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Experimental investigation of pressure loss in rotating curved rectangular channels

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In this article are presented the results of pressure loss measurements in a viscous, incompressible medium. The dependence of pressure loss on the channel curvature and rotational motion was investigated. Of particular interest was the influence of the cross-sectional shape of the channel and the influence of the rotational direction relative to the flow direction on the frictional resistance. The pressure loss, in the form of a dimensionless resistance coefficient, has been determined as a function of the Dean number, the Taylor number and the aspect ratio, respectively, whereby the Dean number has been varied in the region $40 < \mathrm{De} < 4 \cdot 10^4$ and the Taylor number in the region $0 \leqslant \mathrm{Ta} < 2 \cdot 10^4$. The aspect ratio b/h experienced a change of between $1/3 \leqslant b/h \leqslant 3/1$. An observably higher pressure loss appears in the experimental investigations than for the straight channel flow case. In non-rotating channels the measurements show an approximate independence of the cross-section geometry. However, for the case of rotation, for a Rossby number region of $\mathrm{Ro} < 0.1$, the influence of the aspect ratio is of crucial significance insofar as the pressure loss value is concerned.

W pracy przedstawiono wyniki pomiarów strat ciśnienia w lepkim ośrodku nieściśliwym. Rozpatrzono zależność strat ciśnienia od krzywizny kanału i jego ruchu obrotowego. Szczególnie zainteresowanie budzi wpływ kształtu przekroju poprzecznego kanału i kąta nachylenia jego obrotu względem kierunku przepływu na wartość oporów tarcia. Stratę ciśnienia wyrażoną w postaci bezwymiarowego współczynnika oporu określono jako funkcję liczby Deana, liczby Taylora oraz smukłości, przy czym liczbę Deana przyjmowano z zakresu $40 < \mathrm{De} < 4 \cdot 10^4$, a liczbę Taylora z zakresu $0 \leqslant \mathrm{Ta} < 2 \cdot 10^4$. Smukłość b/h zmieniała się w granicach od 1/3 do 3. W doświadczeniach stwierdza się wyraźne podwyższenie się strat ciśnienia w porównaniu z kanałami prostymi. W kanałach nie wirujących pomiary wskazują na praktyczną niezależność tych strat od geometrii przekroju poprzecznego. Jednak w przypadku rotacji i dla liczb Rosby'ego $\mathrm{Ro} < 0{,}1$, wpływ smukłości na wartość strat ciśnienia jest bardzo istotny.

В работе представлены результаты измерений потерь давления в вязконесжимаемой среде. Рассмотрена зависимость потерь давления от кривизны канала и его вращательного движения. Особенный интерес возбуждает влияние формы поперечного сечения канала и угла наклона его вращения по отношению к направлению течения на значени сопротивлений трения. Потеря давления, выраженная в виде безразмерного коэффициента сопротивления, определена как функция числа Дина, числа Тейлора и тонкостие причем число Дина принимается из интервала $40 < \mathrm{De} < 4 \cdot 10^4$, а число Тейлора из интервала $0 < \mathrm{Ta} < 2 \cdot 10^4$. Тонкость b/h изменялась в пределах от 1/3 до 3. В экспериментах констатировано отчетливое повышение потерь давления по сравнению с простыми каналами. В невращающихся каналах измерения указывают на практическую независимость этих потерь от геометрии поперечного сечения. Однако, в случае вращения и для чисел Росби $\mathrm{Ro} < 0{,}1$, влияние тонкости на значение потерь давления очень существенно.

