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Drag reduction of an oscillating flat plate with an interface film

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DRAG reduction of an oscillating plate using an intermediate film of elastico-viscous fluid has been discussed. Expressions for velocity profiles for film and infinite fluid are obtained. Power input to fluid at the plate is significantly reduced by the use of a thin film of an elastico-viscous fluid.

Przedmiotem rozważań jest redukcja oporu oscylującej tarczy uzyskana za pomocą warstwy pośredniej z cieczy sprężysto-lepkiej. Otrzymano wyrażenia na profile prędkości dla warstwy i nieskończonej cieczy. Moc przekazana cieczy na powierzchni tarczy maleje znacznie po wprowadzeniu pośredniej ciekłej warstwy sprężysto-lepkiej.

Предметом рассуждений является редукция сопротивления осциллирующего диска, полученная при помощи промежуточного слоя из упруго-вязкой жидкости. Получены выражения для профилей скорости для слоя и для бесконечной жидкости. Мощность передаваемая жидкости на поверхности диска значительно убывает после введения промежуточного жидкого упруго-вязкого слоя.

1. Introduction

DRAG reduction by long-chain polymers or particulate suspensions is a well-documented phenomenon. Recently, some excellent reviews [1, 2, 3] have appeared which deal with various aspects of drag reduction. The literature on this subject reveals that most of the experimental investigations have been carried out with circular pipes; however, comparatively little work has been done with flat plates.

Studies of drag reduction of flat plate is of practical importance to naval applications. Since the drag reduction by polymer additives is a boundary region phenomenon [4], the concentrated polymer solution is injected as such at the solid boundary to reduce the frictional drag of a plate. Continuous injection of a concentrated polymer solution forms a coating of a non-Newtonian fluid next to the wall.

Some natural lubricants have also been found to reduce the frictional drag. Fish mucus is secreted by cells of the epidermis; this reduces the water resistance up to 60% [5] and assists in fish locomotion. The concentration of slime secreted by hogchoker is about 2000 ppm (weight) and shows a high reluctance to diffuse in water. The reported studies [5, 6] with slime solutions have also been conducted in pipes.

In order to understand the influence of the presence of fish slime on shear stress produced at the fish body, a basic problem of fluid motion of a thin lubricating film between an oscillating flat plate and an infinite fluid has been considered in the present work. The mathematical model considered also simulates the flow of water — wood pulp mixture on the shaking table of Fourdrinier paper making machine. In this machine a layer of wood pulp is separated from the vibrating table by a thin layer of water.

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The problem of an intermediate viscous film introduced between an oscillating plate and another infinite viscous fluid has been treated by DEBLER and MONTOGOMERY [7]. These authors have not discussed the influence of the thickness of the intermediate film on various parameters affecting the friction reduction of the plate. Also, since the fish slime [6] as well as the water — wood pulp mixture contain solid constituents, the results obtained with the Newtonian film do not apply to actual fluid situations.

In the present study, drag reduction of an oscillating flat plate in the presence of an intermediate film has been analysed. For a general approach, an infinite elastico-viscous fluid is separated by a thin layer of another immiscible fluid. Expressions for the velocity profiles for the intermediate film and for the infinite fluid have been obtained. The reduction of power input to fluid at the plate with the intermediate film has been determined. The optimum power ratio obtained with the elastico-viscous fluid film is found to be much less than that when the film is of a Newtonian fluid.

It is shown that the results of the present analysis can also be applied to analogous problems of heat conduction in solids. Variation of temperature in a boiler grate, over which a thin layer of ash acts as an insulating material and is subjected to an oscillating temperature field, can also be determined. Results without the film can be used to find the temperature distribution of the Earth.

