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BRIEF NOTES

Pore structure influence on elastic waves propagation in a fluid-saturated porous solid (*)

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IN THIS PAPER the influence of structure of a porous medium on harmonic waves parameters i.e. phase velocity and attenuation coefficient has been analysed. The disscusion has been carried out based on the theory of a fluid-saturated porous medium with two macroparameters characterising the pore structure. The interaction forces with coefficients dependent on or independent of a frequency have been taken into account. The results have proved a considerable influence of the structure of a porous medium on wave parameters for the mean and higher frequency.

1. Introduction

THE DESCRIPTION of wave propagation phenomena in a deformable fluid-saturated porous solid is related to many engineering applications, particularly in such areas as geophysics, biomechanics or acoustics. The dynamic behaviour of such a material, apart from the physical features of particular components of the medium, strongly depends on the skeleton pore structure. Thus the propagation parameters will depend on the pore structure characteristic used within the theory of porous media.

First, it should be pointed out that the analysis of wave propagation in two-phases media [1] based on the classical mixture theory [2] does not take into account any pore structure influence on wave propagation parameters (velocities, attenuation coefficients). This is due to the fact that in the mixture theory, mostly applied in the miscible mixtures, there is no pore structure parameter describing the internal skeleton structure. An extended discussion of this problem is given in the review article [3].

The use of the volume fraction theory (see e.g. [3]) in which the skeleton pore structure is characterized by the volume porosity allows us to analyse the pore effect on wave parameters ([4, 5]). But since the volume porosity does not reflect all the geometrical features of a porous skeleton, the pore effect is accounted for only partially.

The purpose of this note is to discuss the pore effect on elastic waves propagation in an isotropic fluid-saturated porous solid in the case when the pore structure is characterized by two macroparameters: the volume porosity f_v and the structural permeability parameter λ defined in [6] where the relation between the micro and macro fluid flow was utilized.

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In such a case, relative fluid macro flow satisfies the condition

$$\varrho^f f_v(\mathbf{v}^f - \mathbf{v}^s) = \varrho^f \lambda \overset{*}{\mathbf{u}}, \tag{1.1}$$

where $\mathbf{v}^f$ and $\mathbf{v}^s$ are the volume average fluid and porous solid velocities, respectively, and $\overset{*}{\mathbf{u}}$ can be considered as the area average relative fluid velocity.

This allows to express the fluid momentum $\varrho f_0 \mathbf{v}^f$ as consisting of two parts:

$$\varrho^f f_v \mathbf{v}^f = \varrho^f (f_v - \lambda)\mathbf{v}^s + \varrho^f \lambda \overset{2}{\mathbf{v}} \tag{1.2}$$

and thus to write the total fluid kinetic energy in the following form:

$$E_k^f = \frac{1}{2}\varrho^f (f_v - \lambda)\mathbf{v}^s \cdot \mathbf{v}^s + \frac{1}{2}\varrho^f \lambda \overset{2}{\mathbf{v}} \cdot \overset{2}{\mathbf{v}}, \tag{1.3}$$

where $\overset{2}{\mathbf{v}} = \mathbf{v}^s + \overset{*}{\mathbf{u}}$.

The form of the relation (1.2) shows that on the macroscopic level only part of the fluid can flow unimpeded $(\varrho^f \lambda)$ while the rest is trapped in the skeleton $[\varrho^f (f_v - \lambda)]$. This fluid division is directly related to the virtual mass effect which is well known for a single particle moving in a fluid (see Appendix).

Therefore the two-parametric pore structure characterization allows one to disclose the dynamic coupling between a solid and a fluid provoked by micropores, and then to establish its influence on the velocity and attenuation of elastic waves propagating through a porous material.

2. Basic balance and constitutive equations

Basing on the mixture theory for the immiscible porous solid fluid composition with two-parametric pore structure characterization, the general equations of mass and momentum balance have been developed in [7]. In such a case the solid fluid composition can be considered as composed of two virtual components; the first being the skeleton and the fluid trapped in it, the second one being the unimpeded fluid.

