dot{P}$, the nonlinear differential equations are derived. They describe the change of stress along the ray on the face of the shock wave in the region II and the change of deformation in the region IV.

To solve these equations, the perturbation method is applied. The stress (in the region II) and the deformation (in the region IV) on the wave follow in the form of the finite power series with respect to the small parameter.

This method can be utilized due to the excess function Φ introduced into the evolution equations for the deformation rate $\dot{P}$ in the region II and the viscosity function λ depending on the small parameter — in the region IV. In the numerical calculations, the small parameter can be chosen according to the existing experimental data.

2. Constitutive relations

As we have already mentioned in the introduction, the deformation rate $\dot{e}$ in the elastic-viscoplastic medium is assumed to be of the form of sum $\dot{e} = \dot{e}^e + \dot{P}$. We assume the existence of the yield condition (in our case, it is to be the Huber-Mises condition $\sqrt{\mathrm{II}_s} = \varkappa_1$, where II_s is the second invariant of the deviatoric part of the stress tensor, $\varkappa_1$ — hardening parameter), i.e. for S such that $\sqrt{\mathrm{II}_s} > \varkappa_1$ there appear the viscoplastic deformations P, and for $\sqrt{\mathrm{II}_s} \leqslant \varkappa_1$ the material is elastic, linear and without viscous effects.

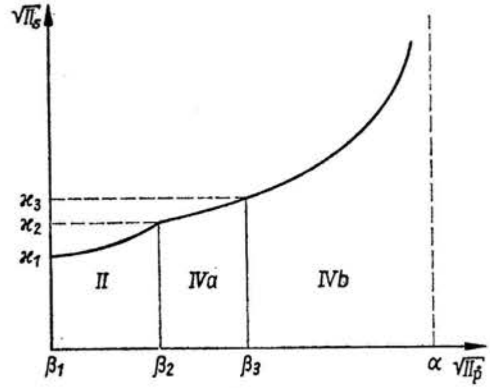

FIG. 1.

We assume that the curve $\sqrt{\mathrm{II}_s}-\sqrt{\mathrm{II}_{\dot{P}}}$ for the complex state of stresses (Fig. 1) is the same as the curve in the test of axial compression (it is confirmed by the experimental investigations). According to the experimental data, it has been noticed that for the rates higher than β_2 (the value of stress is such that $\sqrt{\mathrm{II}_s} > \varkappa_2$), the relation between the stresses and the rate of inelastic deformation is linear. However, it holds true only to a certain value of the rate, which is denoted by β_3 in the figure; it is the region IVa.

For high rates (bigger than β_3), the material is very sensitive to the change of rate; the small variation of $\sqrt{\mathrm{II}_{\dot{P}}}$ is accompanied by the big variation of $\sqrt{\mathrm{II}_s}$ and, for $\sqrt{\mathrm{II}_{\dot{P}}}$ approaching the rate α, the stress tends to infinity. The equality $\sqrt{\mathrm{II}_{\dot{P}}} = \alpha$ corresponds to the dislocation moving with the speed of sound, and $\alpha = \varrho_m b_c c/\sqrt{3}$, where ϱ_m is the density of moving dislocations, b — Burgers vector, c — speed of sound in the material. We have here a certain kind of the relativistic effect related to the speed of sound (PERZYNA [7], DHARAN, HAUSER [1]).

The Fig. 1, describing the influence of the rate $\dot{P}$ on the stress, is right only for the constant permanent deformation. During the whole process, the inelastic deformation changes, and, consequently, the curve $\left(\sqrt{\mathrm{II}_s}-\sqrt{\mathrm{II}_{\dot{P}}}\right)$ is going to be more complex, i.e. in real materials $\varkappa_1$, $\varkappa_2$ and α are functions of P. Taking into account the above considerations, we can write the following evolution equation for P in all ranges of the deformation rate[1].

$$
(2.1)\qquad \dot{P} = \begin{cases} 0 & \sqrt{\mathrm{II}_s} \leqslant \varkappa_1, \\ \gamma_1 \Phi\left(\dfrac{\sqrt{\mathrm{II}_s}}{\varkappa_1}-1\right)\dfrac{S}{\sqrt{\mathrm{II}_s}} & \varkappa_1 < \sqrt{\mathrm{II}_s} \leqslant \varkappa_2, \\ \dfrac{\gamma_2}{\lambda\left(1-\dfrac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)}\left(\dfrac{\sqrt{\mathrm{II}_s}}{\varkappa_2}-1\right)\dfrac{S}{\sqrt{\mathrm{II}_s}}+\gamma_1\Phi\left(\dfrac{\varkappa_2}{\varkappa_1}-1\right)\dfrac{S}{\sqrt{\mathrm{II}_s}}, & \sqrt{\mathrm{II}_s} > \varkappa_2, \end{cases}
$$

[1] The influence of the temperature is neglected in the paper.

where: γ_1, γ_2 — coefficients of viscosity, Φ — the excess function, λ — viscosity function (see PERZYNA [6]).

The function λ satisfies the conditions

$$\lim_{II_{\dot{p}} \to \alpha^2} \lambda = \infty \quad \text{and} \quad \lambda \equiv \lambda_0 \quad \text{for} \quad II_{\dot{p}} \leqslant \alpha^2. \tag{2.2}$$

Both functions Φ and λ are to be fitted to the experimental data (HAUSER, SIMMONS, DORN 1961 [3]) and are chosen according to the physical theory of plasticity.

