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Axisymmetric Stokes flow about a body made of intersection of two spherical surfaces

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ON THE BASIS of a bipolar coordinate solution due to Payne and Pell, the fine structure of Stokes flows about a crescent-type body and a snowman-type body is considered. Only for crescent-type bodies a wake region can exist in the concavity, depending on the relative configuration of inner and outer surfaces. The drag formula is also presented, which is applicable for general bodies from a disk to two spheres in contact.

Posługując się dwubiegunowym rozwiązaniem Payne'a i Pella rozważono strukturę przepływu Stokesa wokół profilu sierpowego i profilu «śniegowego bałwanka». Ślad aerodynamiczny powstać może jedynie w wydrążeniu przekroju sierpowego, w zależności od względnej konfiguracji powierzchni zewnętrznej i wewnętrznej. Przedstawiono również wzór na opór, który stosować można dla dowolnych konfiguracji ciał od tarczy aż do dwóch stykających się kul.

Послуживаясь двухполюсным решением Пейна и Пелла, рассмотрена структура течения Стокса вокруг серповидного профиля и профиля „снежная баба''. Аэродинамический след может возникнуть только в углублении серповидного сечения, в зависимости от относительной конфигурации внешней и внутренней поверхностей. Представлена тоже формула для сопротивления, которую можно применять для произвольных конфигураций тел: от двух плоскостей, вплоть до двух касающихся сфер.

1. Introduction

THERE have been some studies [1, 2] on axisymmetric Stokes flow about a spherical cap. Among those, DORREPAAL *et al.* [2] have considered the fine structure of the flow and found a wake region within the cap's concavity. For the general lens-shaped body, a similar feature of flow near its sharp rim has recently been shown by MICHAEL and O'NEIL [3]. However, as their treatment was only local and qualitative, details of the flow separation seem to be still left to examine. Meanwhile, the two-sphere problem has been solved for separated and touching spheres already. In particular, DAVIS *et al.* [4] have studied the behaviour of flow in the neighbourhood of the spheres.

In order to obtain the solution for axisymmetric Stokes flow about a spherical lens, PAYNE and PELL [5] introduced bipolar coordinates (ξ, η) in a plane of cylindrical coordinates (x, r) through

$$x = h\sin\eta, \quad r = h\,\mathrm{sh}\,\xi,$$

where

$$s = \mathrm{ch}\,\xi, \quad t = \cos\eta \quad \text{and} \quad h = c/(s-t),$$

c being a constant. The body is formed by revolving two arcs $\eta = \eta_1$ and $\eta_2 (2\pi > \eta_2 > \eta_1 > 0)$ about the x-axis. The exterior to that is then given by $\eta_2 < \eta < \eta_1 + 2\pi$ and

$0 \leqslant \xi < \infty$. In case $\eta_1 < \pi$, say, either for $\eta_2 < \pi$ or $\eta_2 > \pi$, bodies may be classified into two types: type I having convex and concave surfaces and type II with two surfaces both convex. Payne and Pell's solution thus allows us to deal with such bodies all together.

The body of type I is crescent-like, having a spherical cap as a limit. Further details for these bodies are to be examined, based on a modified form of their solution. For $\tau = \eta_2 - \eta_1 = \pi$, we have a sphere. Hence for $\tau > \pi$, bodies of type II are snowman-like and of particular interest from the view-point of the general two-sphere problem. When the angle of intersection of two arcs is less than 146.3°, there exists an infinite sequence of ring vortexes surrounding the intersection line. The drag acting on the body will be given in an explicit expression which makes the drag formula for two spheres complete, together with the known equations for separated and touching spheres [6, 7].

