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Boundary problems in nonlocal continuum models of large engineering structures(*)

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THE PAPER continues the considerations concerning nonlocal models of large discrete structures. Using a hexagonal rod structure as an example, the problems related to adequate modelling of boundary value problems have been discussed.

Praca stanowi kontynuację rozważań dotyczących ciągłych nielokalnych modeli dużych struktur dyskretnych. Na przykładzie heksagonalnej struktury prętowej omówiono zagadnienia związane z adekwatnym modelowaniem zagadnień brzegowych. Przedyskutowano różne przypadki warunków brzegowych związanych z różnymi sposobami podparcia bądź obciążenia brzegu.

Работа составляет продолжение рассуждений, касающихся сплошных нелокальных моделей больших дискретных структур. На примере гексагональной стержневой структуры обсуждены вопросы, связанные с адекватным моделированием краевых задач. Обсуждены разные случаи граничных условий, связанных с разными способами опирания или нагружения границы.

Introduction

THE PURPOSE of the paper has been to continue considerations of the possible use of various nonlocal continuum models describing engineering structures. The problem was put forward in the paper [1] where the possibility of describing an infinite plane rod lattice structure was discussed in terms of an integral and a gradient nonlocal model. The example of the structure being analysed (Fig. 1) has been selected so that an effect of short- and

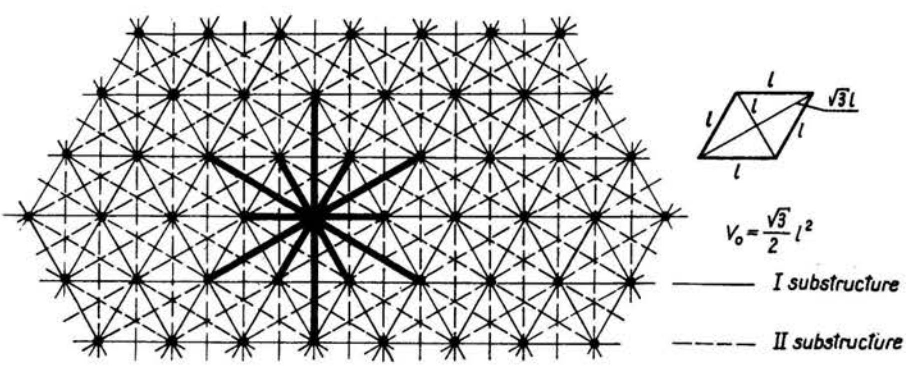

FIG. 1.

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long-range interactions can be obtained without complicating needlessly our study. The structure under analysis can be naturally resolved into substructure I of shorter-range interactions and substructure II of longer-range interactions.

In the previous paper [1] the forms of internal energy of the structure were determined for the cases of the two models under discussion, i.e. integral and gradient.

This paper discusses the formulation of boundary problems for structures with strictly defined edge support and load conditions.

In the case of definite boundary conditions and external loads (acting upon the internal nodes of the structure), it is possible first to determine their continuous distributions being their equivalents in the continuum model and, then, to determine continuous model fields of displacements.

Given the forms of internal energy, external loads and boundary conditions, use can be made of the principle of minimum potential energy to obtain stationary conditions describing displacement fields in the continuous model under consideration.

