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Experimental investigation of the stability of supersonic boundary layer on a flat insulated plate

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THE GROWTH of the "natural" fluctuations in zero pressure gradient, laminar boundary layer at a Mach number 2.0 using the hot-wire anemometry technique is investigated. It has been found that the disturbances of all frequencies increase with the distance from the leading edge of the plate and their maximum value at the position x which is inversely proportional to the frequency, and then decrease slowly. Such a behaviour of disturbances is in qualitative agreement with the theoretical conclusions on the interaction of the supersonic boundary layer with the external sound waves which are generated by the turbulent boundary layer at the walls of the wind tunnel test section. The variation of the fluctuations downstream the maximum follows the linear theory of hydrodynamic stability. The influence of the leading edge thickness and the unit Reynolds number on the growth of the fluctuations are also investigated. On the basis of the experimental neutral stability curves and the measured power spectra of the external fluctuations, the explanation of the influence of the unit Reynolds number on the location of the boundary layer transition is advanced.

Badany jest wzrost "naturalnych" fluctuacji przy zerowym ciśnieniu gradientowo-laminarnej warstwy przyściennej dla liczby Macha 2, za pomocą anemometru cieplno-oporowego. Zaobserwowano, że dla wszystkich częstości zaburzenia rosną z odległością od przedniej krawędzi płyty, osiągają maksymalną wartość dla współrzędnej x odwrotnie proporcjonalnej do częstości, a następnie wolno zanikają. Takie zachowanie się zaburzeń jest w jakościowej zgodzie z rozważaniami teoretycznymi dotyczącymi wzajemnego wpływu naddźwiękowej warstwy przyściennej z zewnętrznymi falami dźwiękowymi, wytworzonymi przez turbulentną warstwę przyścienną w ścianach badanej części tunelu aerodynamicznego. Zmiana fluktuacji od maksymalnej wielkości w dół linii prądu opisana jest za pomocą liniowej teorii stabilności hydrodynamicznej. Badany jest również wpływ grubości prowadzącego brzegu płyty oraz jednostkowej liczby Reynoldsa na wzrost fluktuacji. Na podstawie doświadczalnie wyznaczonych krzywych neutralnej stabilności oraz zmierzonych widmach zewnętrznych fluktuacji dokonana jest próba wytłumaczenia wpływu jednostkowej liczby Reynoldsa na położenie warstwy przyściennej.

Исследуется рост „естественных" флуктуаций при нулевом градиенте давления в ламинарном пограничном слое, для числа Маха 2, при помощи термо-резистивного анемометра. Наблюдалось, что для всех частот возмущения растут с расстоянием от переднего края пластины, достигают максимального значения для координаты x пропорциональной частоте, а затем медленно исчезают. Такое поведение возмущений находится в качественном согласии с теоретическими рассуждениями, касающимися взаимодействия сверхзвукового пограничного слоя с внешними звуковыми волнами, образованными турбулентным слоем на стенках рабочей части аэродинамической трубы. Изменение флуктуации от максимальной величины вниз по потоку описано при помощи линейной теории гидродинамической устойчивости. Исследуется тоже влияние толщины передней кромки пластины и единичного числа Рейнольдса на рост флуктуаций. На основе экспериментально определенных кривых нейтральной устойчивости и измеренных внешних уровней флуктуаций проведена попытка объяснения влияния единичного числа Рейнольдса на положение перехода пограничного слоя.

1. Introduction

THERE are not many experimental works in which the development of fluctuation in the supersonic boundary layer was studied. The investigations carried out by LAUFER and

Vrebalovich [1], Demetriades [2], Kendall [3, 4] are worth noting. However, recently several effects have been predicted theoretically and experimental verification is required. The theory predicts the existence of more then one instability mode and the interaction of the laminar supersonic boundary layer with the external sound waves [3, 5].

Moreover, the study of the initial growth of the disturbances can help to understand the influence of the different factors (leading edge thickness, unit Reynolds number, surface cooling etc.) on the transition of the laminar boundary layer to the turbulent one. An increase of both the model leading edge thickness and the unit Reynolds number leads to the increase of the Reynolds number of transition [6]. A change of the stability characteristics in the laminar boundary layer can be considered as a reason for this increase.