The stream function due to Payne and Pell for a lens-shaped body in a uniform stream U is

$$(1.1) \qquad \psi = \frac{Ur^2}{2}\left\{1 - \frac{(s-t)^{1/2}}{[s-\cos(\eta-\eta_0)]^{1/2}} - (s-t)^{1/2}\int\limits_0^\infty F(\alpha, \eta)K'_\alpha(s)d\alpha\right\},$$

where η_0 is a value of η such that $\eta_1 < \eta_0 < \eta_2$ and can generally be chosen $\eta_0 = (\eta_1 + \eta_2)/2$, as adopted all over the present study. $K'_\alpha(s) = dK_\alpha(s)/ds$, where $K_\alpha(s) = P_{i\alpha-1/2}(s)$, the Legendre function of complex degree. The function $F(\alpha, \eta)$ was determined so as to satisfy the no-slip conditions $\psi = 0$ and $\partial\psi/\partial\eta = 0$ on $\eta = \eta_1$ and η_2. To do this, the integral representation

$$(1.2) \qquad (s-t)^{-1/2} - [s-\cos(\eta-\eta_0)]^{-1/2} = \int\limits_0^\infty f(\alpha, \eta)K'_\alpha(s)d\alpha$$

was used. However, their solution is a lengthy expression and appears, unfortunately, to have some algebraic errors. To give corrected expressions to the major equations involved becomes part of our purpose.

2. Expression near the body of the solution

In order to analyse the flow behaviour near the body, it is suitable to write ψ in the form

$$(2.1) \qquad \psi = \sqrt{2}Ur^2(s-t)^{1/2}\int\limits_0^\infty \frac{\operatorname{th}\alpha\pi}{H(z)}G(\alpha, \eta)K'_\alpha(s)d\alpha.$$

For $2\pi > \eta > \eta_0$, say, $G(\alpha, \eta)$ can be given as follows:

$$(2.2) \quad G(\alpha, \eta) = g_1(z)\operatorname{sh}(\alpha\Delta\eta)\sin\Delta\eta - \frac{g_2(z)}{\alpha^2+1}\{\alpha\operatorname{ch}(\alpha\Delta\eta)\sin\Delta\eta - \operatorname{sh}(\alpha\Delta\eta)\cos\Delta\eta\},$$

where

(2.3) $$g_1(z) = \operatorname{sh} z \operatorname{sh} \sigma z \sin\eta_2 + kz \operatorname{sh}(1-\sigma)z\sin\eta_1,$$

(2.4) $$g_2(z) = \operatorname{sh} z(\alpha \operatorname{ch}\sigma z \sin\eta_2 - \operatorname{sh}\sigma z\cos\eta_2) + kz\{\alpha \operatorname{ch}(1-\sigma)z\sin\eta_1 + \operatorname{sh}(1-\sigma)z\cos\eta_1\},$$

(2.5) $$H(z) = \operatorname{sh}^2 z - k^2z^2, \quad k = \sin(2\pi-\tau)/(2\pi-\tau),$$

and

$$\Delta\eta = \eta - \eta_2, \quad z = \alpha(2\pi-\tau), \quad \sigma = \eta_1/(2\pi-\tau).$$

This expression for $G(\alpha, \eta)$ is the basis of our analyses which follow. For sufficiently small values of $\Delta\eta$, we asymptotically have

(2.6) $$G(\alpha, \eta) = \alpha\left\{g_1(z) - \frac{1}{3}\Delta\eta g_2(z)\right\}(\Delta\eta)^2 + O(\Delta\eta^4).$$

Near $\xi = \infty$, the intersection line of two arcs, we can have the representation

(2.7) $$K'_\alpha(\operatorname{ch}\xi) = \frac{2i}{\sqrt{\pi}}\{C(\alpha, \xi) + C(-\alpha, \xi)\},$$

where

(2.8) $$C(\alpha, \xi) = C_0(\alpha, \xi)\operatorname{cth}\alpha\pi e^{(i\alpha-3/2)\xi},$$

(2.9) $$C_0(\alpha, \xi) = \frac{\Gamma(3/2-i\alpha)}{\Gamma(1-i\alpha)} F\left(\frac{3}{2}, \frac{3}{2}-i\alpha, 1-i\alpha, e^{-2\xi}\right),$$

Γ being a Gamma function and F a hypergeometric function. Using this expression for $K'_\alpha(s)$, we alternatively have

(2.10) $$\psi = 2\left(\frac{2}{\pi}\right)^{1/2} Ur^2(s-t)^{1/2} \int\limits_{-\infty}^{\infty} iC_0(\alpha, \xi)\frac{G(\alpha, \eta)}{H(z)} e^{(i\alpha-3/2)\xi} d\alpha.$$

The above integral may be evaluated by a contour integration in the upper half of the complex α-plane, where the respective roots μ_n and ν_n of $\operatorname{sh} z \pm kz = 0$ are all poles of the integrand, except that $\alpha = 0$ and $\alpha = i$.

