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Mass and energy transport in the atmospheric neutrally stable layer(*)

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IN A CLOUDLESS atmosphere, heated by the hot earth's surface, a neutrally stable layer may appear, with almost adiabatic distribution of temperatures. In this layer an intensive mixing may occur, provoking a convective transport of heat and water vapour. The object of our interest is to determine the instantaneous thickness of this layer and its thermodynamic parameters for known initial structure of the atmosphere and given heat flux and evaporation rate from the earth. The introduced simple theoretical model of the considered phenomenon allows to deduce the equations describing the growth of the layer and its temperature increase on the basis of the principles of mass and energy transfer and of the stability condition at the upper mixing layer. The analysis of the humidity distribution allows also to determine the elevation at which the first condensation clouds may appear.

W podgrzewanej powierzchnią ziemi bezchmurnej atmosferze może powstawać dolna, obojętnie stabilna warstwa o prawie adiabatycznym rozkładzie temperatur. W warstwie tej występuje intensywne mieszanie, powodujące konwekcyjną wymianę ciepła i pary wodnej. Przedmiotem pracy jest wyznaczenie chwilowej grubości tej warstwy i jej parametrów termodynamicznych, przy danej początkowej strukturze atmosfery i przy założeniu strumienia ciepła i wydatku parowania z powierzchni ziemi. Przedstawiony uproszczony model teoretyczny zjawiska pozwala z zasad zachowania masy i energii oraz z warunku stabilności na górnej powierzchni warstwy mieszania wyprowadzić równania opisujące narastanie warstwy i wzrost temperatury przy ziemi. Analiza rozkładu wilgotności pozwala określić wysokość, na której wskutek zapoczątkowania procesu kondensacji mogą powstawać pierwsze obłoki.

В подогреваемой поверхностью земли безоблачной атмосфере может возникать нижний, нейтрально стабильный слой с почти адиабатическим распределением температур. В этом слое выступает интенсивное смешивание, вызывающее конвекционные теплообмен и обмен водяным паром. Предметом работы является определение мгновенной толщины этого слоя и его термодинамических параметров при заданной начальной структуре атмосферы и при заданных потоке тепла и эффективности испарения из поверхности земли. Представленная упрощенная теоретическая модель явления позволяет, из законов сохранения массы и энергии, а также из условия стабильности на верхней поверхности слоя смешивания, вывести уравнения описывающие нарастание слоя и рост температуры при земле. Анализ распределения влажности позволяет определить высоту, на которой, вследствие инициирования процесса конденсации, могут возникать первые облока.

1. Introduction

THE VERTICAL heat transfer through the atmosphere exerts an influence on the stability conditions in stratified atmospheric layers. On the other hand it is much influenced itself by these stability conditions. In consequence, the cooling or heating of the air by the Earth's surface are accompanied by different heat transfer phenomena.

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The stability of the atmosphere is determined by the vertical distribution of the potential temperature θ defined as the temperature of an element transported adiabatically to the reference level. In the stably stratified layer the potential temperature θ increases with the height z (Fig. 1), although the corresponding real temperature T may decrease. The

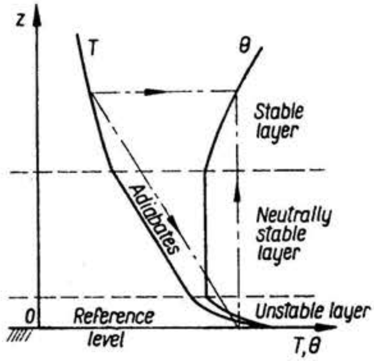

FIG. 1.

decrease of θ with z corresponds to an unstable condition when the relatively hotter and less heavy layers are situated below the cooler and heavier ones. Because of the buoyancy forces, arising then, the unstable stratification provokes vertical mixing during which an almost adiabatic motion of unstably situated elements takes place. The vertical adiabatic distribution of the temperature T corresponds to the neutral stability with $\theta = \text{const}$.