In the present investigation an experimental study is carried out on the development of "natural" small fluctuations in the laminar boundary layer at a Mach number being equal to 2.0 for a flat insulated plate with a various leading edge thickness in a wide range of the wind tunnel operation.

2. Equipment and methods

The experiments were performed in the supersonic wind tunnel of the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the USSR Academy of Sciences. The test section dimensions are 20 cm width, 20 cm height and 60 cm length.

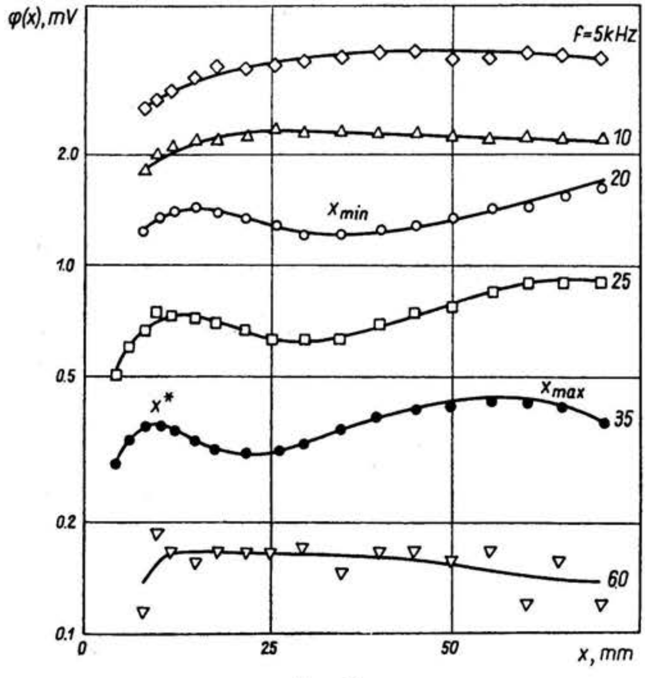

Fig. 1.

All measurements were carried out on a flat steel plate 22 cm long which was placed in the centre of the test section under a zero angle of attack. The plate had different leading edge thicknesses: a sharp leading edge ($b \leqslant 0.02$ mm), $b = 0.2$ mm, $b = 0.4$ mm. Such a bluntness has the strongest effect on the location of the boundary layer transition.

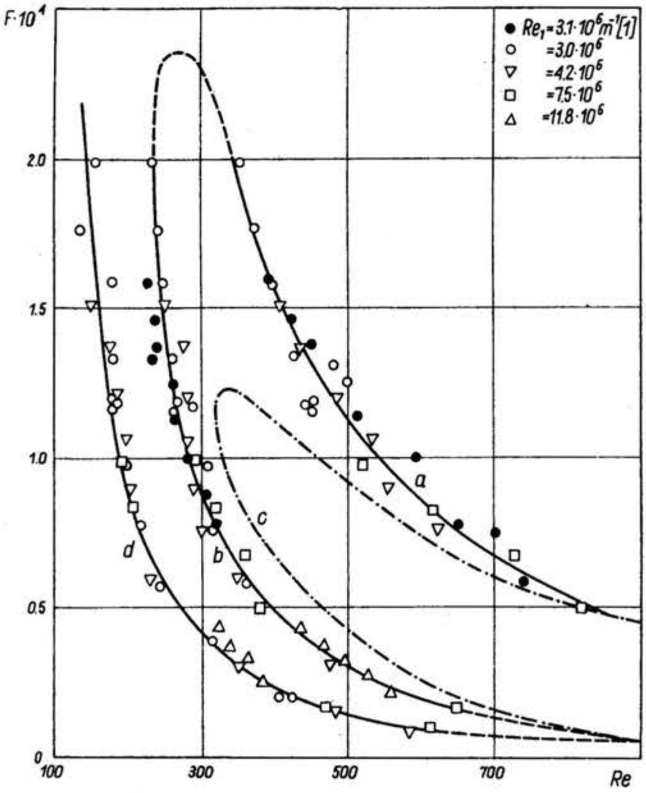

FIG. 2.