For a chemically inert porous solid-fluid mixture the equation of mass continuity for each virtual component has the form

$$\frac{\partial}{\partial t}\overset{k}{\varrho} + \operatorname{div}(\overset{kk}{\varrho \mathbf{v}}) = \overset{k}{g}, \qquad k = 1, 2, \tag{2.1}$$

$$\overset{1}{\varrho} = \varrho^s (1 - f_v) + (f_v - \lambda)\varrho^f; \qquad \overset{2}{\varrho} = \varrho^f \lambda,$$

where $\overset{k}{\varrho}$ represents the density of the virtual k-component, $\overset{k}{\mathbf{v}}$ is the velocity and $\overset{k}{g}$ stands for mass supply of the k-component and where ϱ^s and ϱ^f are effective densities of a porous solid and a fluid, respectively. Mass supply terms $\overset{1}{g}$ and $\overset{2}{g}$ satisfy the condition

$$\overset{1}{g} + \overset{2}{g} = 0,$$

and are defined as follows [8]:

$$g = \overset{1}{g} = -\overset{2}{g} = \bar{\varrho}^s \frac{\overset{1}{D}}{Dt}\left[\left(1-\frac{\lambda}{f_v}\right)\frac{\bar{\varrho}^f}{\bar{\varrho}^s}\right],$$

where

$$\bar{\varrho}^s = \varrho^s(1-f_v), \quad \bar{\varrho}^f = \varrho^f f_v,$$

$$\frac{\overset{1}{D}}{Dt}(\cdot) = \frac{\partial}{\partial t}(\cdot)+\overset{1}{\mathbf{v}}\mathrm{grad}(\cdot).$$

The motion equations for the virtual components are

$$\mathrm{div}\overset{k}{\mathbf{\Pi}}+\overset{k}{\varrho}\overset{k}{\mathbf{b}}+\overset{k}{\boldsymbol{\pi}} = \overset{k}{\varrho}\frac{\overset{k}{D}}{Dt}\overset{k}{\mathbf{v}}+\frac{1}{2}\overset{1}{g}(\overset{k}{\mathbf{v}}\overset{2}{\mathbf{v}}-\overset{1}{\mathbf{v}}), \qquad k = 1, 2, \tag{2.2}$$

$$\frac{\overset{k}{D}}{Dt}(\cdot) = \frac{\partial}{\partial t}(\cdot)+\overset{k}{\mathbf{v}}\mathrm{grad}(\cdot),$$

where $\overset{k}{\mathbf{\Pi}}$ is the symmetric partial stress tensor, $\boldsymbol{\pi} = \overset{1}{\boldsymbol{\pi}} = -\overset{2}{\boldsymbol{\pi}}$ represents the internal interaction forces and $\mathbf{b}$ is the body force per unit mass.

The linear constitutive relations can be written in the following form [9]:

$$\begin{aligned}\overset{1}{\mathbf{\Pi}} &= 2N\overset{1}{\boldsymbol{\epsilon}}+\{(H_1+2N)\overset{1}{\Theta}+H_2\overset{2}{\Theta}\}\mathbf{1},\\ \overset{2}{\mathbf{\Pi}} &= (H_2\overset{1}{\Theta}+H_3\overset{2}{\Theta})\mathbf{1},\end{aligned} \tag{2.3}$$

where $\overset{1}{\boldsymbol{\epsilon}}$ is the strain tensor of the first virtual constituent, $\overset{1}{\Theta}$, $\overset{2}{\Theta}$ represent the dilatation of the constituents and N, H_1, H_2 and H_3 are material constants.