Downward cooling does not disturb stable stratification. The lower parts become cooler and heavier and, in consequence, more stable. This case shall not be the object of our interest here.

On the contrary, upward heating from a hot surface of the Earth may perturb stable stratification of the lowest thin layer adjacent to the Earth. This layer, being relatively hotter and less heavy, should move upward and be replaced by cooler surrounding air going down. Thus, above the Earth's surface a thicker, neutrally stable layer may appear, in which the convective turbulent heat flux directed upward is accompanied by an intensive inner vertical mixing.

A simplified theoretical model of such a neutral mixing layer shall be the object of our attention here. In this model we will avoid going into many details as to the mechanism and structure of vertical mixing, and will deal mainly with the final results concerning transport phenomena of mass, energy and water vapour in the neutral layer. Our considerations could be applied in an approximate description of some atmospheric phenomena which occur during a hot sunny summer day from early morning until the first clouds appear.

In our model the atmospheric air shall be considered as an ideal gas mixture with variable specific humidity ξ defined as the mass ratio of water vapour to the total mixture. Since the value ξ is very small, the changes of humidity shall be neglected in transport equations of total mass and energy.

The phenomenon shall be considered as horizontally uniform, depending on the vertical coordinate z and on the time t only. The initial vertical distribution of the potential temperature $\theta(z, 0)$ and of the specific humidity $\xi(z, 0)$ should correspond to the stable stratification with $\partial\theta(z,0)/\partial z > 0$ and to the unsaturated air conditions with $\xi(z, 0) < \xi_s(z, 0)$, where ξ_s is the saturation specific humidity (Fig. 2). On the hot and humid

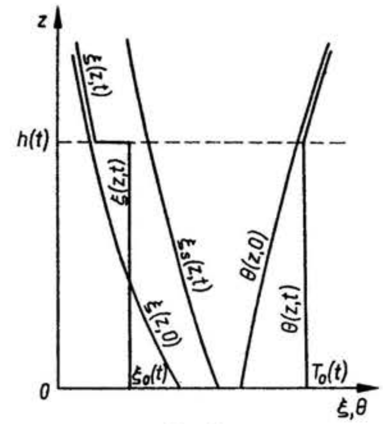

FIG. 2.

Earth's surface $z = 0$ a uniform heat flux $\dot{Q}(t)$ and evaporation rate $\dot{m}(t)$ are directed upward (Fig. 3). In $\dot{Q}$ the convective evaporation term is also contained. Above the Earth's surface a mixing atmospheric layer appears. Ideal mixing in this layer shall be assumed with potential temperature $\theta = T_0(t)$ and specific humidity $\xi = \xi_0(t)$ (Fig. 2), not de-

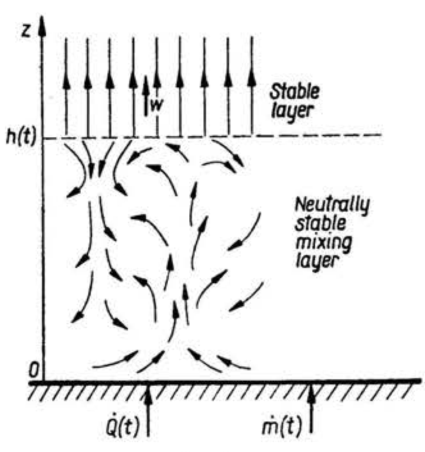

FIG. 3.

pendent on z. The thickness $h(t)$ of the mixing layer is very small in comparison with the thickness of the Earth's atmosphere. Above this layer no vertical mixing of the stably stratified atmosphere takes place. The heat transfer is also negligible there. Due to the thermal expansion of the lower layer, the upper layer moves adiabatically upward with a small velocity $w(t)$ (Fig. 3).

Since an ideal mixing in the neutral layer is assumed, the kinetic energy of the inner turbulent motion is neglected. Also the kinetic energy of the other atmospheric motion, small in comparison with thermal energy, shall be disregarded.