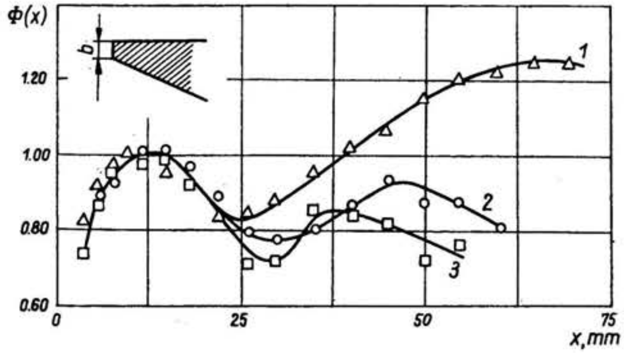

FIG. 3. $1 — b = 0$, $2 — b = 0.2$ mm, $3 — b = 0.4$ mm.

The directcurrent hot-wire anemometer was used to measure the flow fluctuations. The length of the hot-wire was approximately 1.5 mm, the diameter was about 0.006 mm. The probe can move streamwise and normally to the plate. The total rms fluctuation of the voltage on the probe $\langle e \rangle$ and rms fluctuations for the fixed frequency $\langle e_f \rangle$ was measured by using the "Brüel and Kjaer" analyzer of the type 2010.

The technique of measurements was similar to the one developed by LAUFER and VREBALOVICH [1]. The probe was moved in the downstream direction and at the last position it was moved normally to the plate. The total rms fluctuations as a function of a normal coordinate were measured and the maximum of this function was determined. Then the probe was moved towards the leading edge of the plate at the position of maximum fluctuations, the so-called "critical layer", and the distribution of the rms fluactuation at the fixed frequency f was measured. The meaning of these fluctuations as a function of the Reynolds number — the amplication curve was determined. Its maximum and minimum correspond to a point of the neutral stability curve $F = F(\mathrm{Re})$ for the non-dimensional frequency

$$F = \frac{2\pi f}{u_\infty \mathrm{Re}_1}, \quad \mathrm{Re} = (\mathrm{Re}_1 \cdot x)^{1/2}, \tag{2.1}$$

u_∞ is a mean velocity in a free stream.

In order to express the hot-wire output voltages in terms of the flow quantities, the method suggested by KOVASZNAY [7] was applied.

3. Results for different leading edge thicknesses

In the first series of experiments the unit Reynolds number Re_1 was fixed at the value of about $3 \cdot 10^6$ m^{-1} and the leading edge thickness was varied. The typical amplification curves, $\varphi(x) = \langle e_f \rangle$ obtained for a plate with a sharp leading edge are presented in Fig. 1 (x — distance from the leading edge). At the low frequencies (f less than 5 kHz) there was no extremum of the signal for all the measured interval. In the case when the frequencies of fluctuations were increased, the minimum ($x_{\min}$) and the maximum ($x_{\max}$) of this function appeared. The additional maximum of this amplification curve was observed for the low Reynolds number at a point with a coordinate x^* ($x^* < x_{\min}$). This phenomenon will be discussed later. Now we consider the results that are in agreement with the statements of the linear stability theory ($x > x^*$).

At very high frequencies $x_{\min}$ and $x_{\max}$ drew together and the extrema disappeared. For example, when f was equal to 60 kHz, the non-dimensional frequency was equal to about $2.4 \cdot 10^{-4}$; there were no distinguished extrema. It means that this frequency parameter was out of the neutral stability curve. In this case the useful signal can be compared with the hot-wire anemometer "noise", hence the accuracy of the measurements was relatively low.

The points of the neutral stability curve obtained in the present study (open symbols) under the condition of $M = 2$ and a sharp plate leading edge and the results by Laufer and Vrebalovich (dark symbols) at $M = 2.2$ and for similar other conditions [1] are plotted in Fig. 2. The line a corresponds to the $x_{\max}$ and the line b — $x_{\min}$. Brawn's calculations (line *c*) for $M = 2.2$ are also plotted there [8]. Brawn's theoretical results were obtained on the basis of numerical integration of a complete linearized set of Navier-Stokes equations for the fluctuations propagating under 55° to the direction of the main flow.

The comparison of these results allows to conclude that unstable waves such as Tollmien-Schlichting waves are determined by means of extrema at the points $x_{\min}$ and $x_{\max}$. These waves were observed in the Laufer and Vrebalovich's experiments and their characteristics were calculated in Brawn's paper.

