Arch. Mech., 32, 2, pp. 193–199, Warszawa 1980

Shock waves in thermo-viscous fluids with hidden variables

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THIS note investigates the amplitude's evolution of a shock wave entering a thermo-viscous fluid at equilibrium, through a model of fluid with hidden variables. It is shown that the evolution of the shock is determined by the difference between the volume gradient and a critical value reflecting the properties of heat conduction and viscosity.

Rozważono ewolucję amplitudy fali uderzeniowej przenikającej do płynu termolepkiego znajdującego się w równowadze; zastosowano model płynu z wewnętrznymi zmiennymi stanu. Wykazano, że ewolucja fali uderzeniowej określona jest przez różnicę gradientu objętościowego i pewnej wartości krytycznej związanej z właściwościami przewodnictwa cieplnego i lepkości.

Рассмотрена эволюция амплитуды ударной волны проникающей в термовязкую жидкость, находящуюся в равновесии; применена модель жидкости с внутренними переменными состояния. Показано, что эволюция ударной волны определена разницей об емного градиента и некоторым критическим значением, связанным со свойствами теплопроводности и вязкости.

1. Introduction

THIS PAPER deals with the evolution of shock waves propagating in thermo-viscous fluids. The investigation is accomplished by adopting a fluid model where heat conduction and viscosity are accounted for via hidden (or internal) variables (or parameters(1)). With respect to the literature on materials with hidden variables, the present constitutive assumptions are rather unusual as far as the argument of the response function is different from that of the growth equation. This is so since it is assumed that the hidden variables depend on the temperature gradient g and the rate of the strain tensor D whereas such is not the case for the response function. We mention that a distinction like this, though confined to the temperature gradient, is present, for instance, in the paper by KOSIŃSKI and PERZYNA ([2], Sect. 5) in connection with thermal waves.

The present model delivers a description of thermo-viscous fluids which yields Fourier's and Navier-Stokes' laws when g and D are time independent. Moreover, it is compatible with the propagation of shock and acceleration waves. As a result of this, the analysis of the growth or decay of shock waves shows that for a wave entering a fluid at equilibrium, heat conduction and viscosity determine the existence of a critical value of the volume gradient. As it is well known, the existence of such critical values has already been found in other similar contexts (e.g. [3, 4]). Therefore this note provides, in particular, a physically significant example of a critical value for shock propagation in materials with hidden variables.

(1) Such a viewpoint may also be applied to the general case of shock and acceleration waves in thermo-viscous materials [1].

2. Constitutive assumptions and thermodynamic restrictions

Throughout $\mathbf{R}$, $\mathbf{R}^+$, $\mathbf{R}^{++}$ stand respectively for the set of real, positive real, and strictly positive real numbers. The symbols $\mathbf{Y}$, $\mathbf{Z}$, $\mathbf{A}$, $\mathbf{\Sigma}$ denote finite dimensional vector spaces. A superposed dot indicates the material time derivative. A subscripts θ, v, $\tilde{\mathbf{g}}$, $\tilde{\mathbf{D}}$, η, X denotes partial derivatives.

A material with hidden variables $\{\mathbf{y}_0, \mathbf{z}_0, \boldsymbol{\alpha}_0, \hat{\boldsymbol{\sigma}}, \mathbf{h}\}$ on $\mathbf{Y}\times\mathbf{Z}\times\mathbf{A}$ consists of a ground value $(\mathbf{y}_0, \mathbf{z}_0, \boldsymbol{\alpha}_0)$ of the independent variables $(\mathbf{y}, \mathbf{z}, \boldsymbol{\alpha}) \in \mathbf{Y}\times\mathbf{Z}\times\mathbf{A}$ and of the maps

$$\hat{\boldsymbol{\sigma}} \in C^3(\mathbf{Y}\times\mathbf{A}, \mathbf{\Sigma}), \quad \mathbf{h} \in C^2(\mathbf{Y}\times\mathbf{Z}\times\mathbf{A}, \mathbf{A}). \tag{2.1}$$

The growth of the hidden variables $\boldsymbol{\alpha} \in \mathbf{A}$ is determined by the whole set of the independent variables through the evolution function $\mathbf{h}$, whereas the response of the material depends only on the pair $(\mathbf{y}, \boldsymbol{\alpha})$ through the response function $\hat{\boldsymbol{\sigma}}$.