2. Formulation of the problem

The fluid considered is characterized by the constitutive equations [8]

$$s_{ik} = p_{ik} - pg_{ik}, \tag{2.1}$$

$$p^{ik} = 2\int_{-\infty}^{t} \varphi(t-t')\frac{\partial x^i}{\partial x'^m}\frac{\partial x^k}{\partial x'^k} e^{(1)mr}(x', t')\,dt', \tag{2.2}$$

where covariant suffixes are written below, contravariant suffixes above, and the usual summation for repeated suffixes is assumed. Here s_{ik} is the stress tensor, p an arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed coordinate system x^i, x'^i is the position at time t' of the element which is instantaneously at the point x^i at time t, $e^{(1)ik}$ is the rate of the strain tensor and

$$\varphi(t-t') = \int_0^\infty \frac{N(\tau)}{\tau}\exp[-(t-t')/\tau]\,d\tau,$$

$N(\tau)$ being the distribution function of relaxation times. One common approximation [8] is that the memory of the fluid is so small that Eq. (2.2) can be simplified to

$$p^{ik} = 2\eta_0 e^{(1)ik} - 2k_0\frac{\delta}{\delta t}e^{(1)ik}, \tag{2.3}$$

where $\eta_0 = \int_0^\infty N(\tau)\,d\tau$ is the limiting viscosity at small rates of shear, k_0 is the coefficient of elasticity of the fluid,

$$k_0 = \int_0^\infty \tau N(\tau)\,d\tau,$$

and terms involving $\int\limits_0^\infty \tau^n N(\tau)d\tau$ $(n \geqslant 2)$ are neglected. $\delta/\delta t$ denotes the convected derivative [9].

Using the simplified equations of state, the equations of motion for the fluid in a Cartesian frame of reference can be written as

$$(2.4)\quad \varrho\left(\frac{\partial v_i}{\partial t}+v_k\frac{dv_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i}+\eta_0\frac{\partial^2 v_i}{\partial x_k\partial x_k}-k_0\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 v_i}{\partial x_k\partial x_k}\right.\right.$$
$$\left.+v_m\frac{\partial^3 v_i}{\partial x_m\partial x_k\partial x_k}-\frac{\partial v_i}{\partial x_m}\frac{\partial^2 v_m}{\partial x_k\partial x_k}-2\frac{\partial^2 v_i}{\partial x_m\partial x_k}\frac{\partial v_m}{\partial x_k}\right],$$

where ϱ is the density.

Consider the unsteady flow parallel to an infinite oscillating plate over which the normal component of velocity takes a zero value. A thin film of thickness h of another elastico-viscous liquid separates the infinite liquid from the plate. The coordinate system used for this arrangement is shown in Fig. 1. The plate performs oscillations with a velocity $U_0\cos(\omega t-\varepsilon)$ in the direction of the x-axis, and the y-axis is normal to the plate. Let u and v be velocity components in the x- and y-directions, respectively. Under these conditions, the flow is independent of x and, from Eq. (2.1), the governing equation with uniform pressure becomes

$$(2.5)\quad \frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2}-k^*\frac{\partial^3 u}{\partial y^2\partial t},$$

where

$$(2.6)\quad \nu = \eta_0/\varrho \quad\text{and}\quad k^* = k_0/\varrho.$$

The continuity equation for the flow situation is identically satisfied. Let the suffixes 1 and 2 represent the film fluid and infinite fluid, respectively, then

$$(2.7)\quad \frac{\partial u_1}{\partial t} = \nu_1\frac{\partial^2 u_1}{\partial y^2}-k_1^*\frac{\partial^3 y_1}{\partial y^2\partial t}\quad\text{for}\quad -h \leqslant y \leqslant 0^-,$$

$$(2.8)\quad \frac{\partial u_2}{\partial t} = \nu_2\frac{\partial^2 u_2}{\partial y^2}-k_2^*\frac{\partial^3 u_2}{\partial y^2\partial t}\quad\text{for}\quad 0^+ \leqslant y \leqslant \infty,$$