Figure 3 presents the results of comparison of the normalized amplification curves for the fixed frequency $f = 30$ kHz obtained for a flat plate with a different leading edge thickness, $\Phi(x) = \varphi(x)/\varphi(x_0)$, $x_0 = 10$ mm. The increase of the leading edge thickness leads to the displacement of the extrema $x_{\min}$ and $x_{\max}$ of the amplification curves. The location of the additional maximum x^* and the peculiarities of the progress of fluctuations near this maximum were independent of the leading edge thickness.

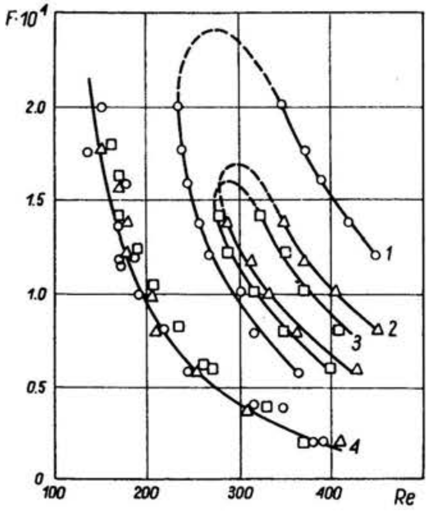

FIG. 4.

Figure 4 presents the neutral stability curves obtained from the measurements carried out on a flat plate with a different leading edge thickness (*1*, *2*, *3*) and the location of the additional maximum of the amplification curve (*4*). The equation of this curve has the form

$$\mathrm{Re}^* \cdot F^{0.5} = 2.0, \qquad \mathrm{Re}^* = \mathrm{Re}_1 \cdot x^*$$

that is, the coordinate of the additional maximum is inversely proportional to the frequency.

These results allow to draw the following conclusion: the disturbances of all frequencies increase when the probe moves from the leading edge of the plate $x = 0$ and reach their maximum value at a point whose coordinate is inversely proportional to the frequency. Then, these disturbances slowly decrease. The change of a mean boundary-layer characteristics such as the distribution of the velocity and temperature due to the increase of the leading edge thickness does not influence the location of this maximum. Such a behaviour of disturbances is in qualitative agreement with theoretical calculations by M. L. MACK [5] on the interaction of the supersonic boundary layer with the external sound waves which are generated by the turbulent boundary layer on the wall of the wind tunnel test section.

The behaviour of the fluctuations in the downstream region from the additional maximum follows the linear theory of hydrodynamic stability. The increase of the leading edge thickness leads to the reduction of the range of unstable frequencies and the instability region and to the increase of the critical Reynolds number of the stability loss. The sensitivity to the change of the boundary layer characteristic is a specific feature of the Tollmien-Schlichting waves. The behaviour of the stability characteristics in this region is in satisfactory agreement with the increase of the transition Reynolds number due to the increase of the leading edge thickness.

4. Results for different unit Reynolds numbers

The next series of experiments were performed using a flat plate with a sharp leading edge and the unit Reynolds number varied in a wide range.

Figure 2 presents the results of the measurements of the neutral stability curves (lines *a* and *b*) and the location of the additional maximum (line *d*) for a unit Reynolds number smaller than $12 \cdot 10^6\ \mathrm{m}^{-1}$. There is no difference between the results obtained for diverse unit Reynolds numbers. However, the increase of the unit Reynolds number leads to the decrease of the range of non-dimensional frequencies which was available in the measurements. For example, at $\mathrm{Re}_1 = 3 \cdot 10^6\ \mathrm{m}^{-1}$ the measurements were performed for $F = 2 \cdot 10^{-4}$, but at $\mathrm{Re}_1 = 12 \cdot 10^6\ \mathrm{m}^{-1}$ the measurements were performed only for $F = 0.5 \cdot 10^{-4}$.

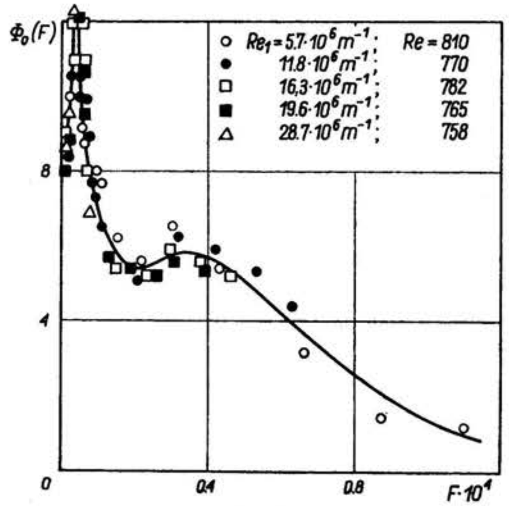

FIG. 5.