Nomenclature

b width of the rectangular channel,

$c_m = \dfrac{\dot{V}}{b \cdot h}$ average velocity,

$d_h = 2\frac{b\cdot h}{b+h}$ hydraulic diameter,

h height,

$L = 2\pi\cdot r_m\cdot n$ length of the channel,

n number of turns,

r_m radius of curvature,

$\dot{V}$ volume flow rate,

Δp static pressure differential,

$\lambda_c = \frac{\Delta p}{\varrho/2\cdot c_m^2}\cdot\frac{d_h}{L}$ friction factor,

μ viscosity,

ϱ density,

ω angular velocity,

$\mathrm{De} = \mathrm{Re}\cdot\left(\frac{d_h}{2\cdot r_m}\right)^{0.5}$ Dean number,

$\mathrm{Re} = \frac{\varrho\cdot c_m\cdot d_h}{\mu}$ Reynolds number,

$\mathrm{Ro} = \frac{c_m}{\omega\cdot d_h}$ Rossby number,

$\mathrm{Ta} = \mathrm{Re}/\mathrm{Ro}$ Taylor number.

1. Introduction

In the literature there are already numerous articles, both experimental and theoretical, about the flow movements in curved channels. Dean [1] has developed a calculation method in the form of a linearization of the equations of motion for which only a slight channel curvature is assumed. Through experimental investigations White [2] confirmed the usefulness of Dean's resistance formula with the restriction that the product of the characteristic Reynolds number with the ratio between the diameter of the tube and the radius of curvature be small. Taylor [3] carried out stability investigations with the result that the influence of curvature delays the transition from laminar to turbulent flow until higher Reynolds numbers are reached and so exercises a stabilizing effect on the laminar flow.

We are indebted to the fundamental work of Adler [4] and Ito [5, 6] that there exist universal mathematical interrelationships for both laminar and turbulent pipe flows, which allow to make assertions about the size of the resistance coefficient for a given pipe geometry, material data and flow rate. In the publication [7, 8] the research was extended to channels with rectangular cross-sections. Here the influence of the aspect ratio on the pressure loss is determined by numerical approximation methods. The first investigations of the effect of rotational motion on the flow resistance in a curved channel with a square cross-section were carried out by Ludwieg [9]. With the assumption that the disturbance resulting from rotational motion is larger than that from the influence of the curvature, a calculation of the laminar flow in the manner of a boundary layer calculation followed. Experimental investigations of the influence of rotation on the resistance coefficient in a curved channel with a circular cross-section have been published by Euteneur and

PIESCHE [10]. The main feature of a flow through curved channel sections is occurrence of the centrifugal force, which causes secondary activity in the flow field. These secondary movements produce an amplification of the resultant wall-stress tension. As a direct consequence of this, there is a change in the pressure loss. If rotation of the channel is superimposed on the curvature, then Coriolis forces also appear. While the centrifugal forces retain their direction for a changing direction of rotation relative to the flow motion and, in the case of an incompressible medium, only have an effect upon pressure distribution, the Coriolis forces change direction constantly, leading to, in some cases, the production of complicated flow movements. Depending on the direction of rotation of the channel in relation to the flow direction, the Coriolis forces will have either a decelerating or an accelerating effect on the flow whereby a reaction on the flow resistance cannot be excluded. The aim of this report is to determine experimentally the pressure loss as a function of the curvature, aspect ratio, angular velocity and its direction in both a laminar and a turbulent flow field and, if possible, to present the results in a universal form. Since rotating channels can be used as heat exchangers in technical areas, it is of primary interest to produce a general statement about the pressure loss as a function of the flow rate and the channel geometry to be able to accomplish the lay-out of a supply system. Recent consideration [11] have produced the evidence that under the influence of rotation an improvement of the heat transfer can be achieved.

2. Experimental apparatus

The geometric model of the channel section is schematically drawn in Fig. 1. The pressure differential Δp along the curved channel section can be determined by measuring

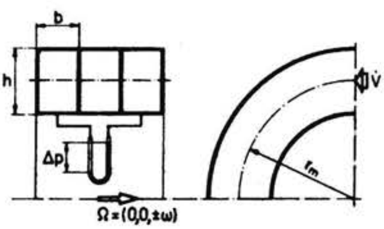

FIG. 1. Schematic diagram of channel.