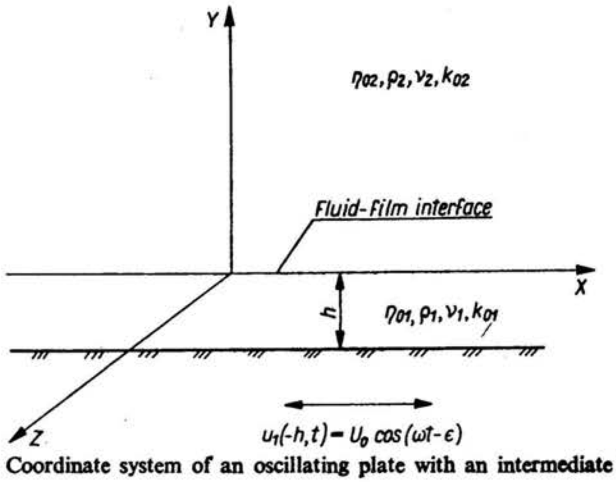

FIG. 1. Coordinate system of an oscillating plate with an intermediate film.

where

$$\nu_1 = \eta_{01}/\varrho_1, \quad k_1^* = k_{01}/\varrho_1;$$

(2.9)

$$\nu_2 = \eta_{02}/\varrho_2, \quad k_2^* = k_{02}/\varrho_2.$$

The boundary conditions are

$$(2.10)_1 \qquad u_1 = U_0\cos(\omega t - \varepsilon) \quad \text{at} \quad y = -h,$$

$$(2.10)_2 \qquad u_2 = 0 \quad \text{as} \quad y \to \infty,$$

and at the fluid-film interface

$$(2.10)_3 \qquad u_1(0^-, t) = u_2(0^+, t),$$

$$(2.10)_4 \qquad [p_{xy}]_{y=0^-} = [p_{xy}]_{y=0^+},$$

where p_{xy} is the shear stress. Equation $(2.10)_4$ in terms of velocity components is

$$(2.11) \qquad \left[\eta_{01}\left(\frac{\partial u_1}{\partial y}\right) - k_{01}\left(\frac{\partial^2 u_1}{\partial y \partial t}\right)\right]_{y=0^-} = \left[\eta_{02}\left(\frac{\partial u_2}{\partial y}\right) - k_{02}\left(\frac{\partial^2 u_2}{\partial y \partial t}\right)\right]_{y=0^+}.$$

3. Method of solution

Since we are interested in the fully established oscillatory flow, we assume the velocity of fluid of the form

$$(3.1) \qquad u(y, t) = U_0 f(y)\exp(i\omega t).$$

Introducing the following non-dimensional parameters

$$(3.2) \qquad \begin{aligned} &\eta = yU_0/\nu_2, \quad \bar{t} = U_0^2 t/\nu_2, \quad \bar{\omega} = \nu_2\omega/U_0^2, \quad \bar{u} = u/U_0, \\ &k = k_2^* U_0^2/\nu_2^2, \quad \lambda = k_1^*/k_2^*, \quad \xi = \nu_1/\nu_2, \quad \theta = \eta_{01}/\eta_{02}. \end{aligned}$$

Eqs. (2.7) and (2.8) when combined with Eqs. (3.1) and (3.2) yield the non-dimensional velocities, after dropping the bars, as

$$(3.3) \qquad u_1(\eta, t) = [A\cosh(a_1 + ib_1)\eta + B\sinh(a_1 + ib_1)\eta]\exp(i\omega t) \quad \text{for} \quad -\beta \leqslant \eta \leqslant 0^-,$$

$$(3.4) \qquad u_2(\eta, t) = [C\exp(a_2 + ib_2)\eta + D\exp\big(-(a_2 + ib_2)\eta\big)]\exp(i\omega t) \quad \text{for} \quad 0^+ \leqslant \eta \leqslant \infty,$$

where

$$(3.5) \qquad \begin{aligned} &a_1, b_1 = \left(1/\sqrt{2}\right)\left[\frac{\omega}{\sqrt{\sigma_1}} \mp \frac{\lambda k\omega^2}{\sigma_1}\right]^{1/2}, \\ &a_2, b_2 = \left(1/\sqrt{2}\right)\left[\frac{\omega}{\sqrt{\sigma_2}} \mp \frac{k\omega^2}{\sigma_2}\right]^{1/2}, \\ &\sigma_1 = (\xi^2 + \lambda^2 k^2\omega^2), \\ &\sigma_2 = (1 + k^2\omega^2). \\ &\beta = \frac{U_0 h}{\nu_2}, \end{aligned}$$