3. Spherical wave

Let us consider an infinite elastic-viscoplastic medium with the spherical hole of the radius r_0 subject to the impact loading on the surface $r = r_0$. In the spherical frame of reference (r, φ, θ), we have the following components of the displacement

$$u_r = u(r, t), \quad u_\varphi = u_\theta = 0, \tag{3.1}$$

of the strain tensor

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}, \quad \varepsilon_{\varphi\varphi} = \varepsilon_{\theta\theta} = \frac{u}{r}, \tag{3.2}$$

and of the stress tensor

$$\sigma_{rr}(r, t), \quad \sigma_{\varphi\varphi}(r, t) = \sigma_{\theta\theta}(r, t). \tag{3.3}$$

The above problem is described by the two systems of equations, which differ one from another by the constitutive equation according to the actual value of the stress, viz. (2.1).

Let us start with the assumption that the applied load is such that the constitutive relation $(2.1)_2$ holds, i.e. it is the region II, where $\dot{e}_{ij} = \frac{1}{2\mu}\dot{S}_{ij} + \gamma_1 \Phi\left(\frac{\sqrt{II_s}}{\varkappa_1} - 1\right)\frac{S_{ij}}{\sqrt{II_s}}$ and $\varepsilon_{ii} = \frac{1}{3K}\sigma_{ii}$. Denoting $v = \frac{\partial u}{\partial t}$, ϱ — density of the material, we obtain

$$\begin{gathered}
-\varrho v_{,t} + \sigma_{rr,r} + 2\frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0,\\
v_{,r} - \frac{1}{2\mu}\sigma_{rr,t} + \frac{1}{2\mu}\sigma_{\varphi\varphi,t} - \frac{v}{r} - \sqrt{3}\gamma_1 \Phi\left(\frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{\sqrt{3}\varkappa_1} - 1\right) = 0,\\
3Kv_{,r} - \sigma_{rr,t} - 2\sigma_{\varphi\varphi,t} + 6K\frac{v}{r} = 0,\\
v_{,r} - \varepsilon_{rr,t} = 0,\\
\varepsilon_{\varphi\varphi,t} - \frac{v}{r} = 0.
\end{gathered} \tag{3.4}$$

It is the system of five partial differential equations of the first order for five unknown functions $(v, \sigma_{rr}, \sigma_{\varphi\varphi}, \varepsilon_{rr}, \varepsilon_{\varphi\varphi})$. The characteristic lines of the system (3.4) have the form

$$r = \text{const}, \quad r = r_0 \pm at + \text{const}, \tag{3.5}$$

where

$$a = \sqrt{\frac{4\mu+3K}{3\varrho}}, \quad \mu \text{ and } K \text{ — elastic constants.}$$

Appropriate conditions have to be fulfilled on each characteristics; in particular for the characteristics $r = r_0+at$, which is the wave of strong discontinuity in our case (the shock wave), we have

$$(3.6) \quad (4\mu+3K)dv-3a\,d\sigma_{rr}+\left[-\frac{6a}{r}(\sigma_{rr}-\sigma_{\varphi\varphi})-(4\mu-6K)\frac{v}{r}-4\mu\sqrt{3}\gamma\Phi\right]dr = 0.$$

Additionally, assuming the unperturbed region ahead of the wave, we obtain the kinematic and dynamic continuity conditions along the discontinuity line for the spherical wave

$$(3.7) \quad \begin{aligned} v+a\varepsilon_{rr} &= 0,\\ \varrho av+\sigma_{rr} &= 0. \end{aligned}$$

Due to the infinite value of $\dot{\varepsilon}_{rr}$ on the wave, the following constitutive relation on the discontinuity line holds

$$(3.8) \quad \sigma_{\varphi\varphi} = \sigma_{rr}\left(1-\frac{2\mu}{\varrho a^2}\right).$$

Taking into account (3.8), (3.7) in (3.6), we arrive at the ordinary differential equation on the front of the shock wave with the initial condition

$$(3.9) \quad \begin{aligned} \frac{d\sigma_{rr}}{dr} &= -\frac{\sigma_{rr}}{r}-\frac{2\mu\gamma_1}{\sqrt{3}a}\Phi\left(\frac{2\mu}{\sqrt{3}\varrho a^2\varkappa_1}\sigma_{rr}-1\right),\\ \sigma_{rr}|_{r=r_0} &= p_0. \end{aligned}$$

The stress on the characteristics $r = r_0+at$ is solely the function of the location $\sigma_{rr} = \sigma_{rr}(r)$.

The hardening parameter $\varkappa_1$ on the wave is constant due to the zero value of the plastic work (see PERZYNA, BEJDA 1964 [8]). To solve the equation (3.9) different forms of the function Φ have been proposed (PERZYNA 1963 [9]). We assume that Φ is the function of the small parameter ε, i.e.:

$$(3.10) \quad \Phi\left(\frac{\sqrt{\mathrm{II}_s}}{\varkappa_1}-1\right) = \overline{\Phi}\left(\frac{\sqrt{\mathrm{II}_s}}{\varkappa_1}-1, \varepsilon\right) = C\left(\frac{\sqrt{\mathrm{II}_s}}{\varkappa_1}-1\right)+\varepsilon\Phi^*\left(\varepsilon, \frac{\sqrt{\mathrm{II}_s}}{\varkappa_1}-1\right),$$

where C is the dimensionless constant.