In Chapter 1 this analysis was carried out for the case of the integral model, in Chapter 2 for the gradient model. Irrespectively of determining the displacement field in continuous models, which allows the displacements of discrete structure nodes ($\tilde{\mathbf{u}} \Rightarrow \mathbf{u}$) to be determined, the problem of determining internal forces in the rods ($\mathbf{P}$) has been discussed. This is very important from the engineering point of view. Internal forces in the rods of substructure I or II ($\mathbf{P}^{\mathrm{I}}, \mathbf{P}^{\mathrm{II}}$) can be determined in two ways. Firstly, using the determined node displacements ($\mathbf{u}$) and employing the constitutive relationships of the discrete model [1] $\left(\tilde{\mathbf{u}} \Rightarrow \mathbf{u} \begin{smallmatrix} \nearrow \mathbf{P}^{\mathrm{I}} \\ \searrow \mathbf{P}^{\mathrm{II}} \end{smallmatrix}\right)$, and secondly, using the auxiliary definition of states of stresses related to the individual substructures ($\tilde{\boldsymbol{\sigma}}^{\mathrm{I}}, \tilde{\boldsymbol{\sigma}}^{\mathrm{II}}$) and determining their corresponding components $\tilde{\mathbf{u}} \begin{smallmatrix} \nearrow \tilde{\boldsymbol{\sigma}}^{\mathrm{I}} \Rightarrow \mathbf{P}^{\mathrm{I}} \\ \searrow \tilde{\boldsymbol{\sigma}}^{\mathrm{II}} \Rightarrow \mathbf{P}^{\mathrm{II}} \end{smallmatrix}$. The second way will be described in the subchapters 1.2 and 2.3.

The discussion of boundary problems associated with various methods of supporting and loading an edge of the structure makes it possible to employ the technique mentioned for calculating the statics of rod structures on the one hand, and allows the concepts used in the theory of nonlocal bodies to be strictly determined and to be given a mechanical sense, on the other hand.

1. Integral nonlocal model

1.1. Description of the state of displacements

Let us consider a structure bounded by the edge $A = A_u \cup A_p$ (Fig. 2).

The potential energy of the system can be written as follows:

$$(1.1) \quad W_\varepsilon = \frac{1}{2} \int_V \int_V C_{ijkl}(\mathbf{r}, \mathbf{r}')\varepsilon_{ij}(\mathbf{r})\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}d\mathbf{r}' - \frac{1}{2} \int_A \int_V \hat{C}_{ijkl}(\mathbf{s}, \mathbf{r}')\varepsilon_{ij}(\mathbf{s})\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}'d\mathbf{s} - \int_V f_i u_i d\mathbf{r} - \int_{A_p} p_i u_i ds$$

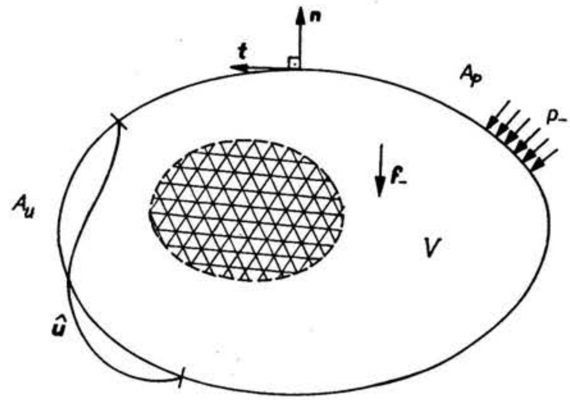

FIG. 2.

for fields of displacements u_i and deformations ε_{ij} conforming to geometric constraints of the system

$$\text{(1.2)} \qquad \begin{aligned} \varepsilon_{ij} &= u_{(i,j)} \quad \text{in} \quad V, \\ u_i &= \hat{u}_i \quad \text{on} \quad A_u. \end{aligned}$$

The stress tensor $C_{ijkl}(\mathbf{r}, \mathbf{r}')$ has been determined for a homogeneous infinite structure in the paper [1]. The expression

$$\text{(1.3)} \qquad W_\varepsilon^A = \frac{1}{2} \int\limits_A \int\limits_V \hat{C}_{ijkl}(\mathbf{s}, \mathbf{r}')\varepsilon_{ij}(\mathbf{s})\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}'d\mathbf{s}$$

appearing in Eq. (1.1) describes equivalent surface energy whose introduction is advantageous in that it provides for the description of the entire heterogeneity of the structure in the boundary layer, leaving the tensor $C_{ijkl}(\mathbf{r}, \mathbf{r}')$ homogeneous within V.

Mechanical interpretation of boundary energy is provided by the total internal energy associated with the rods removed from the boundary layer of the full structure and must be determined individually for every boundary problem.

