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Unsteady one-dimensional extensions and small amplitude longitudinal waves in simple fluids

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IT IS SHOWN that certain unsteady one-dimensional extensions of simple fluids, under the assumption of small amounts of extension, can be treated as particular cases of the motions with proportional stretch history (cf. [4]). Solutions of the governing equations are obtained for the case of harmonic oscillations, leading to one-dimensional longitudinal waves standing or propagating in a compressible viscoelastic fluid. Various properties of such waves, e.g. the damping coefficients, the phase shifts, the maximum amounts of extension, the speeds of propagation etc., are discussed in greater detail for very low and very high (ultrasonic) frequencies. It is shown, among other things, that damping effects in the fluids considered are always weaker than those in purely viscous fluids with the same viscosities at zero frequency. Viscoelastic fluids subjected to high frequency disturbances may be more or less deformable than purely viscous fluids, depending on their limit behaviour at short times.

Pokazano, że pewne nieustalone jednowymiarowe przepływy rozciągające w cieczach prostych, w założeniu małych amplitud rozciągania, można traktować jako szczególne przypadki ruchów z proporcjonalną historią deformacji (por. [4]). Rozwiązania odpowiednich równań, określające jednowymiarowe fale podłużne stojące lub propagujące się w ściśliwej cieczy lepkosprężystej, otrzymano dla przypadku harmonicznych oscylacji. Bardziej szczegółowo przedyskutowano różne własności takich fal, jak np. współczynniki tłumienia, przesunięcia fazowe, maksymalne amplitudy rozciągania, prędkości propagacji itp., zarówno dla bardzo niskich jak i bardzo wysokich (naddźwiękowych) częstości. Wykazano, między innymi, że efekty tłumienia w rozważanych cieczach są zawsze słabsze niż w czysto lepkich cieczach o takich samych lepkościach przy zerowej częstości. Ciecze lepkosprężyste poddane zaburzeniom o wysokich częstościach mogą deformować się mniej lub bardziej niż ciecze czysto lepkie, w zależności od ich zachowania przy krótkich czasach.

Показано, что некоторые неустановившиеся одномерные растягивающие течения в простых жидкостях, в предположении малых амплитуд растяжения, можно трактовать как частные случаи движений с пропорциональной историей деформации (ср. [4]). Решения соответствующих уравнений, определяющие одномерные продольные стоячие или распространяющиеся волны в сжимаемой вязкоупругой жидкости, получены для случая гармонических осцилляций. Более подробно обсуждены разные свойства таких волн, как напр. коэффициенты затухания, фазовые сдвиги, максимальные амплитуды растяжения, скорости распространения и т.п., так для очень низких, как и очень высоких (сверхзвуковых) частот. Показано, между прочим, что эффекты затухания в рассматриваемых жидкостях всегда более слабые, чем в чисто вязких жидкостях с такими самыми вязкостями при нулевой частоте. Вязкоупругие жидкости, подвергнутые возмущениям с высокими частотами, могут деформироваться менее или более чем чисто вязкие жидкости, в зависимости от их поведения при коротких отрезках времени.

1. Introduction

IN OUR PREVIOUS papers [1, 2, 3] certain examples of unsteady shearing flows of incompressible simple fluids were analysed in greater detail. It has been shown, among other things, that various oscillatory shearing flows lead, under the assumption of harmonic time-dependence, to finite amplitude plane shear waves with linear, circular or elliptical polarization [2, 3]. All the flows considered belonged to particular classes of the motions with proportional or superposed proportional stretch histories discussed elsewhere [4].

In the present paper we consider the case of unsteady one-dimensional extensions and one-dimensional longitudinal waves in compressible simple fluids (cf. [5]). Instead of introducing a priori any material restrictions, we assume that the amounts of extension involved are sufficiently small (small-amplitude waves) to linearize the corresponding governing equations. Various properties of such waves, e.g. the damping effects, the phase shifts, the maximum amounts of extension, the speeds of propagation etc., are discussed in the full range of frequencies, that is from zero to infinity. To this end two types of limit behaviour at short times and very long times have been introduced (cf. [6]).