In Ref. [8] the first five (in order of increasing imaginary part) of the eigenvalues have been evaluated for various k values, in which generally $\operatorname{Im}(\mu_n) < \operatorname{Im}(\nu_n)$. Referring to those data, we can see immediately that when $2\pi-\tau$ is less than about 146.3°, Moffatt's angle, solutions to $H(z) = 0$ all have a non-zero real part and therefore for such bodies of type II an infinite sequence of eddies must exist near the line of intersection of two arcs. When $2\pi-\tau$ is greater than the above critical value and $2\pi-\tau < \pi$, the bodies are still snowman-like and at least the first root μ_1 is purely imaginary. For $2\pi-\tau > \pi$, μ_1 and ν_1 are purely imaginary at least. They are surveyed in Fig. 1.

The separation point ξ_* of flow on the surface, if any, is found from $(\partial^2\psi/\partial\eta^2)_{\eta=\eta_2} = 0$, which yields the equation

(2.11) $$E(\xi) \equiv e_1(\xi)\sin\eta_2 + e_2(\xi)k\sin\eta_1 = 2\int\limits_0^{\infty} \frac{g_1(z)}{H(z)}\alpha \operatorname{th}\alpha\pi K'_\alpha(s)d\alpha = 0.$$

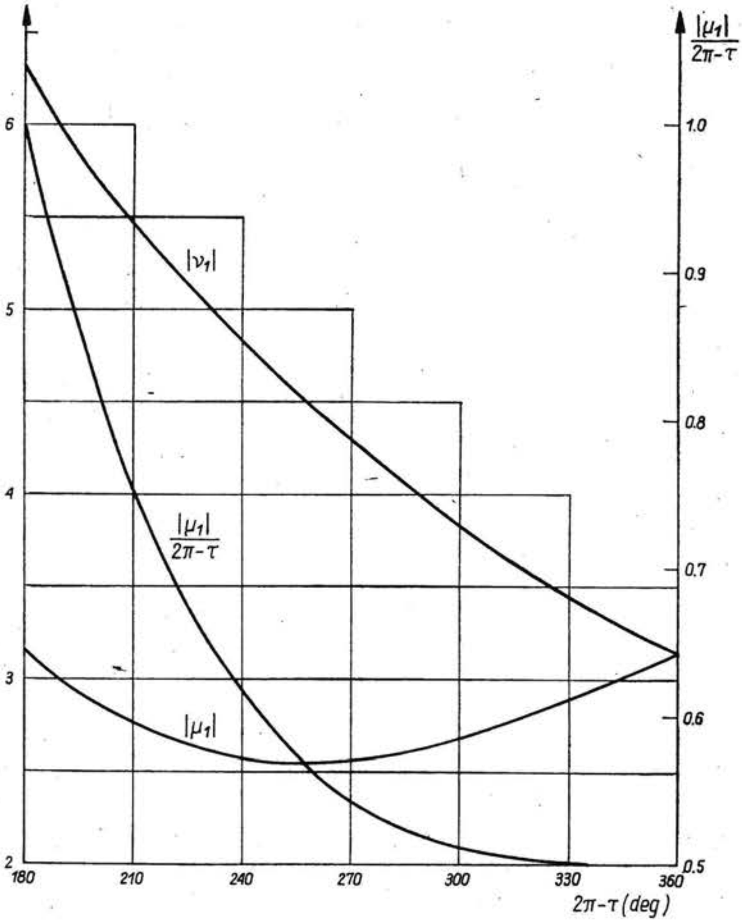

FIG. 1. The first roots μ_1 and ν_1 of $\text{sh}\,z \pm kz = 0$, which are purely imaginary when $2\pi - \tau > \pi$.