Taking into account all these assumptions, our problem consists in finding instantaneous vertical distributions of pressure $p(z, t)$, density $\varrho(z, t)$, temperature $T(z, t)$, potential temperature $\theta(z, t)$ and specific humidity $\xi(z, t)$ for known initial distributions $\theta(z, 0)$, $\varrho(z, 0)$, $\xi(z, 0)$, and for given time-dependent heat flux $\dot{Q}(t)$ and evaporation rate $\dot{m}(t)$ on the Earth's surface. To solve this problem the thickness $h(t)$ of the neutrally stable layer should be also found.

In the method applied here the main parameters θ, ϱ, ξ, shall be found at first from transport equations. Then, from known rules of hydrostatics and thermodynamics, other quantities such as pressure p, temperature T and specific humidity of saturation ξ_s may be obtained. Comparing at last ξ with ξ_s, we may eventually determine the time t_s corresponding to the appearance of the first clouds on the level $h_s = h(t_s)$.

2. Neutrally stable adiabatic layer $z < h$

The atmospheric air considered as a mixture of ideal gases should fulfil the known state equations

$$p = R\varrho T, \qquad \gamma = \frac{c_p}{c_v},$$

(2.1)

$$e = c_v T = \frac{1}{\gamma-1} RT, \quad i = e + \frac{p}{\varrho} = c_p T = \frac{\gamma}{\gamma-1} RT,$$

where e and i are specific internal energy and enthalpy. Moreover, the gas constant R entering here and the specific heats $c_v c_p$ are almost constant

$$R = R_a(1-\xi) + R_w \xi \approx R_a,$$

$$\gamma = \frac{\gamma_a(1-\xi) + \dfrac{R_w}{R_a}\dfrac{\gamma_a - 1}{\gamma_w - 1}\gamma_w \cdot \xi}{(1-\xi) + \dfrac{R_w}{R_a}\dfrac{\gamma_a - 1}{\gamma_w - 1}\cdot \xi} \approx \gamma_a, \quad \xi < \xi_s \ll 1,$$

the subscripts a and w here denote the dry air and water vapour, respectively. The saturation specific humidity ξ_s is determined by the formula

$$\xi_s = \frac{R_a}{R_w} \frac{E(T)}{p - \left(1 - \dfrac{R_a}{R_w}\right) E(T)} \approx \frac{R_a}{R_w} \frac{E(T)}{p}, \tag{2.2}$$

in which the saturation pressure of water vapour E is a known function of the temperature T.

Using the hydrostatic equations for adiabatic atmosphere $\partial p / \partial z = -g\varrho$, $p/\varrho^\gamma = \text{const}$ (g is the gravity acceleration), and taking into account the Clapeyron equation (2.1) $p = R\varrho T$, we obtain the distribution of parameters in the adiabatic layer:

$$p = p_0(1-\zeta)^{\frac{\gamma}{\gamma-1}}, \quad \varrho = \frac{p_0}{RT_0}(1-\zeta)^{\frac{1}{\gamma-1}}, \quad T = T_0(1-\zeta), \tag{2.3}$$

(2.3) [cont.]
$$\theta = T_0, \quad \xi = \xi_0,$$
$$\zeta = \frac{z}{H}, \quad \chi = \frac{h}{H}, \quad H = \frac{\gamma}{\gamma-1}\frac{RT_0}{g}, \quad \zeta < \chi.$$

The values of parameters on the Earth's level $z = 0$ are denoted by the subscript 0. In our case $T(0, t) = T_0(t)$, $\xi(0, t) = \xi_0(t)$ may depend on time and $p(0, t) = p_0 = \text{const}$.

The formula obtained from Eq. (2.3)

(2.4)
$$\theta = T + \frac{\gamma-1}{\gamma}\frac{gz}{R}$$

may also be considered as the definition of the potential temperature θ.