For example, in the case of perfect clamping along a part A_u of the edge (Fig. 3a) and for loading along a part of the edge A_p provided with a membrane keeping the rods in their original position (Fig. 3b), the tensor $\hat{C}(\mathbf{s}, \mathbf{r}')$ vanishes, and the properties of the structure are homogeneous over the entire region V. Essential here is the fact that the rods intersected by the edge cooperate with the entire structure as if they were not intersected.

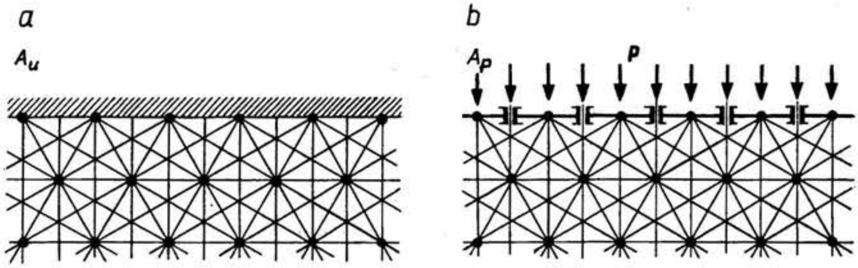

FIG. 3.

In the case of clamping only the nodes located along A_u (Fig. 4a) and loading only the nodes located on A_p (Fig. 4b), the situation is different. The rods intersected by the edge do not cooperate with the entire structure and can be neglected in the analysis.

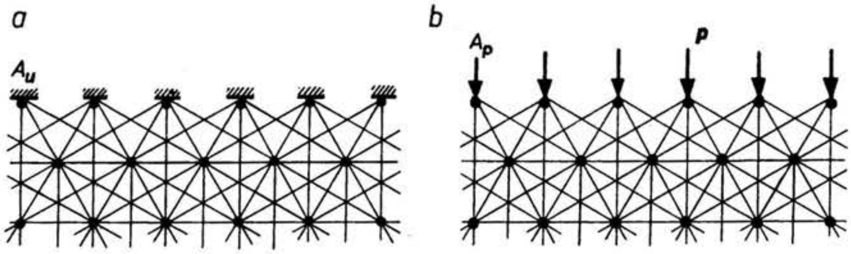

FIG. 4.

The tensor $\hat{C}_{ijkl}(\mathbf{r})$ describing deviation of the elastic features of a real structure in boundary layer from those of a homogeneous structure assumes in these cases the following form (cf. the definition of $C^{II}_{ijkl}(\mathbf{r})$ in [1]).

$$\hat{C}_{ijkl}(\mathbf{r}) = \hat{C}^{II}_{ijkl}(\mathbf{r}) = \frac{3}{\sqrt{3}} a^{II} t_i^{II2} t_j^{II2} t_k^{II2} t_l^{II2} = \begin{cases} \frac{3}{\sqrt{3}} a^{II} & \text{for} \quad i=j=k=l=2 \\ 0 & \text{in the remaining cases} \end{cases} \tag{1.4}$$

for

$$r \in \hat{\Omega}'',$$

where

$\hat{\Omega}''$ the region of the boundary layer (Fig. 4),

$a^{II} = \frac{EA}{l}$ the rigidity of the rods of substructure II,

$t_i^{II2} = [0, 1]$ the directional unit vector of the rods of substructure II orthogonal to the edge.

Using the definition of the distribution of the tensor with respect to the second coordinate [1], we obtain:

$$\hat{C}_{ijkl}(\mathbf{s}, \mathbf{r}) = \frac{\hat{C}_{ijkl}(\mathbf{r}) \hat{h}^{II}(\mathbf{s}, \mathbf{r})}{\Omega^{II}}, \tag{1.5}$$

where

$$s \in A,$$
$$r \in \hat{\Omega}^{II},$$

$\hat{h}^{II}$ — the characteristic function of the region $\hat{\Omega}^{II}$.

Let us locus our attention on the fact that in the case of specifying the volume forces $\mathbf{f}$ and surface forces $\mathbf{p}$ when passing from a discrete to a continuous model ($\mathbf{f} \Rightarrow \tilde{\mathbf{f}}$) we have at our disposal certain arbitrariness in their definition in the neighbourhood of the edge.