For symmetrical bodies with $\eta_1 + \eta_2 = 2\pi$ ($\eta_0 = \pi$), the general expressions are greatly simplified. In Eq. (2.1),

$$\text{(2.12)} \quad \frac{G(\alpha, \eta)}{H(z)} = -\frac{1}{H_1(\alpha)}\Big[\text{sh}(\alpha\Delta\eta)\sin\Delta\eta\,\text{sh}\,\alpha\eta_1\sin\eta_1 - \frac{1}{\alpha^2+1}\{\alpha\,\text{ch}(\alpha\Delta\eta)\sin\Delta\eta - \text{sh}(\alpha\Delta\eta)\cos\Delta\eta\}(\alpha\,\text{ch}\,\alpha\eta_1\sin\eta_1 + \text{sh}\,\alpha\eta_1\cos\eta_1)\Big],$$

where

$$H_1(\alpha) = \text{sh}(2\alpha\eta_1) + \alpha\sin 2\eta_1.$$

In case of contour integration, ν_n does not appear. Thus, for ξ large, $E(\xi)$ is approximated with a small error of order $\exp(-2\xi)$, as

$$\text{(2.13)} \quad E(\xi) \sim 4\sqrt{\pi}\,\frac{\sin\eta_1}{\eta_1^2}\,e^{-3\xi/2}\,\text{Re}\left[\frac{\Gamma(3/2 - i\mu_1/2\eta_1)}{\Gamma(1 - i\mu_1/2\eta_1)}\,\frac{\mu_1\text{sh}(\mu_1/2)}{\text{ch}\,\mu_1 + k}\,e^{i\mu_1\xi/2\eta_1}\right],$$

for μ_1 with a non-zero real part, where $E(\xi) = 0$ has an infinite number of roots.

3. Crescent-type bodies

A crescent-type body is named for a body of type I. In what follows the case $\pi > \eta_2 > \eta_1 > 0$ is considered for $2\pi > \eta > \eta_2$. On the x-axis the velocity of flow is

$$(3.1)\qquad v_x = -(v_\eta)_{\xi=0} = \left(\frac{1}{hr}\frac{\partial\psi}{\partial\xi}\right)_{\xi=0} = 2^{3/2}U(1-t)^{1/2}\int\limits_0^\infty \frac{\operatorname{th}\alpha\pi}{H(z)}G(\alpha,\eta)K'_\alpha(1)d\alpha.$$

For a small value of $\Delta\eta$, this equation tends to

$$(3.2)\qquad v_x \sim \sqrt{2}U(1-t)^{1/2}\{e_1(0)\sin\eta_2+e_2(0)k\sin\eta_1\}(\Delta\eta)^2.$$

Since $k < 0$ for bodies of type I, it is clear that a back current ($v_x < 0$) opposite to the direction of the main stream is possible to occur only for the crescent configuration where $\eta_2 < \pi$. When the back flow streams near the concave surface, a point η_* at which $v_x = 0$ must exist on the x-axis and a stream surface through this point separates the external flow from a sort of wake region within the concavity. Some values of η_* obtained from

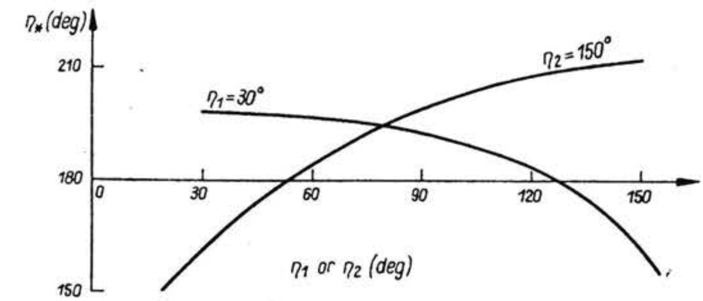

FIG. 2. The change of η_* vs. η_1 for $\eta_2 = 150°$ and vs. η_2 for $\eta_1 = 30°$.

Table 1. Separation stream lines for crescent-type bodies.