The increase of mass and energy in the whole adiabatic layer $0 < \zeta < \chi$ should be equal to the transport of these quantities through the lower and upper boundaries. Applying this rule to the total mass and energy and to the mass of water vapour, we find the integral transport equations[1]

(2.5)
$$\frac{d}{dt}\int_0^t \varrho\, dz = \hat{\varrho}(\dot{h}-w)+\dot{m},$$
$$\frac{d}{dt}\int_0^t \varrho e\, dz = \hat{\varrho}\hat{e}(\dot{h}-w)+\dot{Q}-\hat{p}w,$$
$$\frac{d}{dt}\int_0^t \xi\varrho\, dz = \hat{\xi}\hat{\varrho}(\dot{h}-w)+\dot{m},$$

where, by $\hat{\varrho}$, $\hat{p}$, $\hat{e}$, $\hat{\xi}$, are denoted the limit values of the corresponding parameters ϱ, p, e, ξ on the level $z = h$ from the upper side $z > h$. According to the assumptions mentioned above the kinetic energy of the turbulent mixing has been neglected here.

By omitting humidity terms as well, we may reduce the two first equations (2.5) to the form

(2.6)
$$\dot{T}_0\int_0^h \frac{\partial\varrho}{\partial T_0}\,dz+\hat{\varrho}w = 0,$$
$$\dot{T}_0\int_0^h \frac{\partial(\varrho e)}{\partial T_0}\,dz+\hat{\varrho}\hat{i}\hat{w} = \dot{Q}.$$

3. Stable upper layer $z > h$

Since the upward vertical translatory motion which follows from the dilatation of the heated lower layer does not change the inner structure of the stable layer, its thermodynamic parameters are the same as on the initial level

(3.1)
$$p(z, t) = p\big(z-\delta(t), 0\big), \quad \varrho(z, t) = \varrho\big(z-\delta(t), 0\big),$$
$$T(z, t) = T\big(z-\delta(t), 0\big), \quad \xi(z, t) = \xi\big(z-\delta(t), 0\big).$$

(1) The derivatives in respect to time t shall be denoted by dots.

The vertical displacement

$$\delta = \int_0^t w(\tau)d\tau \tag{3.2}$$

of the upper layer has been introduced here.

The potential temperature θ, according to Eqs. (2.4) and (3.1), is

$$\theta(z,t) = \theta(z-\delta(t),0)+\frac{\gamma-1}{\gamma}\frac{g\delta(t)}{R}, \quad z > h. \tag{3.3}$$

It will be shown later that δ is very small and that, generally, in Eq. (3.1) the influence of the upward motion may be disregarded.

4. The boundary level $z = h$

The boundary level $z = h$ separating the lower mixing layer from the upper stable region moves upward with a velocity $\dot{h}$, not smaller than the velocity w of the upper layer. On this boundary the stability condition should also be fulfilled, what means that the rate of growth of the potential temperature on the lower side $z = h-0$ should not exceed the growth rate on the upper side $z = h+0$. During the heating of the lower layer both quantities should be equal to each other

$$\left.\frac{D\theta(z,t)}{Dt}\right|_{z=h(t)} = \frac{\partial\theta(h\pm 0,t)}{\partial z}\dot{h}+\frac{\partial\theta(h\pm 0,t)}{\partial t}. \tag{4.1}$$

Introducing here

$$\theta(z,t) = \begin{cases}\theta(z-\delta(t),0)+\dfrac{\gamma-1}{\gamma}\dfrac{g\delta(t)}{R}, & z > h,\\ T_0(t), & z < h,\end{cases} \tag{4.2}$$

obtained from Eqs. (3.3) and (2.3), we find

$$\dot{T}_0 = \theta'(\dot{h}-w)+\frac{T_0}{H}w, \tag{4.3}$$

where θ' is the initial value of the derivative $\partial\theta(z,t)/\partial z$ on the level $z = h-\delta$

$$\theta' = \left.\frac{\partial\theta(z,0)}{\partial z}\right|_{z=h-\delta} \approx \frac{\partial\theta(h,0)}{\partial z}. \tag{4.4}$$