Depending on the case under consideration it is more convenient to treat the forces loading the nodes located along the edge as volume forces or surface forces.

It follows from the principle of minimum potential energy (1.1) that among all the geometrically permissible fields ε satisfying Eq. (1.2) the field minimizing W_ε will be actually determined (under the load $\mathbf{f}, \mathbf{p}$).

Stationary conditions of the potential energy functional $\delta W_\varepsilon = 0$ are obtained in the form

$$(1.6)\qquad \begin{aligned} &\Big[\int_V \big(C_{ijkl}(\mathbf{r},\mathbf{r}')-\hat{C}_{ijkl}(\mathbf{r},\mathbf{r}')\big)\varepsilon_{ij}(\mathbf{r}')d\mathbf{r}'\Big]_{,l}+f_k(\mathbf{r}) = 0 \quad \text{in} \quad V,\\ &\Big[\int_V \big(C_{ijkl}(\mathbf{r},\mathbf{r}')-\hat{C}_{ijkl}(\mathbf{r},\mathbf{r}')\big)\varepsilon_{ij}(\mathbf{r}')d\mathbf{r}'\Big]n_l = p_k \quad \text{on} \quad A_p. \end{aligned}$$

The relations (1.2) and (1.6) describe the field of $\mathbf{u}$ displacements of equivalent continuous medium.

1.2. Description of the state of stress of the structure

The analysis of the state of strain of the structure makes it possible to analyse in a formal way its state of stress $\sigma_{ij}(\mathbf{r})$, although the state of stress classically defined here has no direct mechanical interpretation and does not allow internal forces in rods to be determined.

In Chapter 1.3, another method for determining the state of stress will be shown, which allows forces in rods to be directly determined.

By introducing the final definition of the state of stress of the structure

$$(1.7)\qquad \sigma_{ij}(\mathbf{r}) \stackrel{\mathrm{df}}{=} \int_V \check{C}_{ijkl}(\mathbf{r},\mathbf{r}')\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}',$$

where

$$\check{C}_{ijkl}(\mathbf{r},\mathbf{r}') = C_{ijkl}(\mathbf{r},\mathbf{r}')-\hat{C}_{ijkl}(\mathbf{r},\mathbf{r}')$$

it is possible to define complementary energy of the structure

$$(1.8)\qquad W_\sigma = \frac{1}{2}\int_V\int_V \check{C}^{-1}_{\overline{ij}kl}(\mathbf{r},\mathbf{r}')\sigma_{ij}(\mathbf{r})\sigma_{kl}(\mathbf{r}')d\mathbf{r}d\mathbf{r}' - \int_{A_u}\sigma_{ij}(\mathbf{r})n_j(\mathbf{r})u_i(\mathbf{r})d\mathbf{r},$$

for the fields of statically permissible stresses:

$$(1.9)\qquad \begin{aligned} \sigma_{ij,j}+f_i &= 0 \quad \text{in} \quad V,\\ \sigma_{ij}n_j &= p_i \quad \text{on} \quad A_p. \end{aligned}$$

The inverse stress tensor $\check{C}^{-1}_{\overline{ij}kl}$ introduced in the relation (1.8) should be adopted from the definition such that it satisfies the condition

$$(1.10)\qquad \int_V \check{C}^{-1}_{\overline{mn}ij}(\mathbf{r},\mathbf{r}'')C_{ijkl}(\mathbf{r},\mathbf{r}')dr = \delta(\mathbf{r}',\mathbf{r}'')\frac{1}{2}(\delta_{mk}\delta_{nl}+\delta_{ml}\delta_{nk}).$$

It can be proved that by acting with it on the constitutive Eq. (1.7)

$$(1.11)\qquad \int_V \check{C}^{-1}_{\overline{mn}ij}(\mathbf{r},\mathbf{r}'')\sigma_{ij}(\mathbf{r})d\mathbf{r} = \int_V \check{C}^{-1}_{\overline{mn}ij}(\mathbf{r},\mathbf{r}'')\Big[\int_V C_{ijkl}(\mathbf{r},\mathbf{r}')\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}'\Big]d\mathbf{r}$$

we obtain the inverse constitutive relation:

$$\varepsilon_{mn}(\mathbf{r}'') = \int_V \check{C}^{-1}_{mnij}(\mathbf{r}, \mathbf{r}'')\sigma_{ij}(\mathbf{r})d\mathbf{r}. \tag{1.12}$$

