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Hydromagnetic flow of a viscoelastic fluid through a porous medium bounded by a vertical porous plate

M. K. ABDEL-HADY and K. A. KAMEL (CAIRO)

A STUDY IS CARRIED out for a hydromagnetic flow with heat transfer of an electrically-conducting incompressible viscoelastic fluid through a porous medium bounded by a vertical porous plate. Analytical expressions for the velocity and temperature fields are obtained. The effect of the magnetic field, Prandtl number, Grashof number, viscoelastic parameter and suction parameter on the flow characteristics have been studied.

Przeanalizowano przepływ hydromagnetyczny z wymianą ciepła dla płynu nieściśliwego lepkosprężystego i przewodzącego prąd w ośrodku porowatym ograniczonym płytką porowatą. Otrzymano analityczną postać rozwiązania dla pól prędkości i temperatury. Rozważono wpływ pola magnetycznego, liczby Prandtla i Grashofa i charakterystyk lepkosprężystości i ssania na przebieg przepływu.

Проанализировано гидромагнитное течение с теплообменом для несжимаемой, вязкоупругой, проводящей жидкости в пористой среде, ограниченной пористой плиткой. Получен аналитический вид решения для полей скорости и температуры. Рассмотрено влияние магнитного поля, числа Прандтля и Грашофа, характеристик вязкоупругости и отсасывания на ход течения.

1. Introduction

FLOWS ARISING from differences in material constitution with temperature differences have a great significance not only for their own interest but also for the application to geophysics and engineering. There are many interesting aspects of such flows, this is why in recent years analytical solutions to such problems of flow have been presented by many authors.

Flow through a porous medium has been studied by a number of workers employing Darcy's law [1]. YAMAMOTO and YOSHIDA [2], YAMAMOTO and IWAMURA [3], CHAWLA and SINGH [4], VARSHNEY [5] and RAPTIS *et al.* [6–8], have solved problems of the flow of a viscous fluid through a porous medium bounded by a vertical surface. There has been an increasing interest in the flow properties of viscoelastic fluids, especially in technological fields. The introduction of the fluid elastic property will play an important role in modifying the flow fields. POP and SOUNDALGEKAR [9] have studied the thermal boundary layer of a viscoelastic flow past an infinite plate when there is no heat transfer between them. RAPTIS and TZIVANIDIS [10] discussed the flow of a viscoelastic fluid, when there is a constant heat flux between the fluid and the plate. KAMEL and EL–ADAWI [11] studied the flow of a viscoelastic fluid through a porous plate with heat and mass transfer.

The object of this paper is a study of the steady flow of a viscoelastic incompressible fluid through a porous medium bounded by an infinite vertical surface subjected to a constant suction velocity taking into account the influence of the viscoelastic fluid and magnetic field on the energy equation. The effect of the magnetic field, Prandtl number, Grashof number, viscoelastic parameter and the suction parameter on the flow characteristics have been studied.

2. Mathematical analysis

A two-dimensional flow of an incompressible, viscoleastic conducting fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical porous surface in the presence of a magnetic field is considered. The X'-axis is taken along the surface in the upward direction and the y'-axis is taken normal to it. A uniform constant magnetic field B_0 is imposed along the y'-axis. Following YAMAMOTO and IWAMURA [3], RAPTIS *et al.* [6, 7, 8, 10] and KAMEL *et al.* [11], the equations which govern the steady flow of an incompressible viscoelastic fluid through a porous medium in the presence of a magnetic field are governed by the following equations:

$$(1)\qquad \frac{\partial v'}{\partial y'} = 0,$$

$$(2)\qquad V'\frac{\partial u'}{\partial y'} = g\beta(T'-T'_\infty)+\nu'\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2}{\varrho'}u'-\frac{K_0^* V'}{\varrho'}\frac{\partial^3 u'}{\partial y'^3}-\frac{\nu'}{K'}u',$$

$$(3)\qquad V'\frac{\partial T'}{\partial y'} = \frac{\lambda}{\varrho' C_p}\frac{\partial^2 T}{\partial y'^2}+\frac{\nu'}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2+\frac{\sigma B_0^2 u'^2}{\varrho' C_p}-\frac{K_0^* V'}{\varrho' C_p}\frac{\partial u'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2}.$$

In these equations u' is the velocity along the X' direction, V' is the velocity normal to the plate, g acceleration due to gravity, β the coefficient of volume expansion, ν' the kinematic viscocity of the fluid, σ is the electrical conductivity of the fluid, B_0 the magnetic induction, ϱ' the density of the fluid, K_0^* the coefficient of the viscoelastic term, K' the permeability of the porous medium, λ the thermal conductivity, C_p the specific heat at constant pressure and T', T'_∞ are the temperatures in the boundary layer and in the free stream, respectively.