In our model on the upper boundary $z = h$ the partial derivatives $\partial/\partial z$ of the thermodynamical parameters are discontinuous. The parameters themselves may be either continuous or discontinuous. Due to the static equilibrium, the pressure p should be continuous in the whole region. If, initially, the vertical distribution of the temperature $T(z,0)$ is continuous, it should remain continuous all the time and, consequently, the same property concerns the density ϱ, the potential temperature θ and the specific humidity of saturation ξ_s. But the specific humidity ξ due to the mixing effect will be generally discontinuous on the level $z = h$, where its lower side value may exceed its upper limit and may reach the condensation value ξ_s, what should provoke the appearing of clouds.

5. General equations

The thickness of the lower neutral layer being assumed very small

(5.1) $$\zeta < \chi, \quad \chi = \frac{h}{H} \ll 1,$$

we will take into account only the lowest order terms in respect to ζ and χ.

In the second Eq. (2.6), rejecting higher order terms (also the small integral term) we obtain the upward velocity

(5.2) $$w = \frac{\gamma-1}{\gamma}\frac{\dot{Q}}{p_0}.$$

After neglecting higher order terms and eliminating w, the first equation (2.6) and Eq. (4.3) may be transformed to the form

(5.3) $$\chi\dot{\chi} = \frac{\dot{q}}{\vartheta}, \quad \frac{\dot{T}_0}{T_0} = \frac{\dot{q}}{\chi},$$

where

(5.4) $$\dot{q} = \frac{w}{H} = \left(\frac{\gamma-1}{\gamma}\right)^2 \frac{g\dot{Q}}{Rp_0T_0}, \quad \vartheta = \frac{H\theta'}{T_0} = \frac{\gamma}{\gamma-1}\frac{R\theta'}{g}.$$

The function $\dot{q}$ characterizes the intensity of the heat flux and ϑ the initial distribution of the temperature described by θ'. For-stably stratified atmosphere $\vartheta \geqslant 0$.

It should be stressed that w in Eq. (5.2) is really very small. For $\dot{Q}_{max} \approx 500\,\frac{\mathrm{W}}{\mathrm{m}^2}$ being the order of the heat flux coming from the sun on the Earth's surface, $p_0 \approx 10^5\,\frac{\mathrm{kg}}{\mathrm{ms}^2}$ and $\gamma = 1.4$, we may evaluate $w < 1.5\,\frac{\mathrm{mm}}{\mathrm{s}}$. As the upward motion is very slow, it may be disregarded in most formulae. The changes of temperature T_0 are also very small and in $\dot{q}$ and H they may be disregarded.

The two differential equations (5.3) with the initial conditions

(5.5) $$\chi(0) = 0, \quad \Delta T_0(0) = 0, \quad \Delta T_0 = T_0(t)-T_0(0),$$

allow us to find two unknown functions $\chi(t)$, $\Delta T_0(t)$. As T_0 is assumed to be approximately constant, χ may be obtained from the first equation (5.3) alone. Solving of the second equation (5.3) gives $\Delta T_0/T_0$. The solutions $h(t) = H\cdot\chi(t)$ and $T_0(t) = T_0(0)+\Delta T_0(t)$ allow us to find the approximate instantaneous vertical distribution of the thermodynamical parameters

(5.6) $$p = \begin{cases} p(z,0), \\ p_0\left(1-\dfrac{z}{H}\right)^{\frac{\gamma}{\gamma-1}}, \end{cases} \quad T = \begin{cases} T(z,0), \\ T_0\left(1-\dfrac{z}{h}\right), \end{cases} \quad \varrho = \begin{cases} \varrho(z,0), & z > h, \\ \dfrac{p_0}{RT_0}\left(1-\dfrac{z}{H}\right)^{\frac{1}{\gamma-1}}, & z < h. \end{cases}$$

The upward motion and the displacement δ, very small in comparison with h, have also been neglected here.