Allowing for the definition (1.10) and the relation (1.12), (1.4) and (1.5) and neglecting the effect of heterogeneity in boundary layer, i.e. equating $\hat{C}_{mnij}(\mathbf{r}, \mathbf{r}') = 0$ we obtain:

$$\check{C}^{-1}_{mnij}(\mathbf{r}, \mathbf{r}') = \frac{1}{\lambda^{\mathrm{I}}+\lambda^{\mathrm{II}}}(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})h^{\mathrm{I}}(\mathbf{r}, \mathbf{r}') + \frac{1}{\lambda^{\mathrm{II}}}(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})[h^{\mathrm{II}}(\mathbf{r}, \mathbf{r}')-h^{\mathrm{I}}(\mathbf{r}, \mathbf{r}')]. \tag{1.13}$$

It follows from the principle of minimum complementary energy (1.8) that among all the statistically permissible fields σ_{ij} satisfying Eq. (1.9) the field minimizing W_σ will in reality be determined.

The stationary conditions of the complementary energy functional $\delta W_\sigma = 0$ are obtained in the form

$$\begin{gathered}\int_V [C_{ijkl}(\mathbf{r}, \mathbf{r}')-\hat{C}_{ijkl}(\mathbf{r}, \mathbf{r}')]^{-1}\sigma_{ij}(\mathbf{r}')d\mathbf{r}' = u_{k,l} \quad \text{in} \quad V,\\ u_k = \hat{u}_k \quad \text{on} \quad A.\end{gathered} \tag{1.14}$$

The relations (1.9) and (1.14) describe the state of stress of an equivalent continuous medium. However, in order to make it possible to determine directly internal forces in rods from the state of stress, it is necessary to decompose the state σ_{ij} into the components related to substructures σ^{I}_{ij} and $\sigma^{\mathrm{II}}_{ij}$, as described in Chapter 1.3.

1.3. Description of the state of forces in rods of the structure

Making use of the decomposition of the stress tensor of an equivalent continuous medium into components describing the rigidity characteristics of the individual substructures $\check{C}_{ijkl}(\mathbf{r}, \mathbf{r}') = \check{C}^{\mathrm{I}}_{ijkl}(\mathbf{r}, \mathbf{r}')+\check{C}^{\mathrm{II}}_{ijkl}(\mathbf{r}, \mathbf{r}')$ [1], the states can be defined:

$$\begin{gathered}\sigma^{\mathrm{I}}_{ij} \stackrel{\mathrm{df}}{=} \int_V \check{C}^{\mathrm{I}}_{ijkl}(\mathbf{r}, \mathbf{r}')\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}',\\ \sigma^{\mathrm{II}}_{ij} \stackrel{\mathrm{df}}{=} \int_V \check{C}^{\mathrm{II}}_{ijkl}(\mathbf{r}, \mathbf{r}')\varepsilon_{kl}(\mathbf{r}')d\mathbf{r}', \qquad \sigma_{ij} = \sigma^{\mathrm{I}}_{ij}+\sigma^{\mathrm{II}}_{ij}\end{gathered} \tag{1.15}$$

associated with individual sublattices. This allows forces in the rods of sublattices I and II to be determined from σ^{I}_{ij} and $\sigma^{\mathrm{II}}_{ij}$, respectively. Here it is assumed that the states $\boldsymbol{\sigma}^{\mathrm{I}}$ and $\boldsymbol{\sigma}^{\mathrm{II}}$ are locally homogeneous.