The parameter ε and the function Φ^* can be chosen for a given material according to the experimental data(²). We assume that the function Φ^* has the derivatives $\left.\frac{\partial^k\Phi^*}{\partial\varepsilon^k}\right|_{\varepsilon=0}$ for arbitrary k. Denoting

$$(3.11) \quad \begin{aligned} A &\equiv C\frac{4\mu^2\gamma_1}{3\varrho\alpha^3\varkappa_1}; \quad B \equiv \frac{2\mu\gamma_1}{\sqrt{3}a};\\ \varphi^*(\sigma_{rr}, \varepsilon) &\equiv \frac{2\mu\gamma_1}{\sqrt{3}a}\Phi^*\left(\frac{2\mu}{\sqrt{3}\varrho a^2\varkappa_1}\sigma_{rr}-1, \varepsilon\right), \end{aligned}$$

(²) An example of such a choice will be discussed separately.

we can write the initial problem (3.9) in the form

$$\frac{d\sigma_{rr}}{dr} = \left(-\frac{1}{r} - A\right)\sigma_{rr} + B - \varepsilon\varphi^*(\sigma_{rr}, \varepsilon), \tag{3.12}$$
$$\sigma_{rr}|_{r=r_0} = p_0.$$

The solution (3.12) will depend on ε, i.e. $\sigma_{rr} = \sigma_{rr}(r, \varepsilon)$. Substituting $\varepsilon = 0$ in (3.12) we get

$$\frac{d\sigma^0_{rr}}{dr} = \left(-\frac{1}{r} - A\right)\sigma^0_{rr} + B, \tag{3.13}$$
$$\sigma^0_{rr}(r_0, 0) = p_0,$$

where $\sigma^0_{rr} = \sigma_{rr}(r, 0)$.

The equation (3.13) has the unique solution σ^0_{rr} and, hence, there exist (O'MALLEY 1974 [5]) such constants ε and D that the equation (3.12) possesses the solution of the form

$$\sigma_{rr} = \sigma^0_{rr} + \varepsilon\sigma'_{rr} + \frac{\varepsilon^2}{2}\sigma''_{rr} + 0(\varepsilon^3) \quad \text{for } r \text{ such that } |r - r_0| \leqslant D, \tag{3.14}$$
$$\sigma'_{rr} \equiv \left.\frac{d\sigma_{rr}}{d\varepsilon}\right|_{\varepsilon=0}, \quad \sigma''_{rr} \equiv \left.\frac{d^2\sigma_{rr}}{d\varepsilon^2}\right|_{\varepsilon=0}.$$

The functions σ'_{rr} and σ''_{rr} are, respectively, the solutions of the differential equations following from (3.12) by the subsequent differentiation with respect to ε for $\varepsilon = 0$. Integrating (3.13), we have

$$\sigma^0_{rr} = \frac{B}{A} - \frac{B}{A^2 r} + d_1\frac{e^{-Ar}}{r}. \tag{3.15}$$

The constant d_1, is determined from the initial condition $(3.13)_2$. The equation for σ'_{rr}:

$$\frac{d\sigma'_{rr}}{dr} = \left(-\frac{1}{r} - A\right)\sigma'_{rr} - \varphi^*(\sigma^0_{rr}, 0), \tag{3.16}$$
$$\sigma'_{rr}|_{r=r_0} = 0,$$

and, hence,

$$\sigma'_{rr} = \sigma'_{rr}(r, 0) = \left[-\int \varphi^*(\sigma^0_{rr}, 0)re^{Ar}dr + d_2\right]\frac{e^{-Ar}}{r}, \tag{3.17}$$

d_2 is such a constant that the condition $(3.16)_2$ is satisfied. Similarly, for σ''_{rr}

$$\frac{d\sigma''_{rr}}{dr} = \left(-\frac{1}{r} - A\right)\sigma''_{rr} - 2[\varphi^*_{,\sigma_{rr}}(\sigma^0_{rr}, 0)\sigma'_{rr} + \varphi^*_{,\varepsilon}(\sigma^0_{rr}, 0)], \tag{3.18}$$
$$\sigma''_{rr}|_{r=r_0} = 0,$$

and, after the integration

$$\sigma''_{rr} = \sigma''_{rr}(r, 0) = \left\{-2\int [\varphi^*_{,\sigma_{rr}}(\sigma^0_{rr}, 0)\sigma'_{rr} + \varphi^*_{,\varepsilon}(\sigma^0_{rr}, 0)]re^{Ar}dr + d_3\right\}\frac{e^{-Ar}}{r}. \tag{3.19}$$

Now, the solution of (3.12) can be written in the form (3.14)

$$\sigma_{rr}(r,\varepsilon) = \frac{B}{A} - \frac{B}{A^2 r} + d_1 \frac{e^{-Ar}}{r} + \varepsilon\left[-\int \varphi^*(\sigma^0_{rr}, 0) r e^{Ar} dr + d_2\right]\frac{e^{-Ar}}{r} + \varepsilon^2\left\{-\int [\varphi^*_{,\sigma_{rr}}(\sigma^0_{rr}, 0)\sigma'_{rr} + \varphi^*_{,\varepsilon}(\sigma^0_{rr}, 0)] r e^{Ar} dr + d_3\right\}\frac{e^{-Ar}}{r}. \tag{3.20}$$

For the load applied in the hole of the radius r_0 and such that $r = r_0$ for $\sqrt{\mathrm{II}_s} > \varkappa_2$, we are in the region IV and the relation $(2.1)_3$ holds for $\dot{P}$. To utilize $(2.1)_3$ in the governing set of equations, we have to express the function $\dot{P}$ in terms of S and ε. It is possible for the nonlinear function λ, under the additional assumption of the following dependence of λ on the small parameter ε

$$\lambda\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right) = \tilde{\lambda}\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}, \varepsilon\right) = \lambda_0 + \varepsilon\hat{\lambda}\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}, \varepsilon\right), \quad \lambda_0 \text{ — const}, \tag{3.21}$$

where $\tilde{\lambda}$ possesses arbitrary derivatives $\left.\frac{\partial^k \tilde{\lambda}}{\partial \varepsilon^k}\right|_{\varepsilon=0}$. Besides, $\tilde{\lambda}$ has to satisfy the condition

$$\lim_{\mathrm{II}_{\dot{P}} \to \alpha^2} \tilde{\lambda} = \infty, \tag{3.22}$$

following from (2.2).