The function $\mathbf{h}$ is subject to the following conditions.

I. *There exists a map* $\mathbf{E}\colon \mathbf{Y}\times\mathbf{Z} \to \mathbf{A}$ *such that for each pair* $(\mathbf{y}, \mathbf{z}) \in \mathbf{Y}\times\mathbf{Z}$ the hidden variable $\mathbf{E}(\mathbf{y}, \mathbf{z}) \in \mathbf{A}$ yields

$$\mathbf{h}\big(\mathbf{y}, \mathbf{z}, \mathbf{E}(\mathbf{y}, \mathbf{z})\big) = \mathbf{0}$$

and

$$\mathbf{E}(\mathbf{y}_0, \mathbf{z}_0) = \boldsymbol{\alpha}_0.$$

II. *There is a map* $\mathbf{\Lambda} \in \mathbf{L}(\mathbf{A}, \mathbf{A})$ *and a positive constant* δ *such that*

$$|\mathbf{h}(\mathbf{y}, \mathbf{z}, \boldsymbol{\alpha}+\boldsymbol{\beta}) - \mathbf{h}(\mathbf{y}, \mathbf{z}, \boldsymbol{\alpha}) - \mathbf{\Lambda}\boldsymbol{\beta}| \leqslant \delta|\boldsymbol{\beta}|, \quad (\mathbf{y}, \mathbf{z}) \in \mathbf{Y}\times\mathbf{Z}, \quad \boldsymbol{\alpha}, \boldsymbol{\alpha}+\boldsymbol{\beta} \in \mathbf{A},$$

while $\mathbf{\Lambda} + \delta\mathbf{I}_{\mathbf{A}}$ *is negative definite.*

Thus $\mathbf{h}(\mathbf{y}, \mathbf{z}, \cdot\,) \in \mathrm{Lip}(|\mathbf{\Lambda}| + \delta)$ for every $(\mathbf{y}, \mathbf{z}) \in \mathbf{Y}\times\mathbf{Z}$. Property II ensures the asymptotic stability of $(\mathbf{y}_0, \mathbf{z}_0, \boldsymbol{\alpha}_0)$ and the uniqueness of the solution of the evolution equation

$$\dot{\boldsymbol{\alpha}} = \mathbf{h}(\mathbf{y}, \mathbf{z}, \boldsymbol{\alpha}), \quad \boldsymbol{\alpha}(t_0) = \boldsymbol{\alpha}'. \tag{2.2}$$

As we are interested in thermo-viscous fluids, we confine our attention to the choice $\mathbf{y} = (\theta, v)$ and $\mathbf{z} = (\mathbf{g}, \mathbf{D})$, where θ is the temperature and v the specific volume. Meanwhile, we represent by $\boldsymbol{\sigma}$ the set of the (specific) free energy ψ, the entropy η, the Cauchy stress tensor $\mathbf{T}$, and the heat flux $\mathbf{q}$. So the thermo-viscous fluid is described by

$$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\theta, v, \boldsymbol{\alpha}),$$
$$\dot{\boldsymbol{\alpha}} = \mathbf{h}(\theta, v, \mathbf{g}, \mathbf{D}, \boldsymbol{\alpha}).$$

Assume now that $\boldsymbol{\alpha}$ is in fact a pair $(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2)$ of a vector $\boldsymbol{\alpha}_1$ and a symmetric tensor $\boldsymbol{\alpha}_2$. Letting $\dim\mathbf{A} = \dim\mathbf{Z}$ — see, e.g. [5] — property II is certainly satisfied if $\mathbf{h}$ is so chosen that the evolution equations for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ are

$$\begin{aligned} \dot{\boldsymbol{\alpha}}_1 &= \frac{1}{\tau}(\mathbf{g} - \boldsymbol{\alpha}_1), \quad \boldsymbol{\alpha}_1(t_0) = \boldsymbol{\alpha}_1', \\ \dot{\boldsymbol{\alpha}}_2 &= \frac{1}{\tau}(\mathbf{D} - \boldsymbol{\alpha}_2), \quad \boldsymbol{\alpha}_2(t_0) = \boldsymbol{\alpha}_2'. \end{aligned} \tag{2.3}$$