2. Unsteady one-dimensional extensions

Consider the following motions:

$$x = X+\varphi(X)f(\tau), \quad y = Y, \quad z = Z, \tag{2.1}$$

where x, y, z denote Cartesian coordinates of a particle at an arbitrary time τ, X, Y, Z—Cartesian coordinates of the same particle in a reference configuration, φ is a function of X only, and f is a smooth function of time. If the motions considered are harmonic oscillations, f can be taken in the form

$$f(\tau) = \exp(i\omega\tau), \tag{2.2}$$

where ω is a constant angular frequency.

Assuming that the amounts of extension φ' and their derivatives are small enough to disregard terms of order φ'^2 and higher as compared with those of order φ', the deformation gradient at time τ with respect to a reference configuration can be written as

$$\frac{\partial \mathbf{x}}{\partial \mathbf{X}} \equiv \mathbf{F}(\tau) = 1+\mathbf{M}f(\tau) \simeq \exp\big(\mathbf{M}f(\tau)\big), \tag{2.3}$$

where

$$[\mathbf{M}] = \begin{bmatrix} \varphi' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [\mathbf{M}^2] = [\mathbf{O}(\varphi'^2)], \quad \operatorname{tr}\mathbf{M} = \varphi' \tag{2.4}$$

and primes denote the derivatives with respect to X. For the assumed degree of accuracy the history of the right Cauchy-Green relative deformation tensor can be expressed in the form (cf. [5])

$$\mathbf{C}(s) = \exp\big(2\bar{g}(s)\mathbf{M}\big) = \exp\big(2g(s)\mathbf{L}\big), \quad s \in [0, \infty), \tag{2.5}$$

where

$$\bar{g}(s) = f(t-s)-f(t), \quad g(s) = \bar{g}(s)/\dot{f}(t), \quad \mathbf{L} = \mathbf{M}\dot{f}(t) \tag{2.6}$$

and t refers to present instant of time.

Under the assumption of small amounts of extension, the flow considered is a particular case of the motion with proportional stretch history [4]. Thus the constitutive equation of a simple fluid (cf. [5])

$$\mathbf{T}(t) = \underset{s=0}{\overset{\infty}{\mathfrak{F}}} \big(\mathbf{C}(s); \varrho(t)\big), \tag{2.7}$$

where $\mathbf{T}(t)$ is the stress tensor at time t, $\mathfrak{F}$ denotes an isotropic functional of $\mathbf{C}(s)$ being also a function of the time-dependent density $\varrho(t)$, can be written in the form

$$\mathbf{T}(t) = \underset{s=0}{\overset{\infty}{\mathfrak{G}}} \big(\bar{g}(s); \mathbf{M}, \varrho(t)\big) = \underset{s=0}{\overset{\infty}{\mathfrak{H}}} \big(g(s); \mathbf{L}, \varrho(t)\big), \tag{2.8}$$

where $\mathfrak{G}$ and $\mathfrak{H}$ denote functionals of scalars $\bar{g}(s)$ or $g(s)$, being simultaneously isotropic functions of the tensor arguments $\mathbf{M}$ or $\mathbf{L}$, respectively.

Retaining terms of order φ' only, we arrive at the following representation:

$$\mathbf{T}(t) = \underset{s=0}{\overset{\infty}{\mathfrak{a}_0}} \big(\bar{g}(s); \operatorname{tr}\mathbf{M}, \varrho(t)\big)\mathbf{1} + \underset{s=0}{\overset{\infty}{\mathfrak{a}_1}} \big(\bar{g}(s); \varrho(t)\big)\mathbf{M}, \tag{2.9}$$

where the material functional $\mathfrak{a}_0$ is a linear function of tr $\mathbf{M}$.