For $\varepsilon = 0$ we obtain the region IVa.

The functions $\varkappa_2$ and $\varkappa_1$, depending on the constant deformation P, can be expressed in terms of the arguments $\boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$ by use of Hooke's relations, i.e.

$$\begin{aligned} \varkappa_2 &= \varkappa_2(P) = \bar{\varkappa}_2(\boldsymbol{\sigma}, \boldsymbol{\epsilon}), \\ \alpha &= \alpha(P) = \bar{\alpha}(\boldsymbol{\sigma}, \boldsymbol{\epsilon}). \end{aligned} \tag{3.23}$$

As we show further in the paper, the function $\tilde{\lambda}$ is chosen to limit in a certain way the constants appearing in the relation of DHARAN and HAUSER [1] between $\dot{P}$ and S in the region IV. The substitution of (3.21) in the relation $(2.1)_3$ yields the system of equations, describing $\dot{P}_{rr}$ and $\dot{P}_{\varphi\varphi}$:

$$\begin{aligned} &\dot{P}_{rr}\lambda_0 - \frac{2}{\sqrt{3}}\gamma_2 \mathrm{w} - \frac{2}{\sqrt{3}}\gamma_1 \mathrm{v}\lambda_0 = \varepsilon\tilde{\lambda}\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}, \varepsilon\right)\left(\frac{2}{\sqrt{3}}\gamma_1 \mathrm{v} - \dot{P}_{rr}\right), \\ &\dot{P}_{\varphi\varphi}\lambda_0 + \frac{1}{\sqrt{3}}\gamma_2 \mathrm{w} + \frac{1}{\sqrt{3}}\gamma_1 \mathrm{v}\lambda_0 = \varepsilon\tilde{\lambda}\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}, \varepsilon\right)\left(-\frac{1}{\sqrt{3}}\gamma_1 \mathrm{v} - \dot{P}_{\varphi\varphi}\right), \\ &\mathrm{w} \equiv \frac{\sigma_{rr}-\sigma_{\varphi\varphi}}{\sqrt{3}\varkappa_2} - 1, \quad \mathrm{v} \equiv \Phi\left(\frac{\varkappa_2}{\varkappa_1} - 1\right), \quad \text{and} \quad \mathrm{w} = \mathrm{w}(\varepsilon), \quad \mathrm{v} = \mathrm{v}(\varepsilon). \end{aligned} \tag{3.24}$$

Now we demonstrate briefly the procedure, applied to the system (3.24), to find $\dot{P} = f(\boldsymbol{\sigma}, \boldsymbol{\epsilon})$.

For a given system of equations with respect to z

$$\begin{aligned} g_1(\mathrm{z}, \mathrm{u}) &= \varepsilon f_1(\varepsilon), \\ g_2(\mathrm{z}, \mathrm{u}) &= \varepsilon f_2(\varepsilon), \end{aligned} \tag{3.25}$$

where $z = (z_1, z_2)$, $u = (w, v)$ and $z = z(\varepsilon)$, $u = u(\varepsilon)$, we can find the approximate solution of (3.25) in the form

$$
\begin{aligned}
z_1 &= z_1^0 + \varepsilon z_1' + \frac{\varepsilon^2}{2} z_1'' + 0(\varepsilon^3) \\
z_2 &= z_2^0 + \varepsilon z_2' + \frac{\varepsilon^2}{2} z_2'' + 0(\varepsilon^3)
\end{aligned}
\qquad z_1' = \left.\frac{dz_1}{d\varepsilon}\right|_{\varepsilon=0}. \tag{3.26}
$$

The method to be applied is analogous to the perturbation method, which has been used for the differential equation (3.12). We assume that, for $\varepsilon = 0$, the system

$$
\begin{aligned}
g_1(z^0, u^0) &= 0, \\
g_2(z^0, u^0) &= 0,
\end{aligned} \tag{3.27}
$$

with the conditions

$$u^0 \equiv u(0), \quad z^0 = z(0),$$

has the unique solution z^0. Then, for sufficiently small ε and a certain region of the variation of z, we obtain the system (3.25). Differentiating (3.25) with respect to ε at the point $\varepsilon = 0$, we arrive at the linear system of equations for z':

$$
\begin{aligned}
g_{1,1}^0 z_1' + g_{1,2}^0 z_2' &= f_1^0 - g_{1,u}^0 \cdot u', \\
g_{2,1}^0 z_1' + g_{2,2}^0 z_2' &= f_2^0 - g_{2,u}^0 \cdot u'.
\end{aligned} \tag{3.28}
$$

The upper 0 stands for the values of these functions calculated at $\varepsilon = 0$. For instance: $f_1^0 = f_1(0)$, $g_{1,1}^0 = \left.\dfrac{\partial g_1}{\partial z_1}\right|_{\varepsilon=0}$.