and A, B, C and D are the constants of integration to be determined from the rephrased boundary conditions

$$
\begin{aligned}
u_1 &= \exp[i(\omega t-\varepsilon)] \quad \text{at} \quad \eta = -\beta,\\
u_2 &= 0 \quad \text{as} \quad \eta \to \infty,\\
u_1(0^-, t) &= u_2(0^+, t),\\
(\theta/\xi)\left[\xi\frac{\partial u_1}{\partial\eta}-\lambda k\frac{\partial^2 u_1}{\partial\eta\partial t}\right]_{\eta=0^-} &= \left[\frac{\partial u_2}{\partial\eta}-k\frac{\partial^2 u_2}{\partial\eta\partial t}\right]_{\eta=0^+}.
\end{aligned} \tag{3.6}
$$

A combination of Eqs. (3.3) and $(3.6)_1$ gives the phase angle

$$\varepsilon = \tan^{-1}\left[\left(\frac{B\cosh a_1\beta - A\sinh a_1\beta}{A\cosh a_1\beta - B\sinh a_1\beta}\right)\tan b_1\beta\right]. \tag{3.7}$$

Equations (3.4) and $(3.6)_2$ give

$$C = 0.$$

The real parts of Eqs. (3.3) and (3.4) give the velocities of the film fluid and infinite fluid, respectively. Therefore, Eqs. (3.3), (3.4) and $(3.6)_3$ give

$$D = A.$$

Also, when this factor is combined with $(3.6)_4$, we get

$$B = -\alpha A, \tag{3.8}$$

where

$$\alpha = (1/\theta)\left[\frac{a_2+k\omega b_2}{a_1+\lambda\xi^{-1}k\omega b_1}\right]. \tag{3.9}$$

The constant A is found by combining Eqs. (3.7) and (3.8) as

$$A = [(\sinh a_1\beta+\alpha\cosh a_1\beta)^2\sin^2 b_1\beta+(\alpha\sinh a_1\beta+\cosh a_1\beta)^2\cos^2 b_1\beta]^{-1/2}. \tag{3.10}$$

Thus the velocity profiles $u_1(\eta, t)$ and $u_2(\eta, t)$ are completely determined.

In the absence of a fluid film, i.e. $\beta = 0$, then $A = 1$ and $A < 1$ for $|\beta| > 0$. Therefore, the expression on the right hand side of Eq. (3.10) is termed as the amplitude reduction factor, K given by

$$K = [(\sinh a_1\beta+\alpha\cosh a_1\beta)^2\sin^2 b_1\beta+(\alpha\sinh a_1\beta+\cosh a_1\beta)^2\cos^2 b_1\beta]^{-1/2}, \tag{3.11}$$

and $K \leqslant 1$ for $|\beta| \geqslant 0$.

Shear stress

The non-dimensional shear stress at the surface of the plate in the presence of the intermediate film, when non-dimensionalized by $\varrho_2 U_0^2$, is given by

$$p_{xy}]_{\eta=-\beta} = \theta\left(\frac{\partial u_1}{\partial\eta}\right)_{\eta=-\beta} - \frac{\lambda\theta k}{\xi}\left(\frac{\partial^2 u_1}{\partial\eta\partial t}\right)_{\eta=-\beta}. \tag{3.12}$$

Using Eq. (3.3) we obtain

$$p_{xy}]_{\eta=-\beta} = (P_r+M_r)\cos\omega t-(P_i+M_i)\sin\omega t, \tag{3.13}$$