On the other hand, bearing in mind the representation theorem proved in [4], we can also use the following equation:

$$\mathbf{T}(t) = \underset{s=0}{\overset{\infty}{\mathfrak{J}}} \big(g(s); \mathbf{A}_1(t), \mathbf{A}_2(t), \varrho(t)\big), \tag{2.10}$$

where the Rivlin-Ericksen kinematic tensors are defined as follows (cf. [5]):

$$\mathbf{A}_n(t) = (-1)^n \left.\frac{d^n\mathbf{C}(s)}{ds^n}\right|_{s=0}, \quad n = 1, 2. \tag{2.11}$$

Under the assumed order of approximation, Eq. (2.10) leads to

$$\mathbf{T}(t) = (-p + \lambda \operatorname{tr}\mathbf{A}_1)\mathbf{1} + \eta\mathbf{A}_1 + \alpha_2\mathbf{A}_2 + \mathbf{O}(\varphi'^2), \tag{2.12}$$

where p denotes a thermodynamic pressure, λ, η and α_2 are the material functionals of $g(s)$ depending also on $\varrho(t)$.

Since for the flow considered

$$[\mathbf{A}_1] = \begin{bmatrix} 2\varphi' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{f}, \quad [\mathbf{A}_2] = \begin{bmatrix} 2\varphi' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ddot{f} + [\mathbf{O}(\varphi'^2)], \tag{2.13}$$

the continuity equation gives

$$\varrho(t) = \varrho_0 \exp(-\varphi' f) = \varrho_0 \big(1 - \varphi' f + O(\varphi'^2)\big), \tag{2.14}$$

where ϱ_0 denotes the density at reference configuration. Therefore, it can be deduced that, within the assumed order of approximation, λ, η and α_2 are functionals of $g(s)$ depending on a constant parameter ϱ_0.

According to Eq. (2.12), the non-zero stress components

$$\begin{aligned} T^{11} &= -p + 2\lambda\varphi'\dot{f} + 2\eta\varphi'\dot{f} + 2\alpha_2\varphi'\ddot{f}, \\ T^{22} &= T^{33} = -p + 2\lambda\varphi'\dot{f}, \end{aligned} \tag{2.15}$$

substituted into the dynamic equations of motion lead to

$$(2.16)\qquad \frac{\partial}{\partial X}(-p+2\lambda\varphi'\dot{f}+2\eta\varphi'\dot{f}+2\alpha_2\varphi'\ddot{f})-\varrho_0\varphi\ddot{f}=0.$$

If pressure p is a barotropic one, i.e. $p = p(\varrho)$, we have moreover,

$$(2.17)\qquad \frac{\partial p}{\partial X}=\frac{\partial p}{\partial \varrho}\frac{\partial \varrho}{\partial X}=-\varrho_0\varphi''f\frac{\partial p}{\partial \varrho},\quad \frac{\partial p}{\partial \varrho}>0$$

and Eq. (2.16) takes the form linear with respect to φ:

$$(2.18)\qquad \left(\varrho_0 f\frac{\partial p}{\partial \varrho}+2\mu\dot{f}\right)\varphi''-\varrho_0\ddot{f}\varphi=0,$$

where

$$(2.19)\qquad \mu=\lambda+\eta+\alpha_2\frac{\ddot{f}}{\dot{f}}.$$

Equation (2.18) being the governing equation for the flow considered can, in principle, be solved for known functions $p(\varrho)$, $\mu(t, \varrho_0)$, $\dot{f}(t)$, $\ddot{f}(t)$ and appropriate initial and boundary conditions.