The equation of continuity (1) gives

$$(4)\qquad V' = \text{constant} = -V_0,$$

where $V_0(>0)$ is the steady normal velocity of suction of the surface. The boundary conditions are

$$(5)\qquad \begin{aligned} &u' = 0, \quad T' = T'_w, \quad \text{at} \quad y' = 0,\\ &u' \to 0, \quad T' \to T'_\infty, \quad \text{as} \quad y' \to \infty, \end{aligned}$$

where T'_w is the temperature of the surface. Introducing into Eqs. (2) and (3) the following nondimensional parameters:

$$(6)\qquad \eta = \frac{y'V_0}{\nu'}\ \text{(distance)}, \qquad U = -\frac{u'}{V_0}\ \text{(velocity)},$$

(6) [cont.]

$$\theta = \frac{T' - T'_\infty}{T_w - T'_\infty} \quad \text{(temperature)},$$

$$P = \frac{\varrho' \nu' C'_p}{\lambda} \quad \text{(Prandtl number)},$$

$$K = \frac{K_0^* V_0^2 \nu'}{\varrho' \nu'^2} \quad \text{(viscoelastic parameter)},$$

$$M = \frac{\sigma' B_0^2}{\varrho' V_0^2} \quad \text{(magnetic parameter)} \dots,$$

$$E = \frac{V_0^2}{C'_p (T'_w - T'_\infty)} \quad \text{(Eckert number)},$$

$$G = \frac{\nu' g \beta (T'_w - T'_\infty)}{V_0^3} \quad \text{(Grashof number)},$$

$$R = \frac{V_0^2 K'}{\nu'^2} \quad \text{(permeability parameter)},$$

we get

$$KU''' + U'' + U' + LU = -G\theta, \tag{7}$$

$$\theta'' + P\theta' = -EPU'^2 - MEPU^2 - PKEU'U'', \tag{8}$$

where

$$L = M + \frac{1}{R} \tag{9}$$

and the prime denotes differentiation with respect to η.

The corresponding boundary conditions (5) reduce to

$$\begin{aligned} U &= 0, \quad \theta = 1, \quad \text{at} \quad \eta = 0, \\ U &\to 0, \quad \theta \to 0, \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{10}$$

Equations (7) and (8) are nonlinear and in order to obtain a solution we expand U and θ in powers of the Eckert number E, assuming that it is very small. This is justified in low speed incompressible flows. Hence we can write

$$\begin{aligned} U(\eta) &= U_0(\eta) + EU_1(\eta) + O(E^2), \\ \theta(\eta) &= \theta_0(\eta) + E\theta_1(\eta) + O(E^2). \end{aligned} \tag{11}$$

On substituting Eqs. (11) into Eqs. (7) and (8) and equating the coefficients of the same power of E and neglecting terms in E^2 and higher order, we get

$$KU_0''' + U_0'' + U_0' - LU_0 = -G\theta_0, \tag{12}$$

$$KU_1''' + U_1'' + U_1' - LU_1 = -G\theta_1, \tag{13}$$

$$\theta_0'' + P\theta_0' = 0, \tag{14}$$

$$\theta_1'' + P\theta_1' = -PU_0'^2 - MPU_0^2 - KPU_0' U'', \tag{15}$$

while the boundary conditions (10), in view of Eqs. (11), become

$$(16)\qquad \begin{aligned} &U_0 = 0, \quad U_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0 \quad \text{at} \quad \eta = 0,\\ &U_0 \to 0, \quad U_1 \to 0, \quad \theta_0 \to 0, \quad \theta_1 \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned}$$