the pressure at two points in the wall of the rotating channel. To accomplish this, two holes are bored into the inner channel wall and pressure lines are led from these through the rotating hollow shaft to the measuring instruments. The holes lie in a flow region in which it is assured that additional pressure loss as a result of the inlet flow into the curved channel can be excluded. A centrifugal pump forces the fluid through the rotating hollow shaft through an inlet in the channel section, which is set into rotation by means of a direct-current motor and a V-belt drive. The flow rate $\dot{V}$ is determined by a flow rate measuring section. For the experiments water is used exclusively. We use an optical tachometer to determine the angular velocity. The main apparatus unit is the rotating curved channel which essentially consists of a hollow aluminium cylinder into which a spiral contour with only a slight pitch is cut. A hollow plexiglas cylinder is precisely shrunk onto the channel and the connection is made water-proof with a rubber seal. The wetted surface can be considered to be hydraulically smooth. The Table 1 below gives an insight into the dimensions used.

Table 1.

b/h	1/3	1/2	2/3	1	1	3/2	3/1
b[mm]	9.1	9	18.2	18	9	27.3	18
h[mm]	27.3	18	27.3	18	9	18.2	6
r_m[mm]	81.35	86	81.35	86	90.5	85.9	92

3. Friction factors of the curved non-rotating channel

Figure 2 shows the results for the pressure loss as they appear in a laminar flow through a curved channel. The dimensionless friction factor ratio λ_c/λ_s is plotted against the Dean

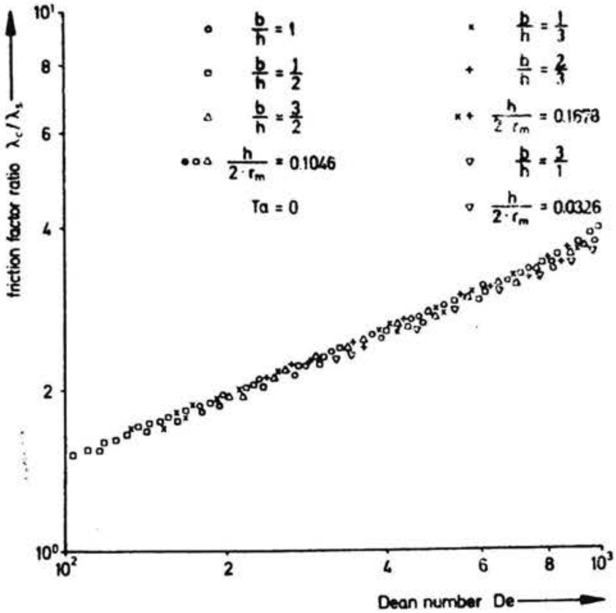

FIG. 2. Resistance curve for the laminar channel flow.

number De. The scale is a log-log scale; λ_s, which represets the resistance coefficient for the straight channel with a square cross-section, is defined as $\lambda_s = 57/\text{Re}$. The two curve parameters, the geometrical curvature ratio $h/(2\,r_m)$ and the aspect ratio b/h, are varied in the range $0.0326 \leqslant h/(2r_m)$ 0.1678 and $1/3 \leqslant b/h \leqslant 3/1$, respectively.

The results proved that there are only insignificant differences in the coefficient ratio λ_c/λ_s, although there appears to be a clear tendency that the aspect ratio b/h has smaller values for $b/h > 1$ than $b/h < 1$. The difference can be partially attributed to experimental errors so the experimental results do not lead to an unequivocal conclusion.