3. One-dimensional longitudinal waves

If function $f(\tau)$ is of the harmonic form (2.2), then

$$(3.1)\qquad g(s)=-\frac{i}{\omega}(e^{-i\omega s}-1),\quad \dot{f}=i\omega\exp i\omega t$$

and the material functionals λ, η and α_2 (cf. (2.12)) become functions of angular frequency ω. The governing equation (2.18) takes the form

$$(3.2)\qquad \left(\varrho_0\frac{\partial p}{\partial \varrho}+2i\omega\mu^*(\omega)\right)\varphi''+\varrho_0\omega^2\varphi=0,$$

where $\mu^*(\omega)$ may be considered as the generalized complex viscosity function. This differential equation can be solved effectively only if $\partial p/\partial\varrho$ does not depend on time. This is the case of isothermal processes for which $p = C\varrho$, C = const. For adiabatic processes $p = C\varrho^k$, C = const, k = const and Eq. (2.14) implies that

$$(3.3)\quad \varrho_0\frac{\partial p}{\partial \varrho}=\varrho_0 kC\varrho^{k-1}=\varrho_0^k kC\exp\big(-(k-1)\varphi'f\big)=kC\varrho_0^k\big(1-(k-1)\varphi'f+O(\varphi'^2)\big).$$

Thus, in both cases the term $\partial p/\partial\varrho$ present in the linearized equation (3.2) can be treated as independent of time.

A general solution of Eq. (3.2) can be written as

$$(3.4)\qquad \varphi(X)=A\exp(\beta+i\gamma)x+B\exp(-\beta-i\gamma)X,$$

where A and B are integration constants, and

$$(3.5)\qquad (\beta+i\gamma)^2 = \frac{i\varrho_0\omega}{2\mu^*(\omega)-i\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial\varrho}} = \frac{-\varrho_0\omega\left(2\mu''+\dfrac{\varrho_0}{\omega^2}\dfrac{\partial p}{\partial\varrho}\right)+2i\varrho_0\omega\mu'}{4\mu'^2+\left(2\mu''+\dfrac{\varrho_0}{\omega^2}\dfrac{\partial p}{\partial\varrho}\right)^2},$$

where $\mu^* = \mu'-i\mu''$.

Any solution of the form (3.4) describes small-amplitude one-dimensional longitudinal waves, standing or propagating along the X-axis with the phase velocity $c(\omega) = \omega/\gamma$. The coefficient of damping (attenuation) β and the phase shift (wave number) γ can be presented as follows:

$$(3.6)\qquad \beta^2 = \frac{\varrho_0\omega}{4\mu'(\omega)}\left[\frac{1}{\sqrt{1+\xi^2}}-\frac{\xi}{1+\xi^2}\right] = \frac{\varrho_0\omega}{4\left(\mu''(\omega)+\dfrac{1}{2}\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial\varrho}\right)}\left[\frac{\xi}{\sqrt{1+\xi^2}}-\frac{\xi^2}{1+\xi^2}\right],$$

$$(3.7)\qquad \gamma^2 = \frac{\varrho_0\omega}{4\mu'(\omega)}\left[\frac{1}{\sqrt{1+\xi^2}}+\frac{\xi}{1+\xi^2}\right] = \frac{\varrho_0\omega}{4\left(\mu''(\omega)+\dfrac{1}{2}\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial\varrho}\right)}\left[\frac{\xi}{\sqrt{1+\xi^2}}+\frac{\xi^2}{1+\xi^2}\right],$$

where the second forms are valid only for $\xi \neq 0$, and

$$(3.8)\qquad \xi(\omega) = \frac{\mu''(\omega)+\dfrac{1}{2}\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial\varrho}}{\mu'(\omega)} = \frac{H'(\omega)+\dfrac{1}{2}\varrho_0\dfrac{\partial p}{\partial\varrho}}{H''(\omega)}.$$

The quantities

$$(3.9)\qquad H'(\omega) = \omega\mu''(\omega),\quad H''(\omega) = \omega\mu'(\omega)$$

can be considered as the generalized dynamic moduli.

For the majority of viscoelastic fluids (polymer solutions and melts) it is reasonable to assume that

$$(3.10)\qquad \lim_{\omega\to 0}\xi(\omega) = \infty,\quad \lim_{\omega\to\infty}\xi(\omega) = \infty$$

and there exists a certain frequency at which $\xi(\omega)$ takes a positive minimum value. For very low frequencies or very long times the fluid considered behaves like a purely viscous compressible liquid while for very high frequencies or very short times its behaviour is almost purely elastic.