For a further discussion we assume that the internal interaction force can be proposed either in the form of the diffusive resistance (quasi-stationary hypothesis)

$$\boldsymbol{\pi} = b_1(\overset{2}{\mathbf{v}}-\overset{1}{\mathbf{v}}), \tag{2.4}$$

where b_1 is the viscous drag coefficient, or in the frequency depending form

$$\boldsymbol{\pi} = b_1(\overset{2}{\mathbf{v}}-\overset{1}{\mathbf{v}})+d_1(\omega)(\overset{2}{\mathbf{v}}-\overset{1}{\mathbf{v}})+c_1(\omega)\frac{\partial}{\partial t}(\overset{2}{\mathbf{v}}-\overset{1}{\mathbf{v}}), \tag{2.5}$$

where the last two terms result from the history of the relative fluid velocity changes influence on the momentum exchange between the constituents [10]. The coefficients d_1 and c_1 can be described through the analysis of a channel-like or globule-like model of a porous material. In the case of a channel-like model they are

$$\begin{aligned} c_1(\omega) &= 2\lambda\varrho^f\frac{1}{Rk}\mathrm{Im}\left[\frac{J_1(j^{3/2}kR)}{j^{3/2}J_2(j^{3/2}kR)}\right],\\ d_1(\omega) &= b_1\left\{\frac{R\omega\varrho^f}{4\mu k}\mathrm{Re}\left[\frac{J_1(j^{3/2}kR)}{j^{3/2}J_2(j^{3/2}kR)}\right]-1\right\},\end{aligned} \tag{2.6}$$

where $k = \sqrt{\frac{\omega\varrho^f}{\mu}}$, $j = \sqrt{-1}$, μ is the fluid viscosity and R stands for the mean channel radius.

3. Pore structure influence on harmonic wave parameters. Example

To analyse the harmonic wave propagation, we make use of the linearized form of the continuity and motion equations (2.1) and (2.2), and we take into account the constitutive relations (2.3) and (2.5). The application of the operation of divergence and rotation yields to both the equations governing the propagation of the dilatational waves:

$$
\begin{aligned}
H_1\nabla^2\overset{1}{\Theta}+H_2\nabla^2\overset{2}{\Theta} &= \overset{1}{\varrho}\frac{\partial^2}{\partial t^2}\overset{1}{\Theta}-(b_1+d_1)\frac{\partial}{\partial t}(\overset{2}{\Theta}-\overset{1}{\Theta})-c_1\frac{\partial^2}{\partial t^2}(\overset{2}{\Theta}-\overset{1}{\Theta}),\\
H_2\nabla^2\overset{1}{\Theta}+H_3\nabla^2\overset{2}{\Theta} &= \overset{2}{\varrho}\frac{\partial^2}{\partial t^2}\overset{2}{\Theta}+(b_1+d_1)\frac{\partial}{\partial t}(\overset{2}{\Theta}-\overset{1}{\Theta})+c_1\frac{\partial^2}{\partial t^2}(\overset{2}{\Theta}-\overset{1}{\Theta}),
\end{aligned}
\tag{3.1}
$$

and the equations governing the propagation of the rotational waves

$$
\begin{aligned}
N\nabla^2\overset{1}{\boldsymbol{\omega}} &= \overset{1}{\varrho}\frac{\partial^2}{\partial t^2}\overset{1}{\boldsymbol{\omega}}-(b_1+d_1)\frac{\partial}{\partial t}(\overset{2}{\boldsymbol{\omega}}-\overset{1}{\boldsymbol{\omega}})-c_1\frac{\partial^2}{\partial t^2}(\overset{2}{\boldsymbol{\omega}}-\overset{1}{\boldsymbol{\omega}}),\\
0 &= \overset{2}{\varrho}\frac{\partial^2}{\partial t^2}\overset{2}{\boldsymbol{\omega}}+(b_1+d_1)\frac{\partial}{\partial t}(\overset{2}{\boldsymbol{\omega}}-\overset{1}{\boldsymbol{\omega}})+c_1\frac{\partial^2}{\partial t^2}(\overset{2}{\boldsymbol{\omega}}-\overset{1}{\boldsymbol{\omega}}),
\end{aligned}
\tag{3.2}
$$

where $\overset{1}{\Theta} = \mathrm{tr}\overset{1}{\boldsymbol{\epsilon}}$, $\overset{1}{\boldsymbol{\omega}} = \mathrm{rot}\overset{1}{\mathbf{u}}$, $\overset{2}{\boldsymbol{\omega}} = \mathrm{rot}\overset{2}{\mathbf{u}}$.