Physically the quantity $\tau \in \mathbf{R}^{++}$ plays the role of relaxation time. The obvious solutions of the initial value problems (2.3) are

$$\tag{2.3'} \begin{aligned} \boldsymbol{\alpha}_1(t) &= \tilde{\mathbf{g}}(t)+\boldsymbol{\alpha}'_1 \exp\{-(t-t_0)/\tau\}, \qquad t-t_0 \in \mathbf{R}^+,\\ \boldsymbol{\alpha}_2(t) &= \tilde{\mathbf{D}}(t)+\boldsymbol{\alpha}'_2 \exp\{-(t-t_0)/\tau\}, \qquad t-t_0 \in \mathbf{R}^+, \end{aligned}$$

the symbol $\tilde{\xi}(t)$ being defined as

$$\tilde{\xi}(t) = \frac{1}{\tau}\int_{t_0}^{t} \exp\{-(t-\zeta)/\tau\}\xi(\zeta)d\zeta.$$

Consider the set of evolutions of hidden variables starting from the vanishing initial values $\boldsymbol{\alpha}'_1 = \mathbf{0}$, $\boldsymbol{\alpha}'_2 = \mathbf{0}$(²). In this case the response function $\hat{\boldsymbol{\sigma}} = \{\hat{\psi}, \hat{\eta}, \hat{\mathbf{T}}, \hat{\mathbf{q}}\}$ is restricted by the Clausius-Duhem inequality expressed by

$$\tag{2.4} -\frac{1}{v}(\hat{\psi}_\theta-\eta)\dot{\theta}+\operatorname{tr}\left\{\left(\mathbf{T}-\psi_v\mathbf{I}-\frac{1}{\tau v}\hat{\psi}_{\tilde{\mathbf{D}}}\right)\mathbf{D}\right\}-\left(\frac{1}{\theta}\mathbf{q}+\frac{1}{\tau v}\hat{\psi}_{\tilde{\mathbf{g}}}\right)\cdot\mathbf{g} +\frac{1}{\tau v}(\hat{\psi}_{\tilde{\mathbf{g}}}\cdot\tilde{\mathbf{g}}+\operatorname{tr}\{\hat{\psi}_{\tilde{\mathbf{D}}}\tilde{\mathbf{D}}\}) \geqslant 0,$$

which must hold for every path $\boldsymbol{\pi}(\zeta) = (\theta, v, \mathbf{g}, \mathbf{D})(\zeta)$, $\zeta-t_0 \in \mathbf{R}^+$. It is a general property within the theory of hidden variables, true here as a particular case, that $\boldsymbol{\alpha}(t)$ is independent of the present value $\boldsymbol{\pi}(t)$. Therefore Eq. (2.4) holds if and only if

$$\tag{2.5} \eta = -\hat{\psi}_\theta, \quad \mathbf{T} = \hat{\psi}_v\mathbf{I}+\frac{1}{\tau v}\hat{\psi}_{\tilde{\mathbf{D}}}, \quad \mathbf{q} = -\frac{\theta}{\tau v}\hat{\psi}_{\tilde{\mathbf{g}}},$$

$$\tag{2.6} \hat{\psi}_{\tilde{\mathbf{g}}}\cdot\tilde{\mathbf{g}}+\operatorname{tr}\{\hat{\psi}_{\tilde{\mathbf{D}}}\tilde{\mathbf{D}}\} \geqslant 0.$$

The application exhibited in Sect. 3 and 4 corresponds to choosing the free energy in the form

$$\tag{2.7} \psi = \hat{\psi}(\theta, v, \boldsymbol{\alpha}) = \Psi(\theta, v)+v\tau\left\{\frac{\varkappa}{2\theta}\tilde{\mathbf{g}}\cdot\tilde{\mathbf{g}}+\mu\operatorname{tr}\{\tilde{\mathbf{D}}\tilde{\mathbf{D}}\}+\frac{\lambda}{2}(\operatorname{tr}\tilde{\mathbf{D}})^2\right\},$$

where $\varkappa$, μ, λ are non-vanishing constants. The function (2.7) is compatible with inequality (2.6) if and only if

$$\tag{2.8} \mu > 0, \quad 3\lambda+2\mu \geqslant 0; \quad \varkappa > 0.$$

Meanwhile Eqs. $(2.5)_{2,3}$ deliver

$$\tag{2.9} \mathbf{T} = -\hat{p}(\theta, v, \boldsymbol{\alpha})\mathbf{I}+2\mu\tilde{\mathbf{D}}+\lambda\{\operatorname{tr}\tilde{\mathbf{D}}\}\mathbf{I}, \quad \mathbf{q} = -\varkappa\tilde{\mathbf{g}},$$