After finding the thermodynamic parameters p, T, ϱ, we may obtain from Eq. (2.2) the vertical distribution of the saturation specific humidity ξ_s. From the last equation(2.5), after rejecting higher order terms in respect to ζ and χ, we obtain the differential equation

$$\frac{d}{dt}(\xi_0 h) = \hat{\xi}(\dot{h}-w)+\frac{RT_0\dot{m}}{p_0}. \tag{5.7}$$

Additionally rejecting here w in comparison with $\dot{h}$, we find after integration

$$\xi_0(t) = \frac{1}{h}\int_0^h \xi(z,0)dz+\frac{1}{h}\int_0^t \frac{RT_0\dot{m}}{p_0}d\tau. \tag{5.8}$$

The interpretation of this formula is very simple. Due to the mixing effect, the specific humidity $\xi(z,t) = \xi_0(t)$ is uniformly distributed in the neutrally stable layer and is equal to the sum of the averaged initial distribution of humidity (the first integral) and of the ratio of the evaporated water to the whole layer's mass (the second integral). If, at a time $t = t_s$, a level $h = h_s$ is reached on which

$$\xi_0(t_s) = \xi_s(h_s), \quad h_s = h(t_s), \tag{5.9}$$

the condensation conditions are obtained and on this level the first clouds should appear.

6. Particular solutions

6.1. Polytropic atmosphere $\theta' = \text{const}$

Let us consider an initially polytropic atmosphere with $\theta' = \text{const}$. The changes of the temperature T_0 being very small, we will take account of them only in $\dot{T}_0 = \Delta\dot{T}_0$.

For $\theta' = \text{const}$, also $\vartheta = \text{const}$, and from Eq. (5.3) we obtain the main unknown functions

$$\chi = \sqrt{\frac{2}{\vartheta}\int_0^t \dot{q}\,d\tau}\,, \qquad \frac{\Delta T_0}{T_0} = \int_0^t \frac{\dot{q}}{\chi}d\tau. \tag{6.1}$$

6.2. Linear growth of heat flux

To obtain some quantitative evaluations let us additionally assume

$$\dot{q} = A^2 t. \tag{6.2}$$

The quantity A^2 characterizes here the rate of growth of the heat flux. Introducing Eq. (6.2) into Eq. (6.1), we obtain

(6.3) $$\chi = \frac{At}{\sqrt{\vartheta}}, \quad \frac{\Delta T_0}{T_0} = \sqrt{\vartheta} At.$$

For the assumed linear growth of the heat flux $\dot{q}$ in Eq. (6.2), we find that the temperature T_0 and the thickness $h = \chi H$ of the mixing layer grow linearly also, Eq. (6.3).

For the Earth's atmosphere $g = 9.81$ ms^{-2}, $\gamma = 1.4$, $R = 286$ m^{2}s^{-2} K^{-1}, $p_0 \approx$ $\approx 10^5$ kgm^{-1} s^{-2}, $T_0 \approx 300$°K and, consequently, Eq. (2.3) $H \approx 30\,000$ m. Assuming that the maximum heat flux $\dot{Q}_{max} \approx 500$ W m^{-2} is reached during the first five hours of linear growth, we may evaluate $A = 1.7 \cdot 10^{-6}$s^{-1} $= 6 \cdot 10^{-3}$ hr^{-1}. The quantity ϑ depends on the initial distribution of the temperature and it increases from zero for the adiabatic atmosphere, through $\vartheta = 1$ for the isothermal case, to higher values $\vartheta > 1$ for inversion cases. The increase rate of the temperature $\dot{T}_0$ at the Earth's level and the rate of growth of the thickness $\dot{h}$ as a function of the vertical temperature gradient

$$\partial T(z, 0)/\partial z = \frac{\gamma - 1}{\gamma} \frac{g}{R} \cdot (\vartheta - 1)$$