In the case when the relation (2.4) instead of Eq. (2.5) is used, one has to assume in Eqs. (3.1) and (3.2) $c_1(\omega) = 0$ and $d_1(\omega) = 0$. The solutions of Eqs. (3.1) and (3.2), for the plane waves, may be written in the following form:

for the dilatational waves

$$
\begin{aligned}
\overset{1}{\Theta} &= A_1\exp[j(lx-\omega t)],\\
\overset{2}{\Theta} &= A_2\exp[j(lx-\omega t)],
\end{aligned}
\tag{3.3}
$$

and for the rotational waves

$$
\begin{aligned}
\overset{1}{\omega} &= A_3\exp[j(kx-\omega t)],\\
\overset{2}{\omega} &= A_4\exp[j(kx-\omega t)],
\end{aligned}
\tag{3.4}
$$

where l and k are complex wave numbers and ω is the frequency.

The requirement of the existence of non-zero solutions (3.3) and (3.4) leads to the dispersion relation for the dilatational waves

$$
\begin{aligned}
l^4(H_1H_3-H_2^2)+l^2[-H_1\overset{2}{\varrho}\omega^2-H_3\overset{1}{\varrho}\omega^2+i\alpha(b_1+d_1)H-c_1\omega^2H]&\\
+\overset{1}{\varrho}\overset{2}{\varrho}\omega^4-i\omega^3(b_1+d_1)\varrho+c_1\varrho\omega^4 &= 0
\end{aligned}
\tag{3.5}
$$

and the dispersion relation for the rotational wave

$$
k^2[-N\overset{2}{\varrho}\omega^2+i\omega N(b_1+d_1)-Nc_1\omega^2]+\overset{1}{\varrho}\overset{2}{\varrho}\omega^4-i\omega^3\varrho(b_1+d_1)+\varrho\omega^4c_1 = 0,
\tag{3.6}
$$

where

$$
H = H_1+H_3+2H_2, \qquad \varrho = \overset{1}{\varrho}+\overset{2}{\varrho}.
$$