being $\hat{p} := -\hat{\psi}_v$. If $\mathbf{D}(\cdot)$ and $\mathbf{g}(\cdot)$ are time independent, i.e. $\mathbf{D}(t) = \mathbf{D}$, $\mathbf{g}(t) = \mathbf{g}$, $t \in \mathbf{R}$, we have the asymptotic condition

$$\lim_{t\to\infty}(\tilde{\mathbf{g}}(t), \tilde{\mathbf{D}}(t)) = (\mathbf{g}, \mathbf{D}).$$

In this instance, except for the dependence of p on $\boldsymbol{\alpha}$, Eqs. (2.9) asymptotically become Navier-Stokes' and Fourier's constitutive equations. Then the conditions (2.8) may be regarded as Stokes-Duhem's and Fourier's inequalities.

(²) This condition is meaningful in view of the asymptotic stability of Eq. (2.3').

It is worth emphasizing that the assumption (2.7) does not significantly reduce the generality of the present note. As it is shown in Sects. 3 and 4, the contributions to the jumps arise from the linear terms.

3. Jump relations

The motion of the fluid is described by a function $\mathbf{x}$ defined on $\mathscr{R}\times\mathbf{R}$, $\mathscr{R}$ being a fixed reference configuration with volume v_0. Labelling the particles by their position in the reference configuration, we say that

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t), \quad (\mathbf{X}, t) \in \mathscr{R}\times\mathbf{R}, \tag{3.1}$$

is the position at time t of the particles $\mathbf{X}$. Consider a motion containing a shock wave $\mathscr{S}(t)$. Adopting the usual definitions, we denote by $[\xi](t) := \xi^-(t)-\xi^+(t)$ the jump across $\mathscr{S}(t)$ of any function ξ defined on $\mathscr{R}\times\mathbf{R}$ and by $U(t)$ the speed of propagation of $\mathscr{S}(t)$. The motion (3.1) is assumed to be continuous across $\mathscr{S}(t)$. Moreover, as $\boldsymbol{\alpha}(t)$ is independent of $\boldsymbol{\pi}(t)$, the same assumption is introduced for the hidden variable $\boldsymbol{\alpha}$. Then, letting ∇ stand for the material gradient operator, we set:

S1. The functions $\mathbf{x}, \boldsymbol{\alpha}$ are continuous on $\mathscr{R}\times\mathbf{R}$.

S2. The functions $\theta, v, \dot{\mathbf{x}}, \nabla\mathbf{x}, \dot{\boldsymbol{\alpha}}, \nabla\boldsymbol{\alpha}$, and the derivatives of higher order suffer jump discontinuities across $\mathscr{S}$ but are continuous functions on $(\mathscr{R}\times\mathbf{R})/\mathscr{S}$.

S3. The body force and the heat supply (per unit mass) are supposed to be $\mathbf{C}^1$ functions on $\mathscr{R}\times\mathbf{R}$.

Although $\mathbf{D}(t)$ has a jump discontinuity the function $\tilde{\mathbf{D}}(t)$ is continuous. This implies that

$$[\mathbf{T}] = -[p]\mathbf{I}.$$

An immediate consequence is that the shock waves are always longitudinal while the same property is not true for the acceleration waves [1]. Thus, in the study of shock waves we may confine our considerations to uni-dimensional waves; henceforth we will refer only to the meaningful component of vectors and tensors relative to the direction of the wave. A standard procedure applied to the balance of momentum (in integral form) leads to the well-known relation

$$[p] = -\frac{U^2}{v_0^2}[v] \tag{3.2}$$

being

$$U[v] = -v_0[\dot{x}]. \tag{3.3}$$

Moreover, since $[q] = 0$, the energy balance makes it possible to write the Hugoniot relation

$$[e] = \overline{T}[v], \tag{3.4}$$

where e is the internal energy and $\overline{T} := (T^- + T^+)/2$. The balance of momentum and energy in differential form yields the further relations

$$[\ddot{x}] = v_0[T_X], \tag{3.5}$$

$$[\dot{e}] = [T\dot{v}] - v_0[q_X]. \tag{3.6}$$

With respect to previous analyses of shock waves in fluids with hidden variables — see, e.g. [3] — the set of jump relations (3.4)–(3.6) exhibits the additional contributions $(\overline{T}+\overline{p})[v]$, $[T_X+p_X]$, and $[(T+p)\dot{v}]$ due to viscosity and $[q_X]$ due to heat conduction. Just these terms are responsible for the existence of a critical value of $[v_X]$ in the evolution of $[v]$.