And thus, for a rod belonging to family I and connecting the nodes $\mathbf{r}$ and $\mathbf{r}'$, we obtain the force

$$P_i(\mathbf{r}, \mathbf{r}') = l^{\mathrm{I}}\sigma^{\mathrm{I}}_{ij}(\mathbf{r})t_j^{(r,r')}. \tag{1.16$_1$}$$

By analogy, for a rod of family II we obtain

$$P_i(\mathbf{r}, \mathbf{r}') = l^{\mathrm{II}}\sigma^{\mathrm{II}}_{ij}(\mathbf{r})t_j^{(r,r')}. \tag{1.16$_2$}$$

As mentioned above, it can be seen that, given the solution to the problem of the continuous model, the technically interesting force values can be arrived at in two manners, i.e. either by using the quantities σ^{I} and σ^{II} (cf. Eq. (1.15)) or by determining the field of displacements oı the nodes of discrete structure in conformity with that of the continuous medium, followed by using the constitutive relationships of discrete description [1].

2. Nonlocal gradient model

2.1. Description of the state of strain

Let us now carry out an analysis analogous to that in the case of the nonlocal integral model.

Considering the body bounded by the edge $A = A_u \cup A_p$ (Fig. 2), the potential energy of the system can be written in the general (three-dimensional) case as follows:

$$(2.1)\qquad W_{\varepsilon} = \frac{1}{2}\int_{V} (C_{ijkl}\beta_{ij}\beta_{kl} + C_{ijklmn}\gamma_{ijk}\gamma_{lmn})dv - \frac{1}{2}\int_{A} (\hat{C}_{ijkl}\beta_{ij}\beta_{kl} + \hat{C}_{ijklmn}\gamma_{ijk}\gamma_{lmn})dA - \int_{V} f_i u_i dv - \int_{A_p} p_i u_i dA - \int_{A_p} m_i u_{i,j} n_j dA.$$

The form of the stress tensors C_{ijkl} and C_{ijklmn} was introduced in the paper [1].

The second integral describes, just as before, the effect of heterogeneity in boundary layer caused by the removal of rods.

In the case shown in Fig. 3 $\hat{C}_{ijkl} = 0$ and $\hat{C}_{ijklmn} = 0$ whereas in that shown in Fig. 4:

$$(2.2)\qquad \begin{aligned} \hat{C}_{ijkl} &= \hat{C}^{II}_{ijkl} = \frac{b^{II}}{2V_0} t_i^{II2} t_j^{II2} t_k^{II2} t_l^{II2},\\ \hat{C}_{ijklmn} &= \hat{C}^{II}_{ijklmn} = \frac{C^{II}}{2V_0} t_i^{II2} t_j^{II2} t_k^{II2} t_l^{II2} t_m^{II2} t_n^{II2}, \end{aligned}$$

where $b^{II} = EAl$, $c^{II} = EAl^3$—the rigidity parameters of the rods of substructure II.

The introduction of the new term $\int_A m_i u_{i,j} n_j dA$ in relation to the integral model notation is suitable for those cases in which the edge is loaded by couples of forces with arms of the order of the lattice parameter l, and where we are interested in a solution in its neighbourhood, m_i then describes the external moment load along the edge.

It follows from the principle of minimum potential energy (2.1) that among all the geometrically permissible fields β_{ij} and γ_{ijk}, i.e. those satisfying the relationships

$$(2.3)\qquad \left.\begin{aligned} \beta_{i(j,k)} &= \gamma_{ijk}\\ \beta_{ij} &= u_{(i,j)} \end{aligned}\right\} \quad \text{in} \quad V, \qquad u_i = \hat{u}_i \quad \text{on} \quad A_u,$$

the field minimizing W_{ε} will in reality be determined (under the load f_i, p_i, m_i).