Solving Eqs. (12)–(15) under the boundary conditions (16) and substituting the solutions obtained into Eqs. (11), we obtain

$$(17)\qquad U = (C_1+EC_{11})C^{g\eta}+(C_2+EC_7)e^{-P\eta}+E(C_8e^{2g\eta}+C_9e^{-2p\eta}+C_{10}e^{(g-P)\eta}),$$

$$(18)\qquad \theta = (1+EC_6)e^{-\varrho\eta}+E(C_3e^{2g\eta}+C_4e^{-2p\eta}+C_5e^{(g-p)\eta})\ldots,$$

where

$$C_1 = -\frac{G}{KP^3-P^2+P+L}, \qquad C_2 = \frac{G}{KP^3-P^2+P+L},$$

$$C_3 = -C_1^2\frac{Pg+KPg^2+(MP/g)}{4g+2P},$$

$$C_4 = -C_2^2\left(P+\frac{M}{P}-KP^2\right)\Big/2,$$

$$C_5 = C_1C_2\frac{2P^2g-2MP-KP^3g+KP^2g^2}{(g-P)^2+P(g-p)},$$

$$C_6 = -\sum_{j=3}^{5}C_j,$$

$$C_7 = \frac{GC_6}{KP^3-P^2+P+L}, \qquad C_8 = -\frac{GC_3}{8Kg^3+4g^2+2g-L},$$

$$C_9 = \frac{GC_4}{8KP^3-4P^2+2P+L},$$

$$C_{10} = -\frac{GC_5}{K(g-p)^3+(g-p)^2+(g-p)-L},$$

$$C_{11} = -\sum_{j=7}^{10}C_j$$

and g is the solution of the equation

$$(19)\qquad Kg^3+g^2+g-L = 0.$$

This equation has three roots. We choose the values of K which gives three real roots, one positive and two negative roots. The positive root is, however, not admissible as U and θ must be finite at ∞. We consider only the negative root which agrees with the result for nonelastic viscous fluid.

Equations (17) and (18) will be used for numerical calculations for the velocity $U(\eta)$ and the temperature $\theta(\eta)$ for $E = 0.01$. From the expression (18), we can calculate the rate of heat transfer in terms of the Nusselt number N. This is given by

$$(20)\qquad \begin{aligned} N &= -(\partial\theta/\partial\eta)_{\eta=0},\\ N &= P+E\big(PC_6-2gC_3+2PC_4-(g-p)C\big)\ldots. \end{aligned}$$

3. Results and discussion

For the purpose of discussing the results, some numerical calculations are carried out for the nondimensional velocity (U), the nondimensional temperature (θ) and the rate of heat transfer (N) taking into account the fact that the Eckert number for incompressible fluids is very small ($E = 0.01$). The values of the magnetic number (M), viscoelastic parameter (K), Prantdl number (P), Grashof number (G) and the permeability parameter (R) take arbitrary positive values in order to investigate their effect on the flow field.

Figure 1 shows that the velocity at any point of the fluid decreases as the magnetic number (M) increases for fixed values of G, P, K and R. A similar effect is observed for the nondimensional velocity U as the permeability parameter (R) increases for fixed values of G, P, K and M, as shown in Fig. 2, and as the Prandtl number (P) decreases for fixed values of G, M, K and R, as shown in Fig. 3. Figure 3 also illustrates the effect of (G) on (U) at constant values of K, M, R and P. The value of (U) increases as (G) increases. A similar effect for the viscoelastic parameter (K) on (U) is observed as shown in Fig. 4.

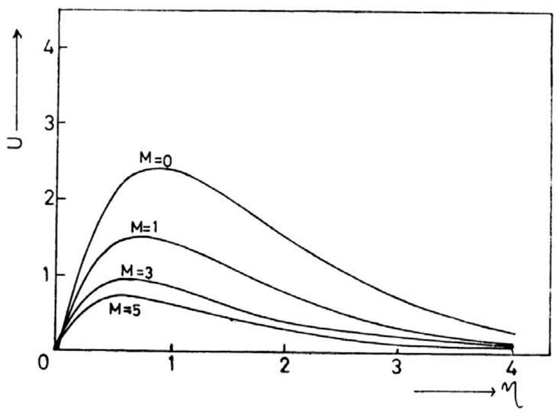

FIG. 1. Velocity profiles for several values of M: $G = 10$, $P = 1$, $K = 0$, $R = 2$.