The investigation of the growth of a wave hinges on the compatibility relation

$$\frac{d[\xi]}{dt} = [\dot{\xi}]+U[\xi_X]. \tag{3.7}$$

On account of Eqs. (3.3), (3.5), and the continuity equation $\dot{v} = v_0\dot{x}_X$, the application of Eq. (3.7) to $\xi = v$ and $\xi = \dot{x}$ gives

$$2U\frac{d[v]}{dt}+[v]\frac{dU}{dt} = U^2[v_X]-v_0^2[T_X]. \tag{3.8}$$

4. The shock amplitude equation

Henceforth we suppose that the fluid ahead of the wave front has been at equilibrium, $(\theta, v, g, D, \boldsymbol{\alpha}) = (\theta^+, v^+, 0, 0, 0)$, at all past times. Moreover, we assume as (uniform) reference configuration that of the fluid before the arrival of the wave. Then we have

$$\dot{\theta}^+ = 0, \quad \dot{v}^+ = 0, \quad g^+ = 0, \quad D^+ = 0, \quad \boldsymbol{\alpha}^+ = 0, \quad \dot{\boldsymbol{\alpha}}^+ = 0, \tag{4.1}$$

and

$$\tilde{g}^{\pm} = 0, \quad \tilde{D}^{\pm} = 0, \quad \dot{\tilde{g}}^{\pm} = \frac{1}{\tau}g^{\pm}, \quad \dot{\tilde{D}}^{\pm} = \frac{1}{\tau}D^{\pm}, \tag{4.2}$$

on the wave. The assumptions (4.2) slightly simplify the jump relations. In fact,

$$T^{\pm} = -p^{\pm}, \quad [T\dot{v}] = -[p\dot{v}],$$

while the compatibility condition $[\dot{\tilde{D}}] = -U[\tilde{D}_X]$ allows us to write

$$[T_X] = -[p_X]-\frac{\lambda+2\mu}{\tau U}[D].$$

Furthermore, since p is a quadratic function of $\tilde{g}$ and $\tilde{D}$, it follows that the derivatives of p with respect to $\tilde{g}$ and $\tilde{D}$ vanish at the wave. Thus, according to Eqs. (4.1), substitution in Eq. (3.8) yields

$$2U\frac{d[v]}{dt}+[v]\frac{dU}{dt} = (U^2+v_0^2p_v^-)v_X^- +v_0p_\theta^-\theta_X^-+\frac{v_0^3(\lambda+2\mu)}{\tau Uv^-}\dot{x}_X^- \tag{4.3}$$

being $\partial_x = (v_0/v)\partial_X$. As we are looking for the evolution equation both of $[v]$ and of U, further informations are required. These may be achieved via the following procedure. First, as a consequence of Eq. (3.6) we obtain

$$\dot{\theta}^- = -\frac{\eta_v^-}{\eta_\theta^-}\left(\frac{d[v]}{dt}-Uv_X^-\right)-\varkappa U\omega\theta_X^-, \tag{4.4}$$

where $\omega = v_0^2/\tau U^2\theta^- v^- \eta_\theta^-$. The jump of p is due only to $[\theta]$ and $[v]$. Then, using Eq. (4.4) we have

$$\frac{dp^-}{dt} = \gamma \frac{d[v]}{dt} + U \frac{p_\theta^- \eta_v^-}{\eta_\theta^-} v_X^- + U p_\theta^- (1-\varkappa\omega)\theta_X^-, \tag{4.5}$$

where $\gamma = p_v^- - p_\theta^- \eta_v^- / \eta_\theta^-$. Applying the compatibility condition (3.7) to $e = \psi + \theta\eta$ we get

$$\frac{d[e]}{dt} = -p^- \frac{d[v]}{dt} + U\theta^- \eta_v^- v_X^- + U\theta^- \eta_\theta^- (1-\varkappa\omega)\theta_X^-.$$