where

$$(3.14)\quad \begin{aligned}
P_r &= \theta a_1\left[\frac{\lambda k\omega}{\xi}(B\sinh a_1\beta - A\cosh a_1\beta)\sin b_1\beta + (B\cosh a_1\beta - A\sinh a_1\beta)\cos b_1\beta\right],\\
P_i &= \theta a_1\left[(B\sinh a_1\beta - A\cosh a_1\beta)\sin b_1\beta - \frac{\lambda k\omega}{\xi}(B\cosh a_1\beta - A\sinh a_1\beta)\cos b_1\beta\right],\\
M_r &= \theta b_1\left[\frac{\lambda k\omega}{\xi}(B\cosh a_1\beta - A\sinh a_1\beta)\cos b_1\beta - (B\sinh a_1\beta - A\cosh a_1\beta)\sin b_1\beta\right],\\
M_i &= \theta b_1\left[(B\cosh a_1\beta - A\sinh a_1\beta)\cos b_1\beta + \frac{\lambda k\omega}{\xi}(B\sinh a_1\beta - A\cosh a_1\beta)\sin b_1\beta\right].
\end{aligned}$$

Power

The power input to the fluid per cycle is given by

$$(3.15)\qquad P = -\int_0^T [u(0,t)]p_{xy}]_{y=0}\,dt$$

in which T is the period of the cycle $= 2\pi/\omega$.

If P_f and P_n denote the power input with and without the intermediate film, then, from Eqs. (3.3), (3.10) and (3.13) through (3.15) we get

$$(3.16)\qquad P_f = (\pi K^2\theta/\omega)\left[\left(a_1+\frac{\lambda}{\xi}b_1k\omega\right)\left(\frac{1}{2}(1+\alpha^2)\sinh 2a_1\beta + \alpha\cosh 2a_1\beta\right) + \frac{1}{2}\left(\frac{\lambda}{\xi}a_1k\omega - b_1\right)(1-\alpha^2)\sin 2b_1\beta\right],$$

$$(3.17)\qquad P_n = (\pi/\omega)(a_2+b_2k\omega).$$

Consider a particular case when both fluids are Newtonian, i.e. $k = 0$ and $\lim_{k_2\to 0}\lambda = 0$ because k_1 approaches to zero more rapidly than k_2. The ratio $(P_f/P_n)_l$ for this case is given by

$$(3.18)\qquad (P_f/P_n) = \left[\frac{1}{2}\alpha(\sinh 2a_1\beta + \sin 2a_1\beta) + \frac{1}{2}\alpha^{-1}(\sinh 2a_1\beta - \sin 2a_1\beta) + \cosh 2a_1\beta\right] \Big/ [\sinh a_1\beta + \alpha\cosh a_1\beta)^2\sin a_1\beta + (\cosh a_1\beta + \alpha\sinh a_1\beta)^2\cos^2 a_1\beta],$$

and for large values of $(a_1\beta)$ the power ratio approaches to α^{-1}.

4. Results and discussion

For a given combination of fluids and oscillating system, the phase angle, ε is found to increase with film thickness and decreases as α increases. For large values of $(a_1\beta)$ the phase angle tends to a value $\tan^{-1}[(-\sin b_1\beta)/\cos b_1\beta]$.

The variation of the amplitude reduction factor, K was plotted with $(a_1 \beta)$ for various values of $(b_1\beta)$ and $\alpha = 3$ and 30. $K = 1$ for $(a_1\beta) = 0$ and decreases asymptotically with $(a_1\beta)$. This decrease is found to be more pronounced in the range $0 < (a_1\beta) < 0.5$, thus indicating that a thin layered film is more effective in reducing the value of the amplitude reduction factor. For the same values of $(a_1\beta)$ and $(b_1\beta)$, the magnitude of K for $\alpha = 30$ is about one-tenth of that for $\alpha = 3$. For large values of $(a_1\beta)$, the amplitude reduction factor becomes independent of $(b_1\beta)$ and decays exponentially as $\left[\frac{1}{2}(1+\alpha)\exp(a_1\beta)\right]^{-1}$.

In general, α is a function of $\theta, k, \lambda, \xi, \omega$ and t. For the Newtonian case, i.e. $\lambda = 0$ and $k = 0$, $\alpha = \left(\frac{\eta_{02}}{\eta_{01}}\frac{\varrho_2}{\varrho_1}\right)^{1/2}$. The variation of $(\alpha\theta/\sqrt{\xi})$ with $k\omega$ for various values of ωt (= $\pi/4, \pi/2, 3\pi/4$) indicated that this parameter is less than unity for $\omega t = \pi/2$ at all values of $k\omega$. Therefore, this type of oscillations of the plate will result in a higher value of K and, consequently, the oscillating system would require higher power input per cycle.