			$v_x(\eta_*) = 0$		$E(\xi_*) = 0$	
η_1(deg)	η_2(deg)	x/c	η_*(deg)	x/c	ξ_*	r/c
30	30	3.732	197.9	−0.157	∞	
30	60	1.732	196.8	−0.148		
30	90	1	193.1	−0.115	10.0	1
30	120	0.333	183.8	−0.033		
30	150	0.268	161.7	0.161	3.06	0.921
60	60		211.1	−0.278	∞	
60	150		185.1	−0.045	4.16	0.922
90	90		219.1	−0.355	∞	
90	120		213.5	−0.301		
90	150		199.7	−0.174	6.44	0.997
90	170	0.087	179.4	0.005	3.35	0.944
120	120		220.4	−0.368	∞	
120	150		208.0	−0.249	15.4	1
150	150		211.7	−0.284	∞	

Eq. (3.1) are exemplified in Table 1 and Fig. 2. Since η_* is unique for a given configuration, the fluid within the wake is in a ring motion as a whole.

Limit values of η_1 and η_2 for which the separation begins to appear is given from

$$E(0) \equiv e_1(0)\sin\eta_2 + e_2(0)k\sin\eta_1 = 0, \tag{3.3}$$

the next term of order $(\Delta\eta)^3$ in v_x then being positive. As $\eta_1 \to 0$,

$$E(0) \sim -\eta_1^2 \int_0^\infty \alpha^2\left(\alpha^2+\frac{1}{4}\right)\frac{\operatorname{th}\alpha\pi}{H(z)}(\operatorname{sh}z\cos\eta_2+\alpha\operatorname{ch}z\sin\eta_2)d\alpha, \tag{3.4}$$

so $\eta_2 = \eta_c > \pi/2$ in order that $E(0) = 0$ is satisfied for the plane with a spherical trough. The value of η_c is estimated to be about 140.3°. In conclusion we can state that for $\eta_2 > \pi$ no wake appears, for $\eta_2 < \eta_c$ the wake always exists in any value of η_1 ($< \eta_2$) and for $\pi > \eta_2 > \eta_c$ the existence of wake depends still on the value of η_1. Their functional relation is shown in Fig. 3 on the curve $E(0) = 0$.

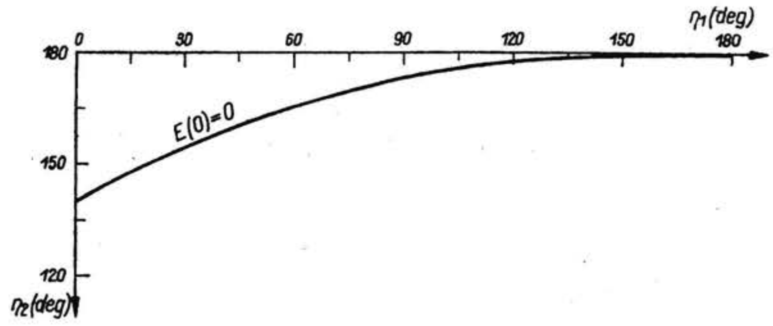

FIG. 3. The relation of η_1 and η_2 for which $E(0) = 0$.

For ξ large, by eigenvalue expansion we have

$$E(\xi) \sim \frac{4\sqrt{\pi}}{(2\pi-\tau)^2}e^{-3\xi/2}\Bigg[\frac{|\mu_1|}{\cos|\mu_1|+k}\{\sin\sigma|\mu_1|\sin\eta_2 - \sin(1-\sigma)|\mu_1|\sin\eta_1\}C_0\left(\frac{i|\mu_1|}{2\pi-\tau},\infty\right)e^{-\xi|\mu_1|/(2\pi-\tau)} + \frac{|\nu_1|}{\cos|\nu_1|-k} \times\{\sin\sigma|\nu_1|\sin\eta_2+\sin(1-\sigma)|\nu_1|\sin\eta_1\}C_0\left(\frac{i|\nu_1|}{2\pi-\tau},\infty\right)e^{-\xi|\nu_1|/(2\pi-\tau)}\Bigg]. \tag{3.5}$$

For the crescent configuration μ_1 and ν_1 are both purely imaginary and therefore $E(\xi) = 0$ has, if any, a unique solution. As $\xi \to \infty$, the first term, say E_1, dominates in Eq. (3.5). A careful examination reveals that $E_1 > 0$ always for bodies of type I. Hence, if the flow separates, the point ξ_* of separation is to be found on the surface η_2 (if $E_1 < 0$, we would have ξ_* at the rim). In most situations, however, numerical calculation yields ξ sufficiently arge (Table 1).