Thus, Eqs. (3.6) and (3.7) imply that

$$(3.11)\qquad \lim_{\omega\to 0}\beta^2 = 0,\quad \lim_{\omega\to 0}\gamma^2 = 0,$$

while for ω tending to infinity we can distinguish the following types of limit behaviour:

a) the Kelvin-like behaviour at short times when the corresponding instantaneous modulus $H(0)$ is infinite and

$$(3.12)\qquad \lim_{\omega\to\infty}\frac{\omega^2}{H'(\omega)+\dfrac{1}{2}\varrho_0\dfrac{\partial p}{\partial\varrho}} = \text{const},$$

then

$$\lim_{\omega\to\infty}\beta^2 = 0, \quad \lim_{\omega\to\infty}\gamma^2 = \lim_{\omega\to\infty}\frac{\varrho_0\omega}{2\mu''(\omega)+\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial \varrho}} = \text{const}; \tag{3.13}$$

b) the Maxwell-like behaviour at short times when the corresponding instantaneous modulus $H(0)$ is finite and

$$\lim_{\omega\to\infty}\left(H'(\omega)+\frac{1}{2}\varrho_0\frac{\partial p}{\partial \varrho}\right) = \text{const}, \tag{3.14}$$

then

$$\lim_{\omega\to\infty}\beta^2 = \text{const or } \infty, \quad \lim_{\omega\to\infty}\gamma^2 = \lim_{\omega\to\infty}\frac{\varrho_0\omega}{2\mu''(\omega)+\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial \varrho}} = \infty. \tag{3.15}$$

The limit values of β^2 may be finite or infinite depending on the rate at which $\xi(\omega)$ tends to infinity for increasing ω.

In both cases the phase shifts γ^2 increase monotonically with ω, while β^2 may reach maximum values for $\xi = 1/\sqrt{3}$. It results from Eq. (3.6) that

$$\beta^2_{\max} = \frac{\varrho\omega_{cr}}{16\left(\mu''(\omega_{cr})+\dfrac{1}{2}\dfrac{\varrho_0}{\omega_{cr}}\dfrac{\partial p}{\partial \varrho}\right)}, \tag{3.16}$$

where the critical frequency ω_{cr} is a solution of the following equation:

$$\mu'(\omega_{cr}) = \sqrt{3}\left(\mu''(\omega_{cr})+\frac{1}{2}\frac{\varrho_0}{\omega_{cr}}\frac{\partial p}{\partial \varrho}\right). \tag{3.17}$$

Existence of a critical frequency or a critical time in various polymer systems may be attributed to a passage from purely liquid states to highly elastic states (cf. [7]).

A diagram illustrating variability of β^2 and γ^2 is schematically shown in Fig. 1. It must be noticed, however, that the scale of abcissa refers to $\xi(\omega)$; the corresponding values

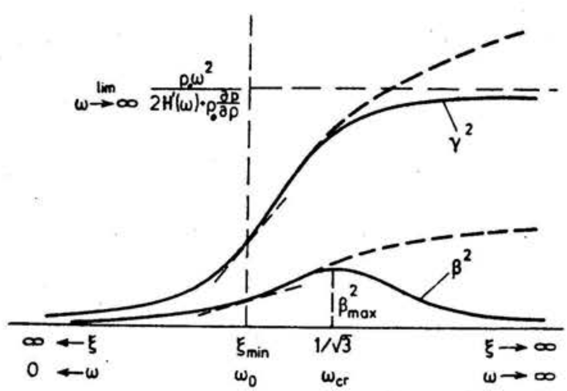

FIG. 1.

of ω are written for comparisons. Since for purely viscous compressible fluids ($\alpha_2 \equiv 0$) we have

$$\beta^2 = \gamma^2 = \frac{\varrho_0 \omega}{4(\lambda+\eta)}, \tag{3.18}$$

where $\lambda+\eta$ is a constant viscosity coefficient, the results (3.13) and (3.15) mean that damping effects in viscoelastic fluids are always weaker as compared with those in viscous fluids for which β^2 increases proportionally to ω.