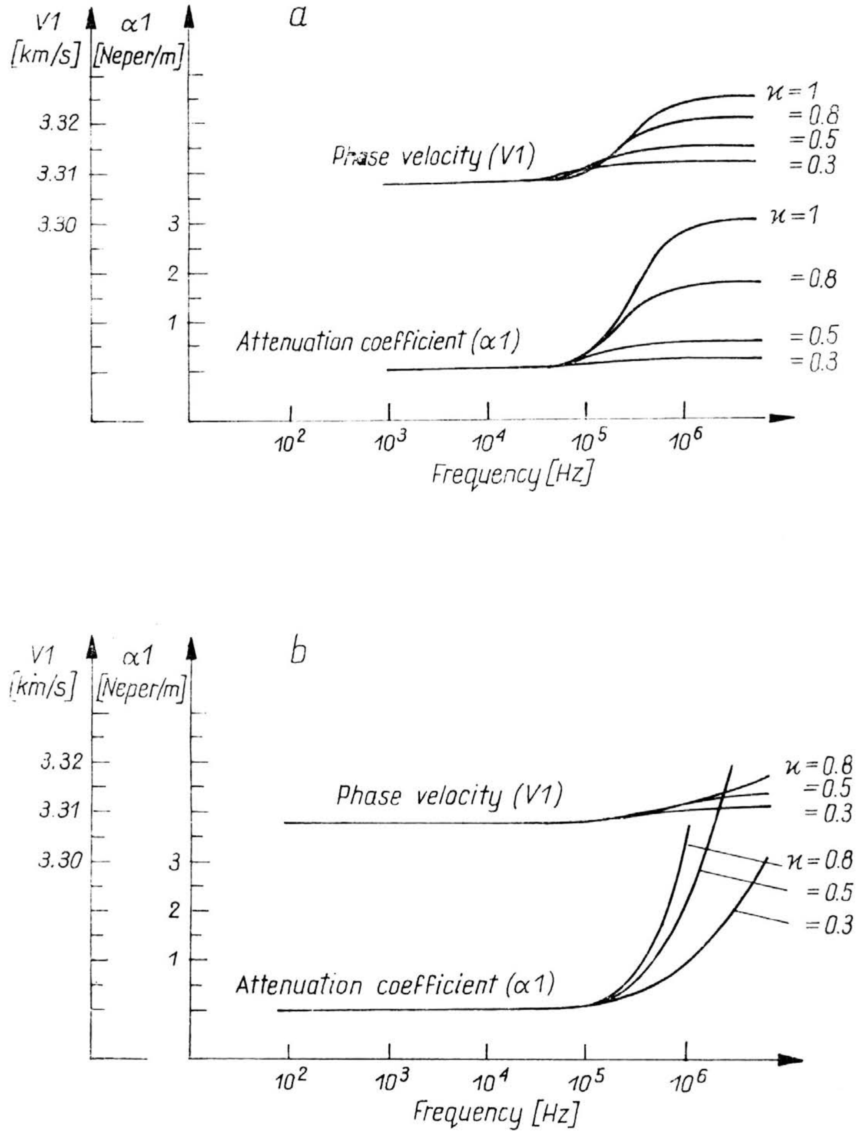

FIG. 1. Phase velocity and attenuation coefficient vs frequency for the fast wave. a — constant flow resis tance, b — frequency-dependent flow resistance.

They allow to analyse a wide range influence of the structural permeability parameter λ on the velocity and attenuation coefficient of harmonic waves while the other material coefficients remain constant.

In Figs. 1, 2 and 3 numerical results are shown for the sandstone Berea filled with water using the material constants determined in [11]. In Figs. 1a, 2a and 3a the phase velocity and attenuation coefficients are plotted upon the assumption that the interaction