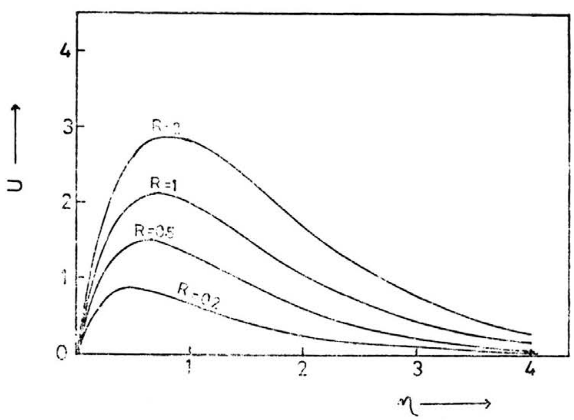

FIG. 2. Velocity profiles for several values of R: $G = 10$, $P = 1$, $K = 0.1$, $M = 0$.

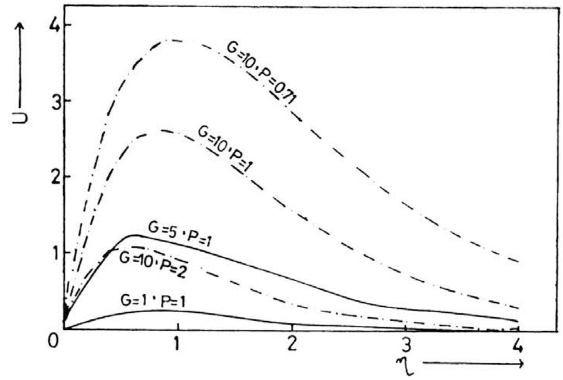

FIG. 3. Velocity profiles for several values of G and P: $K = 0.05$, $M = 0$, $R = 2$.

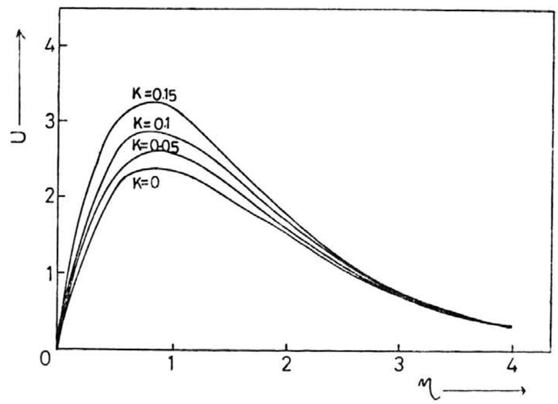

FIG. 4. Velocity profiles for several values of K: $G = 10$. $P = 1$, $M = 0$, $R = 2$.

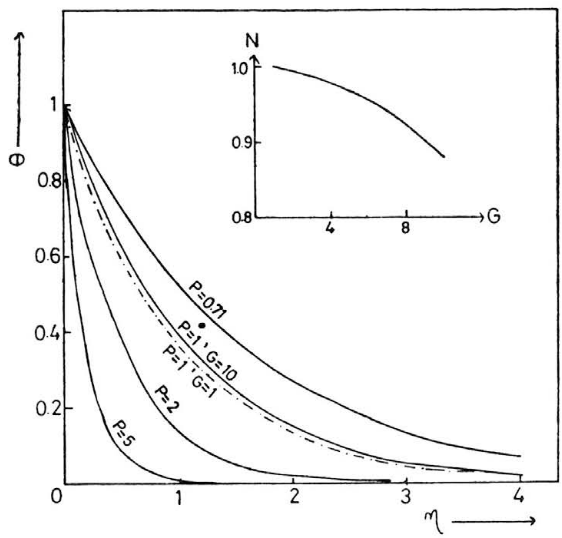

FIG. 5. Temperature profile for several values of P, G and the variation of the Nusselt number N with G: $K = 0.05$, $M = 0$, $R = 2$.