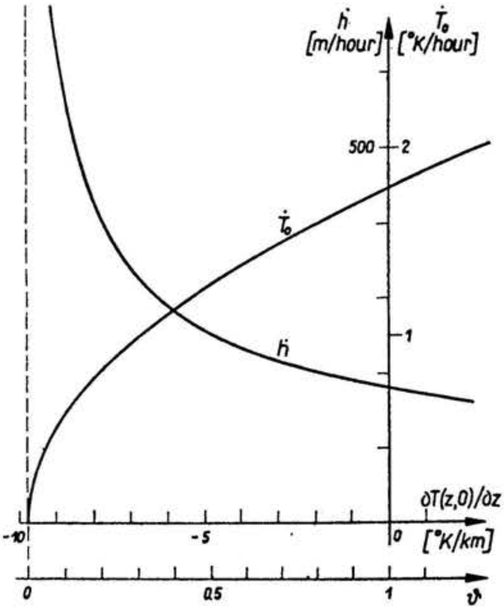

FIG. 4.

are presented in Fig. 4. It is seen that both quantities $\dot{h}$ and $\dot{T}_0$ depend mainly on the initial distribution of the temperature $\partial T(z, 0)/\partial z$ and other factors since the temperature T_0 itself or the pressure p_0 exert secondary influence on them.

6.3. Appearing of first clouds

The humidity ξ_0 of the mixing layer in the formula (5.8) depends not only on its initial distribution $\xi(z, 0)$, but it may also be much influenced by the evaporation rate $\dot{m}$ of the Earth's surface. The data [1] given for mean evaporation rates up to 250 mm/month allow to estimate $RT_0\dot{m}/p_0$, of the order 1 m/hour. The increase of the specific humidity due to such evaporation $\frac{1}{h}\int\limits_0^t \frac{RT_0\dot{m}}{p_0}d\tau$ of the order 10^{-2} is rather not negligibly small. However, in our rough evaluations of the formula (5.8) we will study particularly the first term due to the initial humidity distribution $\xi(z, 0)$, and we will take into account only the linear approximation in respect to the vertical coordinate z. Neglecting $\dot{m}$ we will present an upper evaluation of the condensation level $h_s = h(t_s)$.

In linear approximation

$$\xi(z, 0) = \xi(0, 0)+\frac{\partial\xi(0, 0)}{\partial z}z+ \ldots,$$

with $\dot{m} = 0$ we obtain from the formula (5.8)

$$\xi(z, t) = \xi_0(t) = \xi(0, 0)+\frac{1}{2}H\frac{\partial\xi(0, 0)}{\partial z}\chi+ \ldots, \quad z < h. \tag{6.4}$$

In linear approximation we also obtain

$$\xi_s(h, t) \approx \xi_s(h, 0) = \xi_s(0, 0)+H\frac{\partial\xi_s(0, 0)}{\partial z}\chi+ \ldots. \tag{6.5}$$

Now, according to Eq. (5.9), we may compare both humidities: ξ and ξ_s on the condensation level $z = h_s$. Taking into account Eq. (2.2) and introducing the relative humidity $f \approx \xi/\xi_s$, we find

$$\chi_s = \frac{1-f_0}{\left(1-\frac{f_0}{2}\right)\left[\left(\frac{T}{E}\frac{dE}{dT}\right)_0(1-\vartheta)-\frac{\gamma}{\gamma-1}\right]+\frac{1}{2}\left(\frac{\partial f}{\partial\zeta}\right)_0}. \tag{6.6}$$

In the considered range of temperature $-30°C < T < +30°C$ we obtain an almost constant value $(T/E)(dE/dT) = 20\pm3$. Introducing it with $\gamma/(\gamma-1) = 3.5$ into Eq. (6.6), we find a rough estimation of the condensation height

$$\chi_s \approx \frac{1-f_0}{\left(1-\frac{f_0}{2}\right)[20(1-\vartheta)-3.5]+0.5(\partial f/\partial\zeta)_0}. \tag{6.7}$$