On the other hand, differentiation of the Hugoniot relation $[e] + \bar{p}[v] = 0$ provides

$$\frac{d[e]}{dt} = -\frac{1}{2}[v]\frac{dp^-}{dt} - \bar{p}\frac{d[v]}{dt}.$$

By comparison it follows at once

$$(1-\varkappa\omega)\theta_X^- = \frac{1}{2U\theta^- \eta_\theta^-}\left\{[p]\frac{d[v]}{dt} - [v]\frac{dp^-}{dt} - 2U\theta^- \eta_v^- v_X^-\right\}. \tag{4.6}$$

On account of Eq. (3.2), substitution of Eq. (4.6) into Eq. (4.5) gives

$$\frac{dp^-}{dt} = \gamma \frac{\nu + 2\phi}{1+2\phi}\frac{d[v]}{dt}, \tag{4.7}$$

where $\phi = \theta^- \eta_\theta^- / p_\theta^- [v]$, $\nu = -U^2/v_0^2\gamma$. Another expression of dp^-/dt may be obtained by differentiating Eq. (3.2) with respect to t; it turns out that

$$\frac{dp^-}{dt} = -2\frac{U}{v_0^2}[v]\frac{dU}{dt} - \frac{U^2}{v_0^2}\frac{d[v]}{dt}.$$

A comparison shows that dU/dt and $d[v]/dt$ are related by

$$\frac{dU}{dt} = \frac{U}{[v]}\frac{\phi(1-\nu)}{\nu(1+2\phi)}\frac{d[v]}{dt}. \tag{4.8}$$

Finally, substitution of Eqs. (4.6)–(4.8) into Eq. (4.3) delivers the evolution equation

$$\frac{d[v]}{dt} = \frac{(1-\nu)(1+2\phi)U}{3\nu - 1 + \phi(3\nu+1)}(\Gamma - v_X^-), \tag{4.9}$$

where

$$\Gamma = \frac{v_0}{(\nu-1)U\gamma v^- \tau}\left\{\frac{v_0\varkappa}{U\phi[v]}\theta_X^- + (\lambda+2\mu)\dot{x}_X^-\right\}. \tag{4.10}$$

According to the result (4.9), the evolution of $[v]$ is determined by the difference between the volume gradient v_X^- and the critical value Γ. The coefficient of $\Gamma - v_X^-$ is usually a positive quantity. Indeed, recalling that the medium ahead of the wave has been at equilibrium until the arrival of the wave, we have

$$p = \hat{p}(\theta, v, 0),$$
$$\eta = \hat{\eta}(\theta, v, 0),$$

on the wave. Then, letting $\hat{\eta}_\theta \neq 0$ so that we can write $p = \check{p}(\eta, v)$, it is reasonable to assume the validity of Weyl's inequalities

$$\check{p}_v = \gamma < 0, \quad \check{p}_{vv} > 0, \quad \check{p}_\eta > 0.$$

As a consequence the shock is compressive, $[v] < 0$, and the shock speed is subsonic with respect to the material behind the wave ([6], § 65); this implies that

$$0 < \nu < 1. \tag{4.11}$$

In conjunction with NUNZIATO and HERMANN [7], since $[v] < 0$ and $\check{p}_\eta > 0$ the quantity ϕ is restricted by

$$\phi < -\frac{1}{2}. \tag{4.12}$$

The sought result follows from Eqs. (4.11) and (4.12). So the quantity $|[v]|$ either grows or decays according as v_X^- is greater or smaller than Γ.

In view of Eq. (4.10) the value of Γ is the whole result of heat conduction and viscosity; it vanishes if the fluid is non-viscous and non-conducting, i.e. $\lambda = 0$, $\mu = 0$, $\varkappa = 0$. Moreover, Γ is inversely proportional to the time constant τ. Thus the contribution of heat conduction and viscosity vanishes in the limiting case $\tau \to \infty$. We may interpret this behaviour in terms of an extremely long memory which hides the effects of the shock as far as heat conduction and viscosity are concerned. On the other hand the value Γ, and $d[v]/dt$ as well, increases unboundedly as τ tends to zero. This is hardly surprising since the memory effect due to the hidden variables fades more and more as τ tends to zero; at the limit $\tau \to 0$ the constitutive Eqs. (2.9) deliver Navier-Stokes' and Fourier's equations which are not compatible with wave propagation.

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Received October 28, 1978, *final version May* 25, 1979.