The flow resistance in a turbulent flow field can be sketched using a modified plot diagram. In Fig. 3 the pressure loss factor divided by the square root of $d_h/(2r_m)$ is plotted

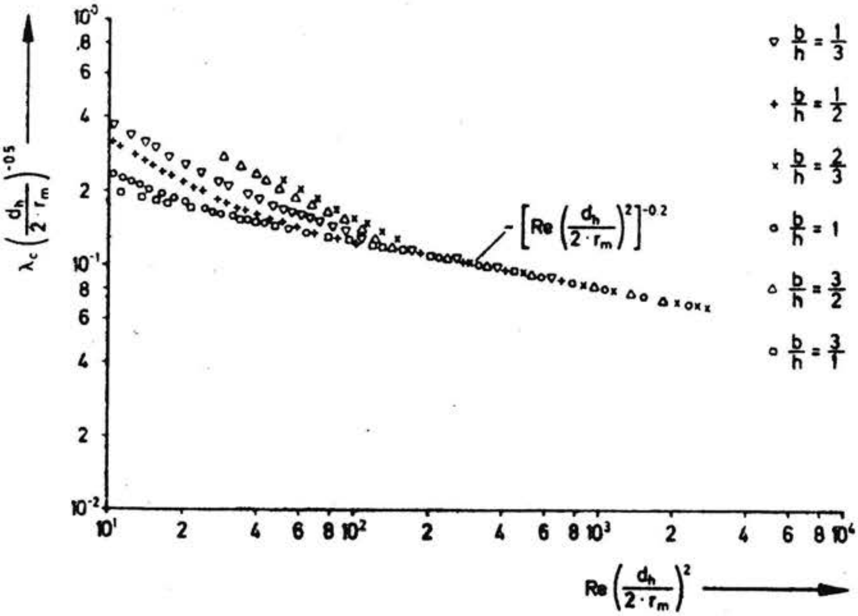

FIG. 3. Resistance curve for the turbulent channel flow.

against the product of the Reynolds number Re and the square of the hydraulic curvature ratio. The aspect ratio is also one of the curve parameters here. When it is thus represented, it is possible to find a universal curve behaviour. It is obvious that the ratio λ_c/λ_s is independent of the aspect ratio. This is also true for channel flows with straight walls. In Fig. 3 the laminar to turbulent transition point is identical with the point at which the curves for different aspect ratios unite into one. This system then describes a boundary curve which separates the laminar region from the turbulent one. Finally, it should be noted that the resistance coefficient can be described by the potential law

$$\lambda_c\left[\frac{d_h}{(2\cdot r_m)}\right]^{-0.5} \sim \left\{\mathrm{Re}\left[\frac{d_h}{(2\cdot r_m)}\right]^2\right\}^{-0.2}.$$

4. Friction factors of the rotating channel

The results for a rotating, curved channel with a square cross-section are presented in Fig. 4 for a Dean number range of $40 < \mathrm{De} < 2\cdot 10^3$. The curve parameters are the Taylor number and the direction of rotation relative to the motion of the fluid. In addition,

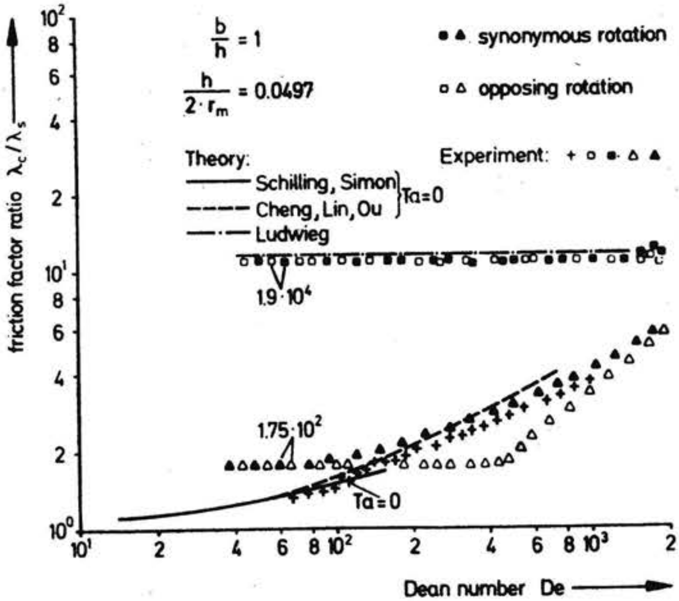

FIG. 4. Pressure loss in a rotating channel with $b/h = 1$.

the experimentally obtained resistance curve for the non-rotating curved channel is plotted in the diagram. These two experimental curves are compared with the theoretical results.