So far, we tacitly assumed that $\xi(\omega)$ was never zero in the full range of frequencies. It may happen, however, that for certain viscoelastic fluids (polymer systems) the equation

$$\mu''(\omega_0)+\frac{1}{2}\frac{\varrho_0}{\omega_0}\frac{\partial p}{\partial \varrho} = 0 \tag{3.19}$$

has one or more positive roots ω_0. For such a root $\xi(\omega_0) = 0$, and

$$\beta_0^2 = \gamma_0^2 = \frac{\varrho_0 \omega_0}{4\mu'(\omega_0)}. \tag{3.20}$$

The above result shows that the corresponding curves of β^2 and γ^2 are mutually crossed. For two or more positive roots the curves of β^2 and γ^2 may cross twice or more times at the points where $\xi(\omega_0)$ are equal to zero. In the case of two roots, for instance, values of ξ are negative in the interval between the roots, and some local extrema may occur for $\xi = \pm 1/\sqrt{3}$. Existence of a frequency ω_0 at which $\xi(\omega_0) = 0$ may be useful to describe an internal structure breakage observed in certain polymer systems.

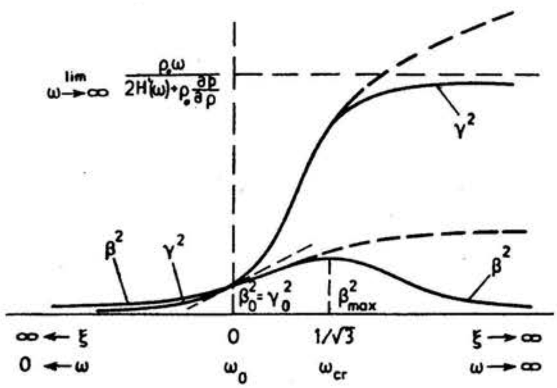

FIG. 3.

Diagrams illustrating the above discussed variability of β^2 and γ^2 are schematically shown in Figs. 2 and 3. It is worthwhile to note that similar pictures are observed for some fluid models with a number of discrete relaxation times (cf. [8]).

In general, the curves presented in Figs. 1, 2 and 3 resemble those usually obtained for mechanical impedances resulting from acoustic measurements (cf. [8]). The relations connecting γ^2 and β^2 with the active R_m and the passive X_m part of the mechanical impedance are as follows:

$$R_m^2 = (\mu'^2+\mu''^2)\gamma^2, \quad X_m^2 = (\mu'^2+\mu''^2)\beta^2. \tag{3.21}$$

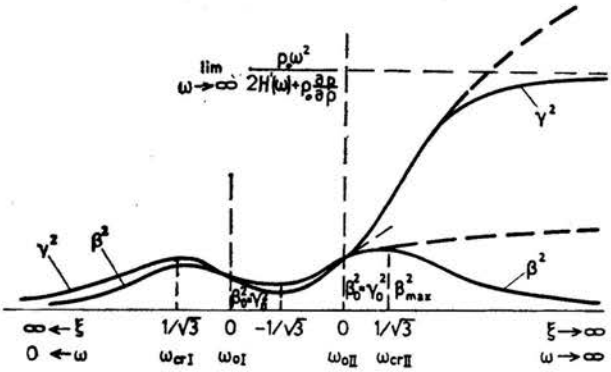

FIG. 2.