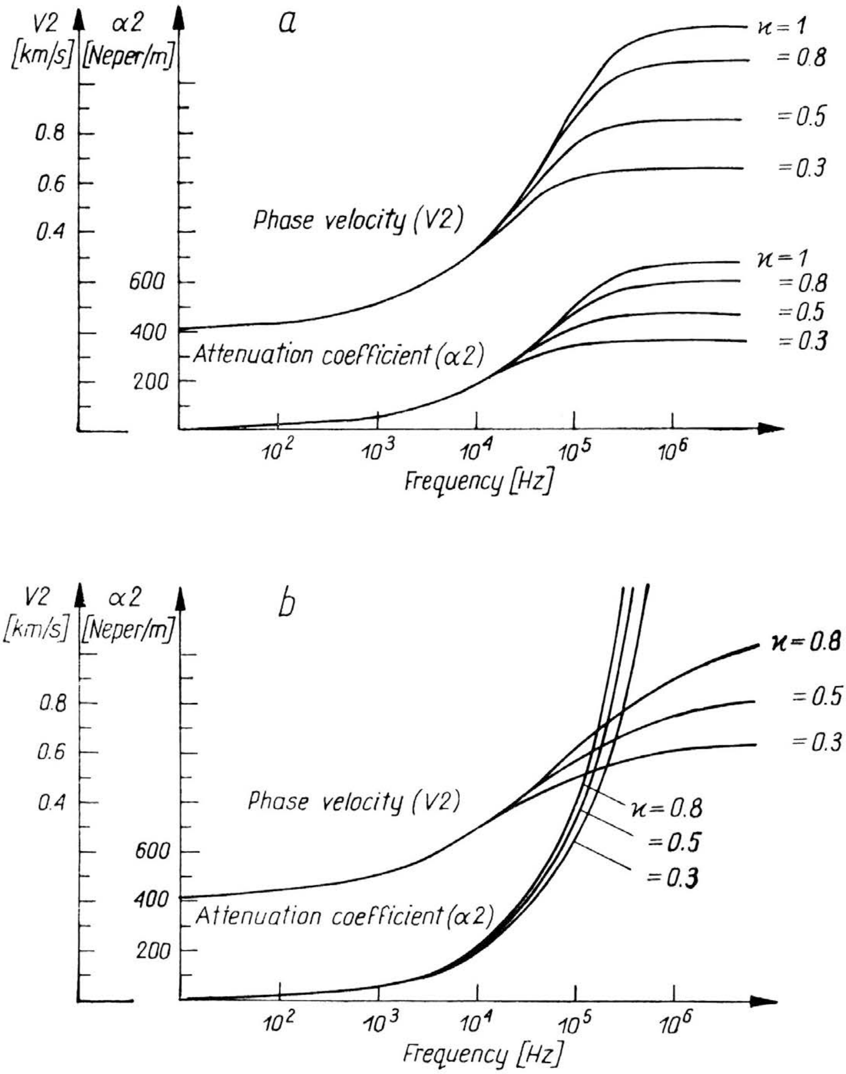

FIG. 2. Phase velocity and attenuation coefficient vs frequency for the slow wave. a — constant flow resistance, b — frequency-dependent flow resistance.

force takes form of the diffusive resistance (2.4) while in Figs. 1b, 2b and 3b they are plotted for the frequency-depending internal interaction force (2.5) applied to the channel-like model of the porous sandstone.

The presented curves of the phase velocity and the attenuation coefficients as the functions of frequency are plotted for different values of $\varkappa$ in the range

$$0 < \varkappa \leqslant 1,$$

where $\varkappa = \lambda/f_v$, taking the volume porosity $f_v = \text{const}$.