From the experimental results we can see that in this Dean number range a laminar motion dominates the flow field. It is interesting that, for example, for the curve path Ta = 175 for a large Dean number range the direction of rotational motion has a decisive

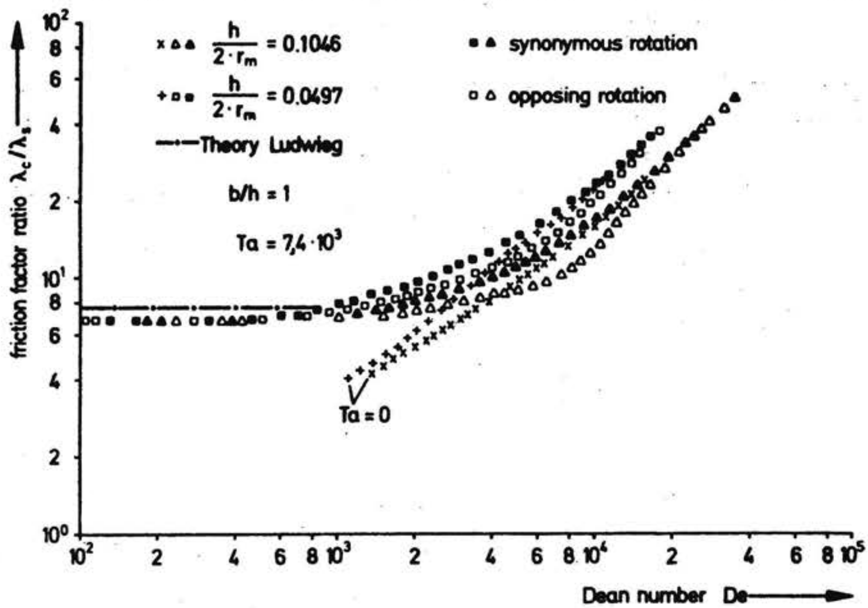

FIG. 5. The pressure loss in a Dean number range $10^2 < \mathrm{De} < 4\cdot10^4$ for $\mathrm{Ta} = 7.4\cdot10^3$.

influence on the size of the pressure loss; there are observable differences between the values when the flow and the rotation have the same direction and when they move in opposite directions. On the other hand, in the range De $< 10^2$ the pressure loss is independent of the direction of rotation. The length of this region, as the plotting of the experimental results for Ta $= 1.9 \cdot 10^4$ showed, is a function of the Taylor number. In addition, we can see that the ratio λ_c/λ_s is independent of the Dean number and from this we can deduce that λ_c/λ_s is also independent of the Reynolds number. Moreover, there is good agreement with the theoretical considerations whereby the result can be deduced, which claims that the curvature of the channel has no influence on the flow resistance at this Taylor number region.

Figure 5 produces the evidence that for an increasing Taylor number the ratio λ_c/λ_s is independent of the geometrical curvature ratio $h/(2r_m)$. This could mean that the disturbance to the secondary flow caused by the rotational motion is far greater than that caused by the curvature. Also there is obviously a stabilizing effect of rotation on the turbulence behaviour. For an increase of the flow rate — the geometry and material values are held constant — the influence of rotational motion disappears and the pressure loss corresponds to the λ-values of a non-rotating, curved channel. The investigations produce the result, in confirmation of the experiments of WHITE [2] and TAYLOR [3], that the curvature exerts a strong stabilizing influence on the flow and thereby leads to an increased critical Dean number. This result is analogous to that found for the rotating straight channel for which the rotation leads to an increase in the critical Reynolds number. The results indicate the width of the transition region. The transition from the laminar to the turbulent flow form occurs gradually within a given region which is, however, a function of the Taylor number. The physical meaning of the experimental results is