From the Cramer's formulae

$$
z_1' = \frac{\begin{vmatrix} f_1^0 - g_{1,u}^0 \cdot u' & g_{1,2}^0 \\ f_2^0 - g_{2,u}^0 \cdot u' & g_{2,2}^0 \end{vmatrix}}{\begin{vmatrix} g_{1,1}^0 & g_{2,1}^0 \\ g_{1,2}^0 & g_{2,2}^0 \end{vmatrix}}; \quad
z_2' = \frac{\begin{vmatrix} g_{1,1}^0 & f_1^0 - g_{1,u}^0 \cdot u' \\ g_{2,1}^0 & f_2^0 - g_{2,u}^0 \cdot u' \end{vmatrix}}{\begin{vmatrix} g_{1,1}^0 & g_{2,1}^0 \\ g_{1,2}^0 & g_{2,2}^0 \end{vmatrix}}. \tag{3.29}
$$

Similarly, the twofold differentiation of the equations (3.25) yields the system for z''. To simplify the further calculations, let us assume that the functions g_i; $i = 1, 2$ are linear with respect to z and u. According to (3.24), it takes place in our case. Then

$$
\begin{aligned}
g_{1,1}^0 z_1'' + g_{1,2}^0 z_2'' &= 2f_{1,\varepsilon}^0 - g_{1,u}^0 \cdot u'', \\
g_{2,1}^0 z_1'' + g_{2,2}^0 z_2'' &= 2f_{2,\varepsilon}^0 - g_{2,u}^0 \cdot u''.
\end{aligned} \tag{3.30}
$$

Hence, the solution is of the form

$$
z_1'' = \frac{\begin{vmatrix} 2f_{1,\varepsilon}^0 - g_{1,u}^0 \cdot u'' & g_{1,2}^0 \\ 2f_{2,\varepsilon}^0 - g_{2,u}^0 \cdot u'' & g_{2,2}^0 \end{vmatrix}}{\begin{vmatrix} g_{1,1}^0 & g_{1,2}^0 \\ g_{2,1}^0 & g_{2,2}^0 \end{vmatrix}}; \quad
z_2'' = \frac{\begin{vmatrix} g_{1,1}^0 & 2f_{1,\varepsilon}^0 - g_{1,u}^0 \cdot u'' \\ g_{2,1}^0 & 2f_{2,\varepsilon}^0 - g_{2,u}^0 \cdot u'' \end{vmatrix}}{\begin{vmatrix} g_{1,1}^0 & g_{1,2}^0 \\ g_{2,1}^0 & g_{2,2}^0 \end{vmatrix}}. \tag{3.31}
$$

In our case, according to the formulae (3.28), $z = (\dot{P}_{rr}, \dot{P}_{\varphi\varphi})$, i.e. we seek $\dot{P}_{rr}$ and $\dot{P}_{\varphi\varphi}$ in the form

(3.32)
$$\dot{P}_{rr}(\varepsilon) = \dot{P}^0_{rr} + \varepsilon\dot{P}'_{rr} + \frac{\varepsilon^2}{2}\dot{P}''_{rr} + 0(\varepsilon^3),$$
$$\dot{P}_{\varphi\varphi}(\varepsilon) = \dot{P}^0_{\varphi\varphi} + \varepsilon\dot{P}'_{\varphi\varphi} + \frac{\varepsilon^2}{2}\dot{P}''_{\varphi\varphi} + 0(\varepsilon^3).$$

Let us notice that

$$g_{1,1} = g_{2,2} = \lambda_0, \quad g_{1,2} = g_{2,1} = 0.$$

Hence

$$\begin{vmatrix} g^0_{1,1} & g^0_{1,2} \\ g^0_{2,1} & g^0_{2,2} \end{vmatrix} = \lambda_0^2.$$

For $\varepsilon = 0$, we obtain $\dot{P}^0_{rr}$ and $\dot{P}^0_{\varphi\varphi}$ from (3.24) in the form:

(3.33)
$$\dot{P}^0_{rr} = \frac{2}{\sqrt{3}}\frac{\gamma_2}{\lambda_0}\mathrm{w}^0 + \frac{2}{\sqrt{3}}\gamma_1\mathrm{v}^0 \equiv \frac{2}{\sqrt{3}}f_0(\boldsymbol{\sigma}^0, \boldsymbol{\epsilon}^0),$$
$$\dot{P}^0_{\varphi\varphi} = -\frac{1}{\sqrt{3}}\frac{\gamma_2}{\lambda_0}\mathrm{w}^0 - \frac{1}{\sqrt{3}}\gamma_1\mathrm{v}^0 \equiv -\frac{1}{\sqrt{3}}f_0(\boldsymbol{\sigma}^0, \boldsymbol{\epsilon}^0),$$

$$\mathrm{w}^0 = \mathrm{w}(0), \quad \mathrm{v}^0 = \mathrm{v}(0) \quad \text{i.e.} \quad \mathrm{w}^0 = \frac{\sigma^0_{rr} - \sigma^0_{\varphi\varphi}}{\sqrt{3}\,\varkappa_2(\boldsymbol{\sigma}^0, \boldsymbol{\epsilon}^0)} - 1, \quad \mathrm{v}^0 = \Phi\left(\frac{\varkappa^0_2}{\varkappa^0_1} - 1\right) \text{ (}^3\text{)}.$$

Similarly to $\dot{P}$, we assume here that $\boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$ can be expressed in the form of the series with respect to ε:

(3.34)
$$\sigma_{rr}(\varepsilon) = \sigma^0_{rr} + \varepsilon\sigma'_{rr} + \frac{\varepsilon^2}{2}\sigma''_{rr} + 0(\varepsilon^3),$$
$$\sigma_{\varphi\varphi}(\varepsilon) = \sigma^0_{\varphi\varphi} + \varepsilon\sigma'_{\varphi\varphi} + \frac{\varepsilon^2}{2}\sigma''_{rr} + 0(\varepsilon^3),$$
$$\varepsilon_{rr}(\varepsilon) = \varepsilon^0_{rr} + \varepsilon\varepsilon'_{rr} + \frac{\varepsilon^2}{2}\varepsilon''_{rr} + 0(\varepsilon^3),$$
$$\varepsilon_{\varphi\varphi}(\varepsilon) = \varepsilon^0_{\varphi\varphi} + \varepsilon\varepsilon'_{\varphi\varphi} + \frac{\varepsilon^2}{2}\varepsilon''_{\varphi\varphi} + 0(\varepsilon^3).$$