In order to evaluate the extent of power reduction by the use of a thin lubricating film, the following particular cases are examined:

(i) When both fluids over the oscillating surface are Newtonian, then (P_f/P_n) given by Eq. (3.18) is in agreement with that obtained by DEBLER and MONTOGOMERY [7].

(ii) In another case, the infinite elastico-viscous fluid is separated from the oscillating plate by a thin layer of a Newtonian fluid. The power ratio (P_f/P_n) for this case is Z times greater than that given by Eq. (3.18), where

$$Z = (\sigma_2)^{1/4}\left[\left(1-\frac{k\omega}{\sqrt{\sigma_2}}\right)^{1/2} + k\omega\left(1+\frac{k\omega}{\sqrt{\sigma_2}}\right)^{1/2}\right]^{-1}.$$

The variation of Z with $k\omega$ is shown Fig. 2.

(iii) The power input ratio with and without a film, for $\lambda/\xi = 1$ and the optimum condition for the amplitude reduction factor, is obtained from Eqs. (3.16) and (3.17). The optimum condition for K is obtained from the expression resulting from $dK/d\beta = 0$. The case of $\lambda/\xi = 1$ represents the condition for which the coefficient of elasticity of each fluid is proportional to its kinematic viscosity. The variation of $(P_f/P_n)_{opt}$ with $(a_1\beta)$ for various values of $k\omega$ (= 0, 0.5, 1.0) and $\alpha = 3$ is shown in Fig. 3. A similar plot for $\alpha = 30$ is shown in Fig. 4. In these plots the curves for $k\omega = 0$ correspond to the Newtonian case. These plots show that power reduction is most rapid in the range $0 < (a_1\beta) < 0.25$.

A comparison of Figs. 3 and 4 reveals that the power ratio is reduced significantly by using the intermediate fluid that yields a higher value of α. For such fluids, even a film of small thickness can reduce the power input dramatically. Thus we may conclude that power reduction by an elastico-viscous film is much greater than that by a Newtonian film.

If $k = 0$ and $\lambda = 0$ are substituted in the results given in Sect. 3, then the analysis contained in Eqs. (2.9) through (3.11) can be applied to analogous problems of heat conduction in solids. The results of the analysis when $\beta = 0$ are used to determine the temperature distribution in the earth due to the periodic fluctuations of the temperature at the surface, for example, from day to day or over the seasons in the year. The analysis

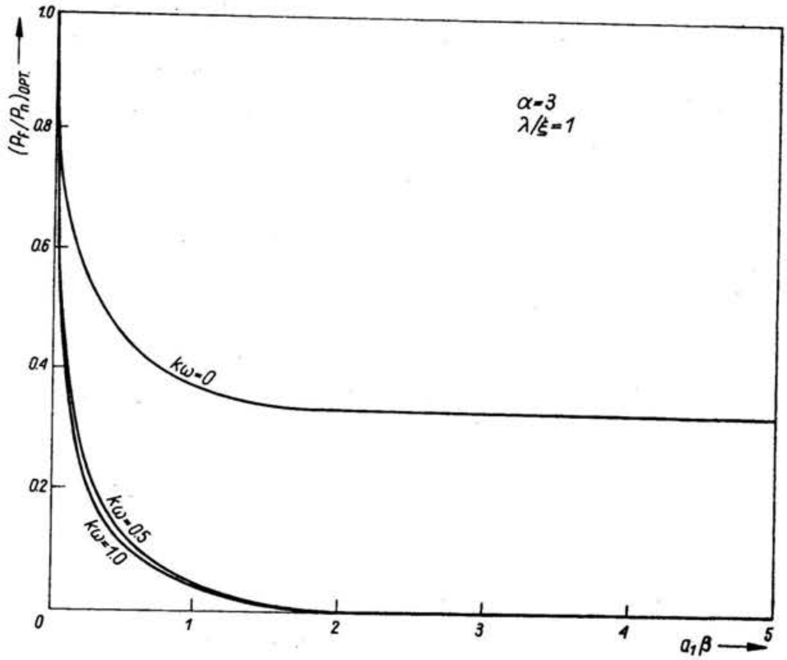

FIG. 2. Variation of Z with $k\omega$ for $\lambda = 0$.