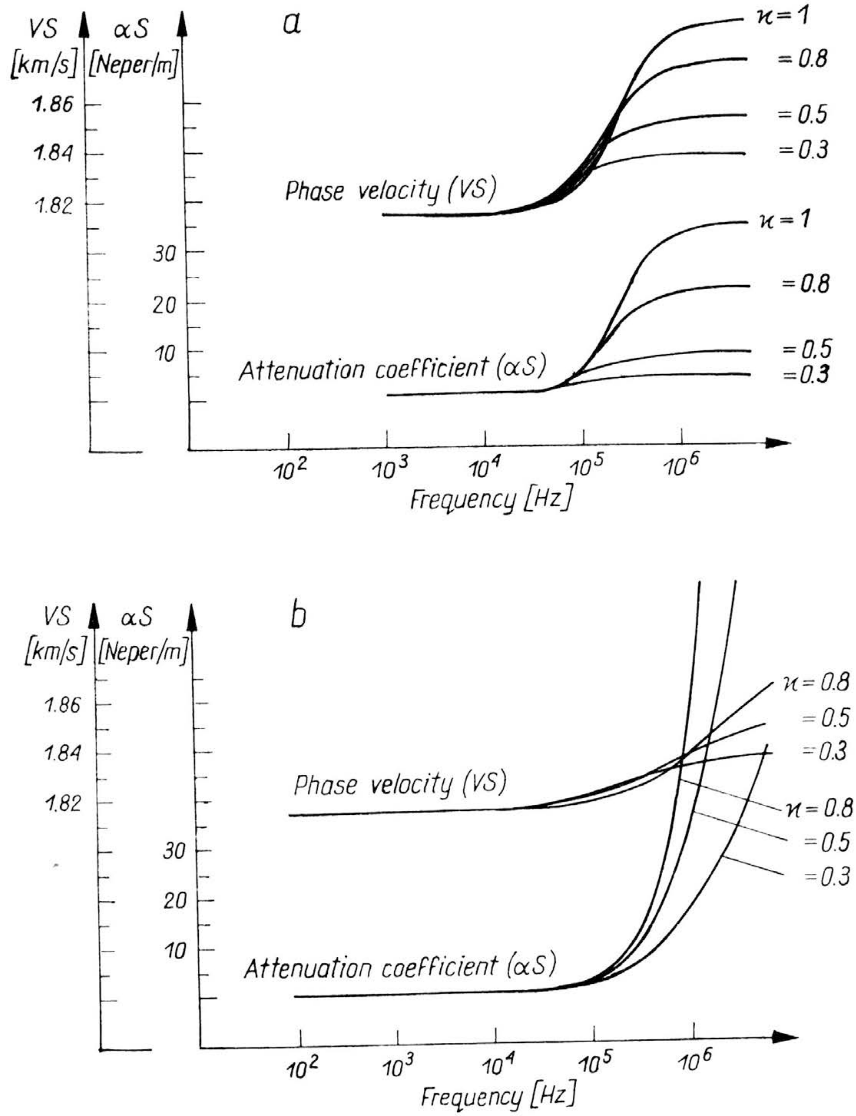

FIG. 3. Phase velocity and attenuation coefficient vs frequency for the shear wave. a — constant flow resistance, b — frequency-dependent flow resistance.

4. Conclusions

On the ground of the numerical results shown in Figs. 1, 2 w and 3, one can state that the wave propagation parameters depend on the skeleton pore structure and this is most visible within the high frequency range This dependence is most evident in the case of slow compressional wave propagation.

One can note that the use of the measured wave parameters, especially at the high frequency range, and the considered relations between the pore structure and the propagation parameters allow one to determine the structure parameter λ.

This suggests that in dealing with the dynamic behaviour of a porous material, the self-consistent theory with the two-parametric characterization, preserving all the basic features of the immiscible porous solid-fluid composition, proves to be very useful.

Appendix

A. Virtual mass effect for a moving particle in an unbounded fluid

It is well known that for a relative motion of a solid particle moving at velocity $\mathbf{v}$ in an unbounded fluid, the motion equation can be written in the following form [12, 13]:

$$M\frac{d\mathbf{v}}{dt}+\left\{\mathbf{R}+m\frac{d\mathbf{v}}{dt}\right\} = \mathbf{F}, \tag{A.1}$$

where M is the mass of a particle, $\mathbf{R}+m\frac{d\mathbf{v}}{dt}$ is the total drag force exerted by the fluid on the particle and $\mathbf{F}$ is an external force.

Although Eq. (A.1) is the dynamic equation for the solid particle, when written in the form

$$(M+m)\frac{d\mathbf{v}}{dt}+\mathbf{R} = \mathbf{F} \tag{A.2}$$

it may be treated as the momentum balance of a system still moving at velocity $\mathbf{v}$ which consists of the particle and some part of the fluid the mass of which is m [13]. The total mass of the system $(M+m)$ is called the virtual mass and m an added mass.

B. Virtual constituents of a porous solid-fluid composition

It is reasonable to expect the effect of the added mass appearing in the description of the relative motion of a solid particle with respect to a fluid in an analysis of the relative motion of any solid and fluid.

Thus this effect should also be accounted for in any consistent description of an unsteady motion of fluid-saturated porous media.

A consequence of this fact is that from the kinematic point of view the fluid phase is divided into two parts: the fluid trapped in the porous skeleton and moving at the skeleton velocity, and the unimpeded fluid moving at its own velocity.

This fluid division allows for the considering a fluid-porous solid mixture as composed of two virtual components; the first being the skeleton and the fluid trapped in it, the second one being the unimpeded fluid.

Such an approach to the description of fluid-filled porous solids has been developed within a theory having a two-parametric pore structure description, the basis of which is given in [7, 14].

The densities of the virtual constituents are

(B.1)
$$\overset{1}{\varrho} = \varrho^s(1-f_v)+\varrho^f(f_v-\lambda),$$
$$\overset{2}{\varrho} = \varrho^f\lambda,$$

where ϱ^s and ϱ^f are the effective densities of the porous skeleton and the fluid, respectively, f_v is the volume porosity and λ stands for the structural permeability parameter [6].

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