The application of the formulae (3.29) yields

(3.35)
$$\dot{P}'_{rr} = -\frac{2}{\sqrt{3}}\frac{\tilde{\lambda}^0}{\lambda_0}\gamma_2\mathrm{w}^0 + \frac{2}{\sqrt{3}}\frac{\gamma_2}{\lambda_0}\mathrm{w}' + \frac{2}{\sqrt{3}}\gamma_1\mathrm{v}' = \frac{2}{\sqrt{3}}f_1(\boldsymbol{\sigma}^0, \boldsymbol{\epsilon}^0, \boldsymbol{\sigma}', \boldsymbol{\epsilon}'),$$
$$\dot{P}'_{\varphi\varphi} = \frac{1}{\sqrt{3}}\frac{\tilde{\lambda}^0}{\lambda_0}\gamma_2\mathrm{w}^0 - \frac{1}{\sqrt{3}}\frac{\gamma_2}{\lambda_0}\mathrm{w}' - \frac{1}{\sqrt{3}}\gamma_1\mathrm{v}' = -\frac{1}{\sqrt{3}}f_1(\boldsymbol{\sigma}^0, \boldsymbol{\epsilon}^0, \boldsymbol{\sigma}', \boldsymbol{\epsilon}'),$$
$$\tilde{\lambda}^0 = \tilde{\lambda}\left(1 - \frac{\mathrm{II}_{\dot{P}0}}{(\alpha^0)^2}, 0\right), \quad \mathrm{w}' = \left.\frac{d\mathrm{w}}{d\varepsilon}\right|_{\varepsilon=0}, \quad \mathrm{v}' = \left.\frac{d\mathrm{v}}{d\varepsilon}\right|_{\varepsilon=0}.$$

(3) w^0 and v^0 are not constant. They depend either on $(\boldsymbol{\sigma}^0, \boldsymbol{\epsilon}^0)$ or on $(\boldsymbol{\sigma}^0, P^0)$ (see (3.23)).

In the same way, we have from (3.31)

$$\dot{P}''_{rr} = \frac{2}{\sqrt{3}}\left\{\frac{\gamma_2}{\lambda_0}w''+\gamma_1 v''-2\frac{\gamma_2}{\lambda_0^2}\left[w^0\frac{d\tilde{\lambda}}{d\varepsilon}\bigg|_{\varepsilon=0}-\tilde{\lambda}^0\left(\frac{\tilde{\lambda}^0}{\lambda_0}w^0-w'\right)\right]\right\} = \frac{2}{\sqrt{3}}f_2(\sigma^0,\epsilon^0,\sigma',\epsilon',\sigma'',\epsilon''),$$

(3.36)

$$\dot{P}''_{\varphi\varphi} = -\frac{1}{\sqrt{3}}\left\{\frac{\gamma_2}{\lambda_0}w''+\gamma_1 v''-2\frac{\gamma_2}{\lambda_0^2}\left[w^0\frac{d\tilde{\lambda}}{d\varepsilon}\bigg|_{\varepsilon=0}-\tilde{\lambda}^0\left(\frac{\tilde{\lambda}^0}{\lambda_0}w^0-w'\right)\right]\right\} = -\frac{1}{\sqrt{3}}f_2(\sigma^0,\epsilon^0,\sigma',\epsilon',\sigma'',\epsilon'').$$

Finally, taking into account (3.33), (3.35) and (3.36), we can write the expansion (3.32) of $\dot{P}$ with respect to ε up to the term $0(\varepsilon^3)$ in the form

$$\dot{P} = \left(f_0+\varepsilon f_1+\frac{\varepsilon^2}{2}f_2\right)\frac{S}{\sqrt{\mathrm{II}_s}}, \quad \dot{P} = \dot{P}(\sigma^0,\epsilon^0,\sigma',\epsilon',\sigma'',\epsilon''). \tag{3.37}$$

Consequently, we are able to write the whole system of equations in the region IV, in the same manner as for the region II:

(3.38)

$$-\varrho v_{,t}+\sigma_{rr,r}+2\frac{\sigma_{rr}-\sigma_{\varphi\varphi}}{r}=0,$$

$$v_{,r}-\frac{1}{2\mu}\sigma_{rr,t}+\frac{1}{2\mu}\sigma_{\varphi\varphi,t}-\frac{v}{r}-\sqrt{3}\left(f_0+\varepsilon f_1+\frac{\varepsilon^2}{2}f_2\right)=0,$$

$$3Kv_{,r}-\sigma_{rr,t}-2\sigma_{\varphi\varphi,t}+6K\frac{v}{r}=0,$$

$$v_{,r}-\varepsilon_{rr,t}=0,$$

$$\varepsilon_{\varphi\varphi,t}-\frac{v}{r}=0.$$

The characteristic equation for this system is the same as for (3.4). Therefore we obtain the differential equation for the stress on the face of the shock wave Σ: $r = r_0+at$

(3.39)

$$\frac{d\sigma_{rr}}{dr} = -\frac{\sigma_{rr}}{r}-\frac{2\mu}{\sqrt{3}a}\left(f_0+\varepsilon f_1+\frac{\varepsilon^2}{2}f_2\right)\bigg|_{\Sigma},$$

$$\sigma_{rr}|_{r=r_0} = P_0.$$

Due to the lack of jump of the inelastic deformation P on the wave Σ and the assumption on the unperturbed state ahead of the wave, we have $P|_\Sigma = 0$. Hence $P'|_\Sigma = P''|_\Sigma = 0$. The simple consequence of this fact and of the equation (3.8) is