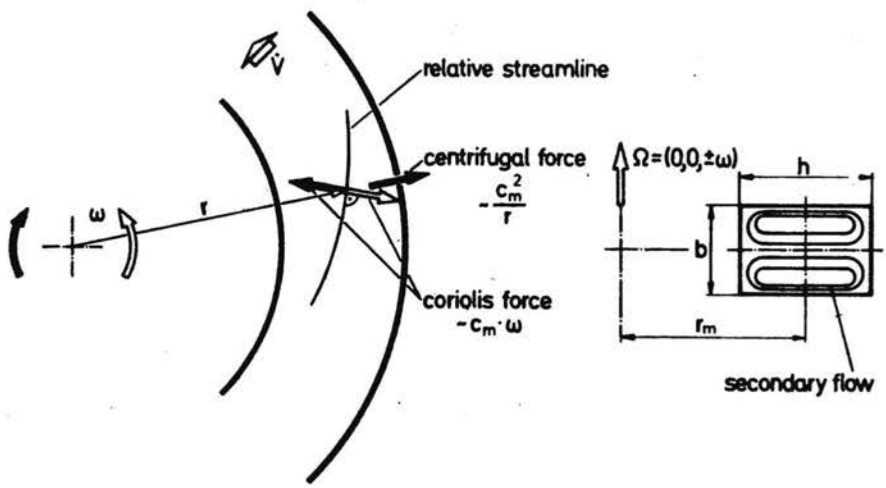

FIG. 6. Direction of the Coriolis forces.

explained with the help of the schematic diagram in Fig. 6. TAYLOR's investigations [3] have shown how strongly the secondary motion influences flow conditions. It then immediately follows that the secondary effects are either indirectly or directly the causes of the discovered increase in resistance. For a static curved channel two influences can essentially be distinguished. One of them is that the secondary flow is continuously carrying flow particles with a higher velocity out of the middle stream to the channel walls, where they decelerate thereby losing an appreciable amount of their kinetic energy as thermal energy. Continuity dictates that lower velocity fluid flows back into the middle of the channel and will there once again accelerate at the cost of the static pressure. The second influence depends on the fact that secondary flow, like every other flow, is subject to friction which also leads to loss of pressure energy and therefore results in an increase of the flow resistance. If, in addition, one observes the effects from the rotational motion, then not only does the centrifugal force resulting from the channel curvature have an effect, but there is also a Coriolis force acting perpendicular to the relative motion. The energy balance must then be completed by the force from the flow subject to friction. The sum of these forces are in equilibrium with the resulting pressure force. Near the wall this equilibrium of forces is disturbed and so the flow in the wall zones moves towards areas of lower pressure. For the case of the rotation being in the same direction as the main flow movement, the force fraction originating from the Coriolis force intensifies the secondary motion and so the pressure loss is insreased. For larger angular velocities and a smaller flow rate the centrifugal force is small relative to the Coriolis force. In this case the secondary motion changes direction and will be re-directed towards the axis of rotation [12]; from this point it is independent of the rotational direction. If the centrifugal force predominates over the Coriolis force, then the pressure loss is no longer dependent on the rotation. This behaviour is characterized by the Rossby number which reflects the relationship between the mass forces and the Coriolis forces (Fig. 7).