The spherical cap, $\eta_2 = \eta_1 = \eta_0$, is a limit of the crescent-type body and for this degenerated case all the integral expressions can be obtined in closed form. For instance, on the x-axis Eq. (3.1) becomes

$$(3.6)\quad v_x = \frac{\sqrt{2}}{4\pi} U(1-t)^{1/2}\Bigg[\frac{1}{\sin[(\varDelta\eta-\eta_0)/2]}\{(\varDelta\eta-\eta_0)-\sin(\varDelta\eta-\eta_0)\} \times\left(\frac{\sin^2[(\varDelta\eta+\eta_0)/2]}{\sin^2[(\varDelta\eta-\eta_0)/2]}-1\right)-\frac{2}{\sin[(\varDelta\eta-\eta_0)/2]}\{(\varDelta\eta-\eta_0)+\sin(\varDelta\eta-\eta_0)\} +\frac{2}{\sin[(\varDelta\eta+\eta_0)/2]}\{(\varDelta\eta+\eta_0)+\sin(\varDelta\eta+\eta_0)\}\Bigg].$$

Since $k = 0$, Eq. (3.3) gives $\eta_0 = \pi$ and for $\eta_0 < \pi$ Eq. (2.11) is only satisfied by $\xi_* = \infty$. Sufficiently near this point, the contour integration leads to

$$(3.7)\quad \psi \sim \frac{16}{\pi}\sqrt{2}Ur^2(s-t)^{1/2}e^{-2\xi}\sin^2(\eta_0/2)\sin^2(\varDelta\eta/2) \times\left\{\frac{1}{3}\sin(\eta_0/2)\sin(\varDelta\eta/2)-\cos(\eta_0/2)\cos(\varDelta\eta/2)\right\},$$

as shown by Dorrepaal et al.

4. Drag of the body

The force D acting on the body may be obtained from the formula due to Payne and Pell, though they did not show that in general form. Since $K'_\alpha(1) = -(\alpha^2+1/4)/2$, we have

$$(4.1)\qquad \frac{D}{8\pi\mu} = \lim_{\substack{\xi=0\\ \eta=2\pi}}\left(\frac{Ur^2}{2}-\psi\right)\frac{1}{r^2}\left(\frac{s+t}{s-t}\right)^{1/2} = \frac{cU}{2}\left\{\frac{1}{\sin(\eta_0/2)}+(F_1+F_2)\right\},$$

where

$$(4.2)\quad F_1 = \int_0^\infty \frac{\alpha^2+1/4}{\alpha^2+1}\big[\{\alpha^2\sin^2\eta_1(\mathrm{sh}[2(1-\sigma)z]+\alpha\sin 2\eta_2) +\alpha^2\sin^2\eta_2(\mathrm{sh}[2\sigma z]-\alpha\sin 2\eta_1)+\alpha(\mathrm{sh}^2(1-\sigma)z\sin 2\eta_1-\mathrm{sh}^2\sigma z\sin 2\eta_2) -2\,\mathrm{sh}\,z\,\mathrm{sh}\,\sigma z\,\mathrm{sh}(1-\sigma)z\}\mathrm{sh}\,\alpha\pi+H(z)\mathrm{ch}\,\alpha\pi\big]\frac{d\alpha}{H(z)\mathrm{ch}\,\alpha\pi},$$

$$(4.3)\quad F_2 = -\int_0^\infty \frac{\alpha^2+1/4}{\alpha^2+1}\{\alpha\,\mathrm{sh}[\alpha(\pi-\eta_0)]\sin\eta_0+\mathrm{ch}[\alpha(\pi-\eta_0)]\cos\eta_0\}\frac{d\alpha}{\mathrm{ch}\,\alpha\pi}.$$

The above is the result produced directly from Payne and Pell's solution. However, the integral for F_2 can be obtained in closed form by a contour integration in the upper half of the complex α-plane, in which poles of the integrand are $\alpha = i$ and $i(n+1/2)$, 1, 2,.... As a result,

$$(4.4)\qquad F_2 = \frac{3}{8}\pi - \frac{1}{\sin(\eta_0/2)},$$

then we may have

$$(4.5)\qquad D = 4\pi\mu c U\left(\frac{3}{8}\pi + F_1\right),$$

μ being the coefficient of viscosity.