(3.40)

$$\varkappa_2^0|_\Sigma = \text{const},\quad v^0|_\Sigma = \text{const},\quad \varkappa_2'|_\Sigma = \varkappa_1''|_\Sigma = v'|_\Sigma = v''|_\Sigma = 0,$$

$$w^0|_\Sigma = \frac{2\mu}{\sqrt{3}\varrho a^2\varkappa_2^0|_\Sigma}\sigma_{rr}^0-1,\quad w'|_\Sigma = \frac{2\mu}{\sqrt{3}\varrho a^2\varkappa_2^0|_\Sigma}\sigma'_{rr},\quad w''|_\Sigma = \frac{2\mu}{\sqrt{3}\varrho a^2\varkappa_2^0|_\Sigma}\sigma''_{rr}.$$

Substituting the relations (3.40) in the function, appearing in the parentheses of the formula $(3.39)_1$, we can write the initial problem (3.39) in the different form

$$\frac{d\sigma_{rr}}{dr} = -\frac{\sigma_{rr}}{r} - A_1\sigma^0_{rr} + B_1 + \varepsilon(-A_1\sigma'_{rr} + \psi_1(\sigma^0_{rr})) + \frac{\varepsilon^2}{2}(-A_1\sigma''_{rr} + \psi_2(\sigma^0_{rr}, \sigma'_{rr})), \tag{3.41}$$

$$\sigma_{rr}|_{r=r_0} = p_0,$$

where

$$A \equiv \frac{4\mu^2\gamma_2}{3\varrho a^3\lambda_0\varkappa^0_2|_\Sigma} = \text{const}, \quad B_1 \equiv \frac{2\mu}{\sqrt{3}a}\left(\frac{\gamma_2}{\lambda_0} - \gamma_1 \mathrm{v}^0|_\Sigma\right) = \text{const},$$

$$\psi_1(\sigma^0_{rr}) = \frac{2\mu\gamma_2}{\sqrt{3}a\lambda_0^2}\tilde{\lambda}^0|_\Sigma \mathrm{w}^0|_\Sigma,$$

$$\psi_2(\sigma^0_{rr}, \sigma'_{rr}) = \frac{4\mu\gamma_2}{\lambda_0^2}\left[\frac{d\tilde{\lambda}}{d\varepsilon}\bigg|_{\varepsilon=0}\mathrm{w}^0 - \frac{\tilde{\lambda}^0}{\lambda_0^2}\gamma_2\left(\frac{\tilde{\lambda}^0}{\lambda_0}\mathrm{w}^0 - \mathrm{w}'\right)\right]\bigg|_\Sigma.$$

Substituting $\varepsilon = 0$ in the equation (3.41), we obtain

$$\frac{d\sigma^0_{rr}}{dr} = \left(-\frac{1}{r} - A_1\right)\sigma^0_{rr} + B_1, \quad \sigma^0_{rr}|_{r=r_0} = p_0. \tag{3.42}$$

The equation $(3.42)_1$ differs from the equation $(3.13)_1$ by the constants only; hence (see (3.15))

$$\sigma^0_{rr} = \frac{B_1}{A_1} - \frac{B_1}{A_1^2 r} + C_1\frac{e^{-A_1 r}}{r}. \tag{3.43}$$

Differentiating (3.41) with respect to ε at the point $\varepsilon = 0$, we get

$$\frac{d\sigma'_{rr}}{dr} = \left(-\frac{1}{r} - A_1\right)\sigma'_{rr} + \psi_1(\sigma^0_{rr}), \quad \sigma'_{rr}|_{r=r_0} = 0. \tag{3.44}$$

The integration of (3.44) (see (3.16) and (3.17)) yields

$$\sigma'_{rr} = \sigma'_{rr}(0, r) = \left[\int \psi_1(\sigma^0_{rr}) r e^{A_1 r} dr + C_2\right]\frac{e^{-A_1 r}}{r}. \tag{3.45}$$

Similarly to (3.18) obtained from (3.12), we arrive at the equation for σ''_{rr} by use of (3.41)

$$\frac{d\sigma''_{rr}}{dr} = \left(-\frac{1}{r} - A_1\right)\sigma''_{rr} + \psi_2(\sigma^0_{rr}, \sigma'_{rr}), \tag{3.46}$$

$$\sigma''_{rr}|_{r=r_0} = 0,$$

and then

$$\sigma''_{rr} = \sigma''_{rr}(0, r) = \left[\int \psi_2(\sigma^0_{rr}, \sigma'_{rr}) r e^{A_1 r} dr + C_3\right]\frac{e^{-A_1 r}}{r}. \tag{3.47}$$

The solution of the problem (3.41) can be written in the form $((3.41)_1)$:

$$\sigma_{rr}(\varepsilon, r) = \frac{B_1}{A_1} - \frac{B_1}{A_1 r} + C_1\frac{e^{-A_1 r}}{r} + \varepsilon\,[\psi_1(\sigma^0_{rr}) r e^{A_1 r} dr + C_2]\frac{e^{-A_1 r}}{r} + \frac{\varepsilon^2}{2}\left[\int \psi_2(\sigma^0_{rr}, \sigma'_{rr}) r e^{A_1 r} dr + C_3\right]\frac{e^{-A_1 r}}{r}, \tag{3.48}$$

C_1, C_2, C_3 are such constants that the initial conditions are satisfied.

4. Final remarks

The solutions (3.20) and (3.48) obtained in the previous Section gives the values of the stress on the face of the shock wave in terms of the assumed initial condition p_0.