4. Other properties of longitudinal waves

When the waves considered are caused by harmonic disturbances applied at time $t = 0$ to the yz-plane ($x = 0$), the amount of extension or compression φ' essentially depends on angular frequencies ω. Since φ' is a complex function of X, we obtain from Eq. (3.4) the following maximum value of extension:

$$q^2_{\max} = |\varphi'|^2 = C(\beta^2+\gamma^2), \tag{4.1}$$

where C is composed of integration constants. Substituting Eqs. (3.6) and (3.7) into Eq. (4.1), we arrive at

$$q^2_{\max} = \frac{1}{2}C\frac{\varrho_0\omega}{\mu'(\omega)\sqrt{1+\xi^2}} = \frac{1}{2}C\frac{\varrho_0\omega}{\left(\mu''(\omega)+\dfrac{1}{2}\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial\varrho}\right)}\sqrt{1+\frac{1}{\xi^2}}. \tag{4.2}$$

Taking into account Eq. (3.8) and passing to the corresponding limits, we have

$$\lim_{\omega\to 0} q^2_{\max} = 0, \quad \lim_{\omega\to\infty} q^2_{\max} = \frac{1}{2}C\lim_{\omega\to\infty}\frac{\varrho_0\omega}{\mu''(\omega)+\dfrac{1}{2}\dfrac{\varrho_0}{\omega}\dfrac{\partial p}{\partial\varrho}}. \tag{4.3}$$

According to Eqs. (3.13) and (3.15) the last limit is finite only in the case a), i.e. for the Kelvin-like behaviour at short times. In the case b), i.e. for the Maxwell-like behaviour at short times, $q^2_{\max}$ tends to infinity almost proportionally to ω^2. In the case a) $q^2_{\max}$ for very high frequencies is less and in the case b) greater than the maximum amount of extension in purely viscous fluids, viz.

$$q^2_{\max} = \frac{1}{2}C\frac{\varrho_0\omega}{\lambda+\eta}, \tag{4.4}$$

where $\lambda+\eta$ is a constant viscosity.

The speed of propagation for one-dimensional longitudinal waves in a viscoelastic fluid which had been at rest in a fixed reference configuration can be derived from the following relation: [9]:

$$U_{||}^2 = c_\infty^2 = \lim_{\omega\to\infty} c^2(\omega) = \lim_{\omega\to\infty} \frac{\omega^2}{\gamma^2(\omega)}, \tag{4.5}$$

where c_∞ denotes the ultrasonic velocity. Taking into account Eq. (3.7), we arrive at

$$U_{||}^2 = c_\infty^2 = \frac{1}{\varrho_0} \lim_{\omega\to\infty} \left(2\omega\mu''(\omega) + \varrho_0 \frac{\partial p}{\partial \varrho}\right). \tag{4.6}$$

If $H(0) = H'(\infty)$ denotes the instantaneous extension modulus (initial value of the stress relaxation function), Eq. (4.6) leads to the following final result:

$$U_{||}^2 = \frac{2H(0)}{\varrho_0} + \frac{\partial p}{\partial \varrho}. \tag{4.7}$$

The speed of propagation $U_{||}$ (called the speed of sound in acoustics) is finite only for the Maxwell-like behaviour at short times, i.e. for fluids for which there exists finite instantaneous extension modulus $H(0)$. It can easily be seen that one-dimensional longitudinal waves propagate with a finite speed even in such ideal fluids for which $H(0) \equiv 0$.

At the end of the present considerations let us briefly summarize the most important properties of viscoelastic waves as compared with those in purely viscous compressible fluids.

In the case of the Kelvin-like behaviour at short times the damping effects are much weaker than in purely viscous fluids and sometimes may be neglected for sufficiently high frequencies. The corresponding maximum amounts of extension or compression tend to constant values when frequency increases. Such fluids are less deformable than purely viscous fluids subjected to the same harmonic disturbances. Finite speeds of propagation do not exist at all.

In the case of the Maxwell-like behaviour at short times the damping effects are weaker than those in purely viscous fluids but not so small as in the previous case. The corresponding maximum amounts of extension or compression may increase unlimitedly with increasing frequencies, and such fluids are more deformable as compared with purely viscous fluids subjected to similar initial disturbances. There exist finite speeds of propagation (speeds of sound), and because of weaker damping any disturbances can propagate at longer distances than in purely viscous fluids.

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