Application to the special case of spherical cap will serve as a test of validity for the result. For the cap,

$$(4.6)\qquad F_\Gamma = \int_0^\infty \frac{\alpha^2+1/4}{\alpha^2+1}\left[\alpha\sin\eta_0\{\alpha\sin\eta_0\operatorname{ch}[2\alpha(\pi-\eta_0)] + \cos\eta_0\operatorname{sh}[2\alpha(\pi-\eta_0)]\} + \operatorname{ch}^2[\alpha(\pi-\eta_0)]\right]\frac{d\alpha}{\operatorname{ch}^2\alpha\pi}.$$

The integral can be obtained in closed form, giving

$$(4.7)\qquad \frac{D}{\mu U} = \frac{2c}{\sin\eta_0}\{(4-\cos\eta_0)\sin\eta_0 + 3(\pi-\eta_0)\}.$$

As $a = c/\sin\eta_0$, the radius of the base sphere, it is essentially the same as the drag form given by COLLINS [1], though Payne and Pell failed for a hemispherical cup.

Equation (4.5) is regarded as a generalization of the drag forms for two spheres which have been known already. For comparison we shall consider the simpler case of equal spheres. For this symmetrical body,

$$(4.8)\qquad F_1 = \int_0^\infty \frac{\alpha^2+1/4}{\alpha^2+1}\{(\alpha^2+1)\operatorname{sh}\alpha\pi + \operatorname{sh}[\alpha(2\eta_1-\pi)] - \alpha(\alpha\operatorname{sh}\alpha\pi\cos 2\eta_1 - \operatorname{ch}\alpha\pi\sin 2\eta_1)\}\frac{d\alpha}{H_1(\alpha)\operatorname{ch}\alpha\pi}.$$

In the particular case of $\eta_1 = \pi/2$, this becomes

$$(4.9)\qquad F_1 = \int_0^\infty \left(\alpha^2+\frac{1}{4}\right)\left(2-\frac{1}{\alpha^2+1}\right)\frac{d\alpha}{\operatorname{ch}\alpha\pi} = \frac{3}{2} - \frac{3}{8}\pi,$$

and for $\eta_1 = \pi$,

$$(4.10)\qquad F_1 = \int_0^\infty \frac{\alpha^2+1/4}{\alpha^2+1}\frac{d\alpha}{\operatorname{ch}^2\alpha\pi} = \frac{4}{\pi} - \frac{3}{8}\pi.$$

Clearly these results produce the drag for a sphere and for a disk, respectively. For $\eta_1 \to 0$, taking $\eta_1 \alpha = \beta$ and $c = a \sin\eta_1$, we have

$$\frac{D}{4\pi\mu a U} = \int_0^\infty \left(1 - 2\frac{\operatorname{sh}^2\beta - \beta^2}{\operatorname{sh}2\beta + 2\beta}\right) d\beta. \tag{4.11}$$

This is just the form given by FAXEN [7] for a doublet of equal spheres (with radius a) in contact. Sample cases are shown in Table 2.

Table 2. Drag coefficients for symmetrical bodies, $c = a \sin \eta_1$.

η_1(deg)	$D/(6\pi\mu Uc)$	$D/(6\pi\mu Ua)$	η_1(deg)	$D(6\pi\mu Uc)$	$D/(16\mu Uc)$
0	(due to Faxen)	1.290	90	1	1.178
15	4.956	1.283	105	0.9237	1.088
30	2.517	1.258	120	0.8810	1.038
45	1.723	1.218	135	0.8599	1.013
60	1.342	1.162	150	0.8514	1.003
75	1.128	1.089	165	0.8491	1.000
90	1	1	180	0.8488	1

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