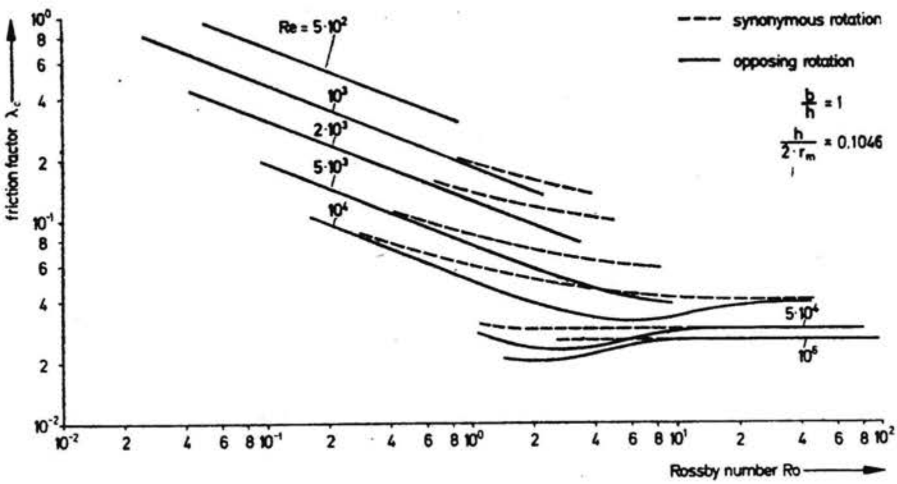

FIG. 7. Resistance coefficient as a function of the mass forces and Coriolis forces.

By specifying the order of magnitude of this characteristic factor the region can be separated into one in which the influence of the direction of rotation disappears and one in which an influence is present. Figure 7 allows us to recognize that in spite of the considerable Re value, e.g. Re $\sim 10^6$ and the certainly high turbulence intensity, regions can be defined in which the Coriolis force, influenced by the rotational direction, causes changes in the resistance coefficient.

Figure 8 shows the influence of the aspect ratio for $Ta = 1.9 \cdot 10^4$. In this Taylor number region the resistance ratio, as the experimental investigations have revealed, is

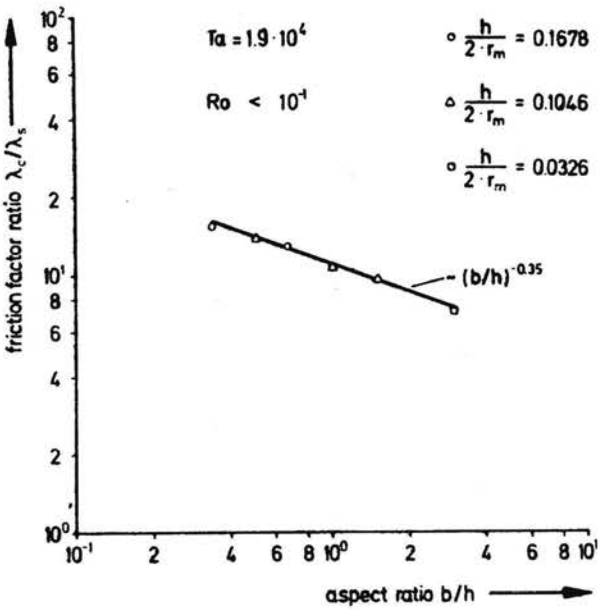

FIG. 8. The relative resistance coefficient as a function of the aspect ratio.

independent of the curvature ratio $h/(2r_m)$. In addition, a Rossby number region has been chosen in which no influence of the direction of rotating is apparent. The resistance measurements reveal that as the b/h value increases, the ratio λ_c/λ_s diminishes and is proportional to $(b/h)^{-0.35}$. Nothing more can be said within the scope of this report about the further development of this curve for different values of the aspect ratio.

5. Conclusions

For the stationary curved channel a universal pressure loss statement was achieved for both the laminar and the turbulent flow regions whereby, in a somewhat engineer — like way of thinking, the independence of the aspect ratio was determinied. In a flow field with rotating channel sections the relationships are more complicated. However, through an estimation of the order of magnitude of the Rossby number, regions can be separated, thereby restricting the variety of parameters. In the region $Ro < 0.1$ and for large Taylor numbers the pressure loss is only a function of the aspect ratio and the Taylor number and also independent of the curvature ratio $h/(2r_m)$ with the restriction that the investigations only considered $h/(2r_m)$ values which were less than 0.1678. In the region $Ro \sim 1$ the direction of rotation makes itself noticeable. For a rotational direction opposite to that of the flow direction there is a smaller pressure loss factor compared to a rotational motion in the same direction as the flow. For the case Ro > 20 the pressure loss is independent of the Taylor number and identical with that of the stationary curved channel.

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