At very low frequencies the region of the initial growth of the disturbances was combined with the neutral stability curve and the increasing fluctuations were observed within the whole measured interval.

Figure 5 presents the results of measurements up to $\mathrm{Re}_1 \leqslant 30 \cdot 10^6\ \mathrm{m}^{-1}$. Here we have the spectrum of the fluctuations in the "critical layer" for the $\mathrm{Re} \approx 770$ and for the different Re_1 values. This spectrum is normalized on the outer spectrum of the fluctuations, $\Phi_0(F) = \varphi(F)/\varphi_\infty(F)$. This result shows how much the fixed frequency fluctuations are increased when reaching $\mathrm{Re} \approx 770$. There is no difference between the results obtained for different Re_1.

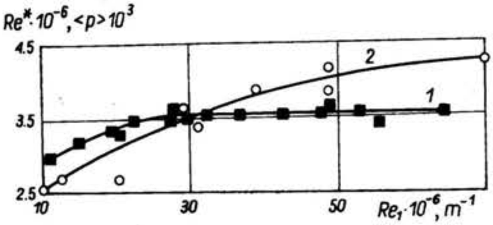

FIG. 6.

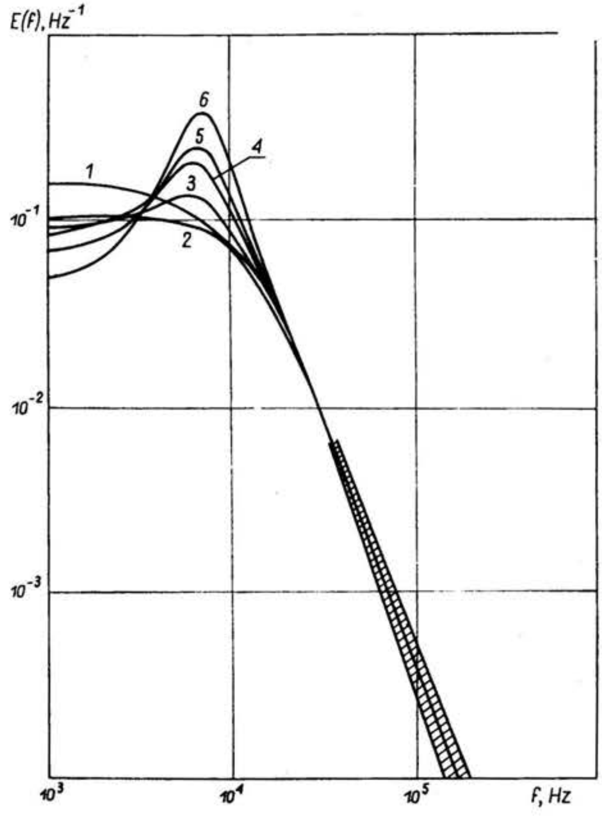

FIG. 7. 1 — $\mathrm{Re}_1 = 6.3 \cdot 10^6\ \mathrm{m}^{-1}$, 2 — $\mathrm{Re}_1 = 15 \cdot 10^6\ \mathrm{m}^{-1}$, 3 — $\mathrm{Re}_1 = 19 \cdot 10^6\ \mathrm{m}^{-1}$, 4 — $\mathrm{Re}_1 = 30 \cdot 10^6\ \mathrm{m}^{-1}$, 5 — $\mathrm{Re}_1 = 40 \cdot 10^6\ \mathrm{m}^{-1}$, 6 — $\mathrm{Re}_1 \geqslant 48 \cdot 10^6\ \mathrm{m}^{-1}$.

The next conclusion can be drawn. At the fixed Mach number M and the total temperature T_0 the regime of the wind tunnel operation has no influence on the evolution of the small fluctuations in the supersonic laminar boundary layer. However, the regime of wind tunnel operation influences the locations of the laminar boundary layer transition to the turbulent one.