The stationary conditions $\delta W_\varepsilon = 0$ will assume the form

$$
(2.4)\qquad
\begin{aligned}
&[(C_{ijkl}-\hat{C}_{ijkl})\beta_{kl}]_{,j}+[(C_{ijklmn}-\hat{C}_{ijklmn})\gamma_{lmn}]_{,kj}+f_i = 0 \quad \text{in} \quad V,\\
&\left.\begin{aligned}
&[(C_{ijkl}-\hat{C}_{ijkl})\beta_{kl}]n_j+[(C_{ijklmn}-\hat{C}_{ijklmn})\gamma_{lmn}]_{,k}n_j = p_i\\
&[(C_{ijklmn}-\hat{C}_{ijklmn})\gamma_{lmn}]_{,k}n_j = m_i
\end{aligned}\right\} \quad \text{on} \quad A_p.
\end{aligned}
$$

The relations (2.3) and (2.4) allow for the determination of $u_i, \beta_{ij}, \gamma_{ijk}$. The boundary displacement conditions (for the part A_u of the edge, cf. Figs. 3a and 4a) are taken into account by the condition $(2.3)_3$.

The boundary load conditions for the part A_p of the edge, (cf. Figs. 3b and 4b) are taken into account by the conditions $(2.4)_{2,3}$.

2.2. Description of the state of forces in rods of the structure

By analogy to the considerations given for the integral model, a direct way will be discussed to determine the internal forces in rods of the structure by defining states of stresses in the substructures.

And thus, we will define the states of stresses:

$$
(2.5)\qquad
\begin{aligned}
\sigma^{\mathrm{I}}_{ij} &= C^{\mathrm{I}}_{ijkl}\beta_{kl}, & \mathfrak{M}^{\mathrm{II}}_{ijk} &= C^{\mathrm{I}}_{ijklmn}\gamma_{lmn},\\
\sigma^{\mathrm{II}}_{ij} &= C^{\mathrm{II}}_{ijkl}\beta_{kl}, & \mathfrak{M}^{\mathrm{II}}_{ijk} &= C^{\mathrm{II}}_{ijklmn}\gamma_{lmn},
\end{aligned}
$$

the stress tensors having been resolved into the components related to the features of substructures I and II $C_{ijkl} = C^{\mathrm{I}}_{ijkl}+C^{\mathrm{II}}_{ijkl}$, $C_{ijklmn} = C^{\mathrm{I}}_{ijklmn}+C^{\mathrm{II}}_{ijklmn}$, in the paper [1].

By assuming local homogeneity of distributions σ_{ij} and $\mathfrak{M}_{ijk}$, it is possible to determine the forces in rods belonging to substructure I by projecting them suitably onto their directions:

$$
(2.6)_1\qquad P^{\mathrm{I}}_j = V_0/l^{\mathrm{I}}\left(\sigma^{\mathrm{I}}_{ij}t^{\mathrm{I}}_j+\frac{1}{l^{\mathrm{I}}}\mathfrak{M}^{\mathrm{I}}_{ijk}t^{\mathrm{I}}_j t^{\mathrm{I}}_k\right).
$$

By analogy, for rods belonging to substructure II

$$
(2.6)_2\qquad P^{\mathrm{II}}_j = V_0/l^{\mathrm{II}}\left(\sigma^{\mathrm{II}}_{ij}t^{\mathrm{II}}_j+\frac{1}{l^{\mathrm{II}}}\mathfrak{M}^{\mathrm{II}}_{ijk}t^{\mathrm{II}}_j t^{\mathrm{II}}_k\right).
$$

3. Conclusions

In the examples discussed for formulating boundary value problems in nonlocal continuous models describing cases of rod structures, the term of equivalent surface energy has been introduced to account for all the heterogeneity of material properties appearing in the boundary layer of the structure due to the perturbation of regular rod distribution by boundary conditions. It is convenient to isolate this term since, given its mechanical interpretation it is possible without any trouble whatsoever to complete the definition of boundary conditions by determining the heterogeneity tensors (1.5) and (2.2).

By analogy, using nonlocal models to describe other objects, such as crystal lattices, physical interpretation of the tensor $\check{\mathbf{C}}$ should be determined and then its components should be specified depending on concrete forms of boundary conditions.

In the case of describing an object for which it is interesting to determine internal forces, it is convenient to introduce, just as in the present paper, states of stresses related to individual substructures provided that they can be isolated. The determination of internal forces is then instantaneous. On the other hand, equilibrium equations for the object comprise complete states of stresses obtained by superposition of stresses in the individual substructures.

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