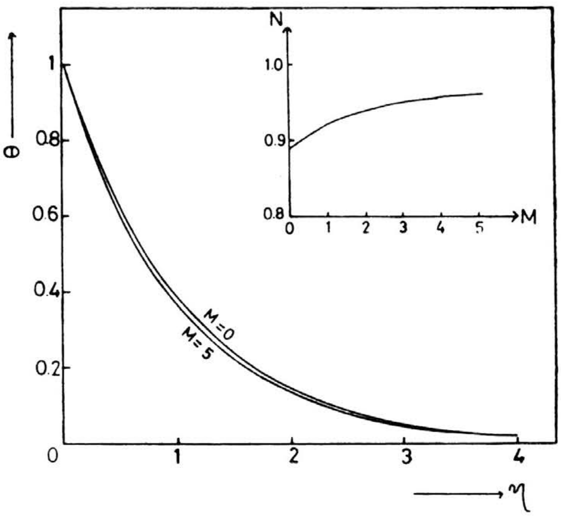

FIG. 6. Temperature profile for several values of M and the variation of the Nusselt number N with M: $G = 10,\ P = 1,\ K = 0,\ R = 2.$

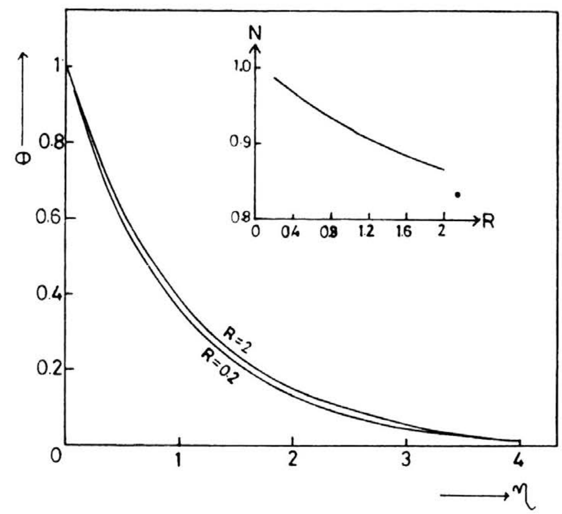

FIG. 7. Temperature profile for several values of R and the variation of the Nusselt Number N with R: $G = 10,\ P = 1,\ K = 0.1\ M = 0$

Figure 5 shows that the nondimensional temperature θ is highly affected by the change of the Prandtl number P, while the change in θ is very small as K, G, M and R change as shown in Figs. 6–8. Figures 5–8 show the variation of heat treansfer with

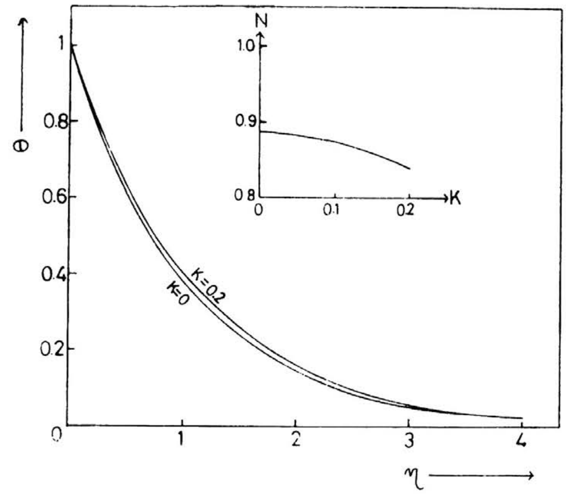

FIG. 8. Temperature profile for several values of K and the variation of the Nusselt number N with K: $G = 10$, $P = 1$, $M = 0$, $R = 2$.

the Grashof number (G), magnetic number (M), viscoelastic parameter K and suction parameter R. We conclude from these figures that an increase in (M) corresponds to an increase in the Nusselt number (N), while (N) decreases as G, K and R increase.

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UNIVERSITY COLLEGE FOR WOMEN, AIN SHAMS UNIVERSITY
and
FACULTY OF SCIENCES (FOR GIRLS), AL-AZHAR UNIVERSITY
CAIRO, EGYPT.

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