The Reynolds number of transition Re^* increases with the increase of the Re_1. These were attempts to connect this effect with the decrease of the total pressure fluctuations $\langle p \rangle$ in the test section of the wind tunnel. Figure 6 presents the transition (*1*) and the measurements of total pressure fluctuations (*2*) which were performed in our wind tunnel at $M = 2.0$. The pressure fluctuations increase when increasing Re_1. Therefore, the measurements of the energy spectrum of the fluctuations were carried out.

Figure 7 presents a spectrum for different Re_1 in dimensional form; it differs only for low frequencies. Figure 8 presents this spectrum in a non-dimensional form. The stability measurements of the present study and the theoretical calculations [5] show that fluctuations with $F > 10^{-5}$ can be caused only by the transition of the laminar boundary layer to the turbulent one. The low-frequency fluctuations have small amplification coefficients and cannot be a reason of transition; this holds true only in the case when the fluctuations

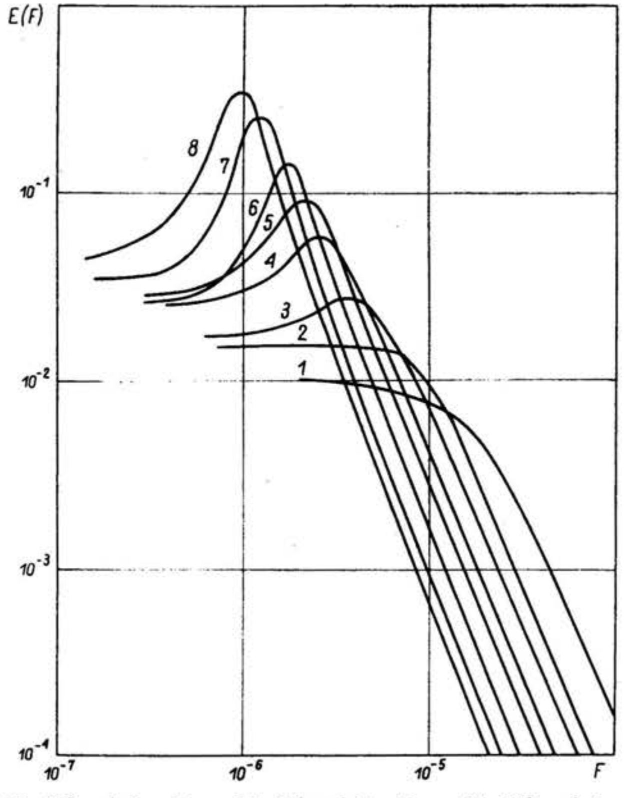

FIG. 8. *1* — $\mathrm{Re}_1 = 6.3 \cdot 10^6\,\mathrm{m}^{-1}$, *2* — $\mathrm{Re}_1 = 15 \cdot 10^6\,\mathrm{m}^{-1}$, *3* — $\mathrm{Re}_1 = 19 \cdot 10^6\,\mathrm{m}^{-1}$, *4* — $\mathrm{Re}_1 = 30 \cdot 10^6\,\mathrm{m}^{-1}$, *5* — $\mathrm{Re}_1 = 40 \cdot 10^6\,\mathrm{m}^{-1}$, *6* — $\mathrm{Re}_1 = 48 \cdot 10^6\,\mathrm{m}^{-1}$, *7* — $\mathrm{Re}_1 = 70 \cdot 10^6\,\mathrm{m}^{-1}$, *8* — $\mathrm{Re}_1 = 89 \cdot 10^6\,\mathrm{m}^{-1}$.

are small. Figure 8 shows that for $F > 10^{-5}$ the initial level of the fluctuations decreases with the increase of Re_1.

Thus we can advance the next explanation of the influence of the unit Reynolds number Re_1 on the location of the transition. If the M and T_0 parameters are fixed, the spectrum of fluctuations formed in the test section of the wind tunnel depends on the size, geometry and construction of the tube, but not much on the regime of the wind tunnel operation for frequencies which stimulate the transition of the laminar boundary layer to the turbulent one. The hydrodynamic nature of the fluctuation development in the boundary layer depends on the non-dimensional frequency (2.1), that is, it is inversely proportional to Re_1.

This fact leads to a decrease in the initial level of the fluctuation intensity for frequencies which stimulate the transition of the laminar boundary layer; moreover, the transition Reynolds number is also increased.

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