The diagram of this approximate upper evaluation, for constant initial relative humidity $(\partial f/\partial\zeta = 0)$, is presented in Fig. 5. It shows a strong dependency of the condensation level on the initial distribution not only of relative humidity but also of temperature. For temperature distribution approaching inversion cases the presented considerations may not be valid because the condensation may occur not at the upper level $z = h$ but at the Earth's surface $z = 0$.

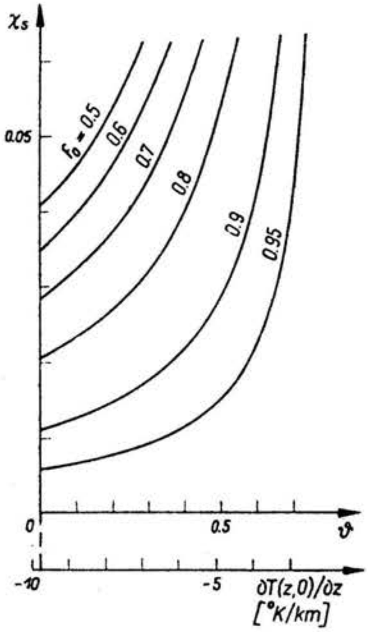

FIG. 5.

7. Final remarks

The phenomenon of convective heat transfer in the lower, neutrally stable, adiabatic layer of the atmosphere is well known and some theoretical models of its description are presented [2] to [12]. It is also observed that the separation level is surrounded by a thinner sublayer of intensive mixing with inversion of temperature. In some theoretical considerations of the existence of such a sublayer or of its influence on the final distribution of temperature, simplified theoretical models were proposed. Here we will be only concerned with the conditions introduced at this separation level.

In the theoretical model presented here the inversion sublayer and its internal structure is disregarded. It is assumed that a horizontal surface $z = h$ exists, separating the upper stable region from the lower, neutrally stable layer. The mixing phenomena within the layer do not influence the stable region. No heat flux penetrates through the separating surface downwards, an upward convective transport of energy being there only possible. As a consequence, the condition imposed at the surface is the stability condition (4.3) only. Although in such an assumption the complex phenomena accompanying the inversion sublayer are extremely simplified, the essential physical factors seem to be taken into account in an appropriate manner. It should be emphasized that in other theoretical models the magnitude of downward heat flux is generally postulated almost arbitrarily (cf. [2] to [6]) and some criticism of such assumptions is presented also [7].

Analyzing the second equation (2.6) of energy transport, we should stress that the first integral term entering there is negligibly small in comparison to the second term which is proportional to a rather generally disregarded small upward velocity w. The heat flux Q

directed upward from the Earth's surface provokes an increase of internal energy beneath the level $z = h$ and at the level $z = h$ an upward convective transport of energy, due to the dilatation of the mixing layer and to the translatory motion of the air with the upward velocity w. As the velocity w is very small, its influence in the whole transport of energy is rather disregarded. However, the presented considerations show that the dilatation exerts here a prevailing influence and that not dilatation but the increase of internal energy, should be rather neglected.

The vertical upward heat transfer through the atmosphere is accompanied by many complex physical phenomena. By the Earth's surface an unstable sublayer appears, provoking convective processes and turbulent mixing through the whole mixing region, including the upper inversion sublayer. In the theoretical model considered here it is not our intention to study the mechanism of all these processes. We do not dwell on a detailed analysis of the inner structure of atmospheric layers either. We evaluate only in a simple way the thickness and mean thermodynamic parameters of the neutrally stable layer, but we intend to take into account the most important factors and to present the results in, possibly the simplest form. By comparing the results with corresponding meteorological data, it could be quantitatively verified whether they have practical applications.

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