Due to the geometrical dispersion and the considered viscoplastic model, the stress on the wave is the decreasing function of the radius r. Hence, for sufficiently large p_0, the stress $\sigma_{rr}(r)$ reaches the values corresponding to all regions. The solution (3.48) holds for r taken from the interval: $r_0 \leqslant r \leqslant r_1$, where r_1 satisfies the condition

$$\sqrt{\mathrm{II}_s(r_1)} = \varkappa_2,$$

or, on the face of the shock wave

$$\frac{2\mu}{\sqrt{3}\varrho a^2}\sigma_{rr}(r_1) = \varkappa_2|_\Sigma,$$

σ_{rr} being the solution (3.48).

For $r \geqslant r_1$, the equation $(2.1)_2$ for $\dot{P}$ holds (the region II) and the value of the stress on the discontinuity line follows from (3.12) with the different initial condition

$$\sigma_{rr}|_{r=r_1} = p_1,$$

where

$$p_1 = \frac{\sqrt{3}\,\varrho a^2 \varkappa_2|_\Sigma}{2\mu}.$$

Similarly, the relation (3.20) holds for r from the interval $r_1 \leqslant r < r_2$, being such that

$$\sqrt{\mathrm{II}_s(r_2)} = \varkappa_1, \quad \left(\frac{2\mu}{\sqrt{3}\,\varrho a^2}\sigma_{rr}(r_2) = \varkappa_1\right).$$

For $r > r_2$, the elasticity equations hold $\left(\sqrt{\mathrm{II}_s} \leqslant \varkappa_1\right)$. Then the solution is

$$\sigma_{rr} = \frac{\sqrt{3}\varrho a^2 \varkappa_1}{2\mu}\frac{r_2}{r}.$$

The calculation of σ_{rr} and, hence, v and ε_{rr} (from (3.7)) on the wave $r = r_0 + at$ yields the possibility of approximate solutions of the equations (3.4). It may be, for instance, the method of finite differences along characteristics.

The conditions on characteristics $r = \text{const}$ and $r = r_0 \pm at + \text{const}$

$$3K\left(\sqrt{3}\gamma\Phi + 3\frac{v}{r}\right)dt + \left(\frac{3K}{2\mu} - 1\right)d\sigma_{rr} = \left(\frac{3K}{2\mu} + 2\right)d\sigma_{\varphi\varphi},$$

$$d\varepsilon_{rr} - \frac{1}{2\mu}d\sigma_{rr} + \frac{1}{2\mu}d\sigma_{\varphi\varphi} - \left(\frac{v}{r} + \sqrt{3}\gamma\Phi\right)dt = 0,$$

(4.1)

$$d\varepsilon_{\varphi\varphi} = \frac{v}{r}dt,$$

$$\left[\pm\frac{6a}{r}(\sigma_{rr} - \sigma_{\varphi\varphi}) + 4\sqrt{3}\mu\gamma\Phi + \frac{v}{r}(4\mu - 6K)\right]dr - (4\mu + 3K)dv \pm 3a\,d\sigma_{rr} = 0,$$

the boundary condition

$$\sigma_{rr}(r_0, t) = -p(t), \qquad p(t) > 0, \quad p(0) = p_0, \tag{4.2}$$

and the known functions σ_{rr}, $\sigma_{\varphi\varphi}$, v, ε_{rr}, $\varepsilon_{\varphi\varphi}$ on $r = r_0+at$ yield the unique solution in the region $D = \{(r, t): r > r_0+at\}$(4). For the constant or monotonically increasing function $p(t)$, the region D is the region of viscoplastic strains. For different forms of loading $p(t)$, both the elastic and viscoplastic strain regions will appear in D.

Let us determine now the function λ, appearing in the formula (3.21). Choosing the function $\tilde{\lambda}$, we apply the hint delivered by the results of Dharan and Hauser. Starting from the experimental data and the physical theory of plasticity, these authors have derived the relation between the stress and the deformation rate. Bearing in mind this relation we arrive at the form of the viscosity function λ (P. PERZYNA [7])

$$\lambda\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right) = A_1\left[\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{\frac{1}{2}}+\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{-\frac{3}{2}}\right]+A_2\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{-1}. \tag{4.3}$$

The first term in this formula for λ corresponds to the dumping mechanism of the motion of dislocation, while the second one is connected with the phonon scattering.

For the deformation rate satisfying the condition $\sqrt{\mathrm{II}_{\dot{P}}} > \beta_3$ (for alumium $\beta_3 \sim 10^4\,\mathrm{sec}^{-1}$) the quantities A_1 and A_2 are related by the inequality $A_1 \ll A_2$ and both are smaller than unity in the room temperature. Neglecting the influence of temperature, the quantity A_2 is constant while A_1 is the decreasing quantity with the increasing speed of moving dislocations (i.e. with the growth of $\dot{P}$). It follows from the diminishing influence of the dumping mechanism on the motion of dislocation for high rates (the region IVb) and the dominant influence of the phonon scattering mechanism. For this reason, assuming A_1 and A_2 to be the constants of the order ε^2 and ε, respectively, we take the function λ in the form

$$\lambda\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right) = \hat{\lambda}\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}, \varepsilon\right) = \lambda_0+\varepsilon\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{-1}+\varepsilon^2\left[\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{\frac{1}{2}}+\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{-\frac{3}{2}}\right],$$

i.e.

$$\tilde{\lambda}\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}, \varepsilon\right) = \left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{-1}+\varepsilon\left[\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{\frac{1}{2}}+\left(1-\frac{\mathrm{II}_{\dot{P}}}{\alpha^2}\right)^{-\frac{3}{2}}\right].$$

For $\varepsilon = 0$, we have $\lambda = \lambda_0$, and it is the region IVa.

This approach allows to obtain the analogous differential equation for σ_{rr} both in the region II and in the region IV. Hence, it allows to apply the same perturbation procedure along the characteristics in both cases and to calculate the actual values of the stress σ_r on the waves of strong discontinuity.

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