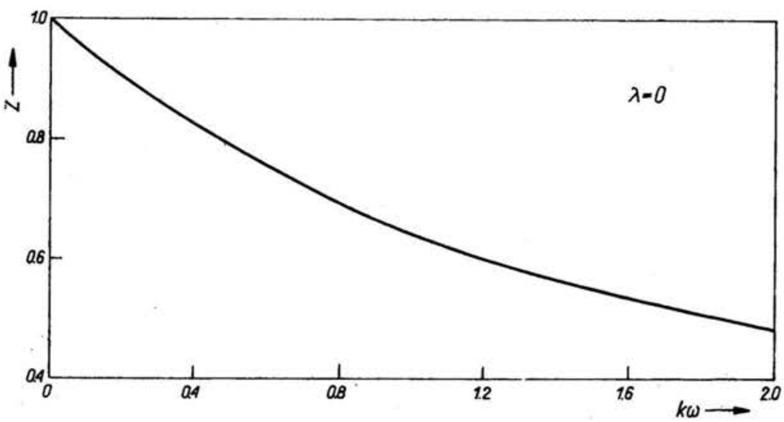

FIG. 3. Optimum power ratio for various values of $k\omega$ and $\alpha = 3$.

for the cases in which an intermediate film exists can be applied to a solid surface at a depth $y = -h$ (Fig. 1) which would be exposed to a fluctuating temperature. We can thus determine the extent to which a thin layer of one material reduces the temperature in a semi-infinite solid of another material. Temperature variation in a boiler grate subjected to a fluctuating temperature field and over which a thin layer of ash acting as an insulating

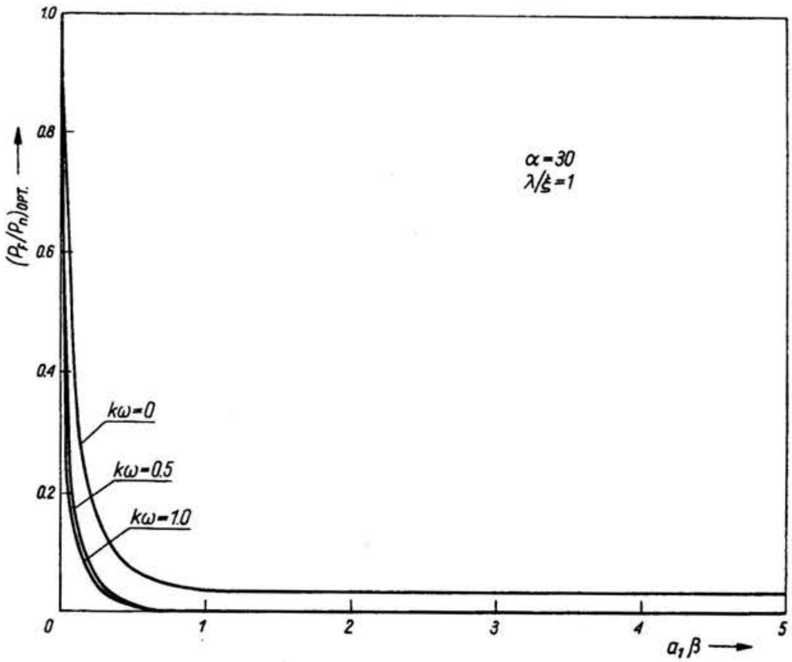

FIG. 4. Optimum power ratio for various values of $k\omega$ and $\alpha = 30$.

film is spread could also be determined. The use of the relations in the present analysis requires simply a replacement of ϱ and η by the specific heat and thermal conductivity of the material, respectively.

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