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BRIEF NOTES

Effects of erythrocytes on the flow characteristics of blood in an indented tube

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An approximate solution is presented for the flow of blood through an indented tube. It is assumed that blood flowing in the tube is a suspension of red cells in plasma and the red cells are spherical in shape. Theoretical results obtained in this analysis are given for the axial velocity, wall shear stress and the pressure gradient. The numerical solutions of these results are explained graphically for better understanding of the problem.

1. Introduction

In the cardiovascular system the normal blood flow is sometimes disturbed by some unnatural growth (medically called stenosis) formed in the lumen of an artery. The important flow characteristics, such as velocity, wall shear stress, pressure, etc. which have medical significance, are also disturbed and reveal some alterations in the flow caused by the stenosis. The effects of stenosis on these flow characteristics have been studied by many workers in the cases of steady flow of blood through indented arteries, and a number of papers concerning these steady flow problems were listed in the paper by Haldar [1].

The present investigation is concerned with the problem of steady blood flow through an indented tube where the fluid is considered to be a suspension of red cells in plasma. The general approach in this analysis is to study the effects of red cells, which have been considered to be spherical particles, on the flow characteristics.

2. Mathematical formulation

Consider steady, laminar and axial flow of blood past a mild stenosis formed in the lumen of an artery in the cardiovascular system. It is assumed that blood flowing in the tube is a suspension of red blood cells in plasma. The density of the fluid is also assumed to be constant, but the viscosity varies radially. The stenosis develop symmetrically with respect to the radial coordinates and its geometry is described by

$$\frac{R(x)}{R_0} = 1-\frac{\varepsilon}{2R_0}\left[1+\cos\frac{2\pi}{L_0}\left(x-\frac{L_0}{2}\right)\right], \quad 0 \leqslant x \leqslant L_0, \tag{1}$$

$$= 1,$$

where $R(x)$ is the radius of the tube in the constricted region; R_0 is the unconstricted tube radius; L_0 is the length of stenosis and ε is the maximum height of stenosis such that $\varepsilon/R_0 \ll 1$ (Fig. 1).

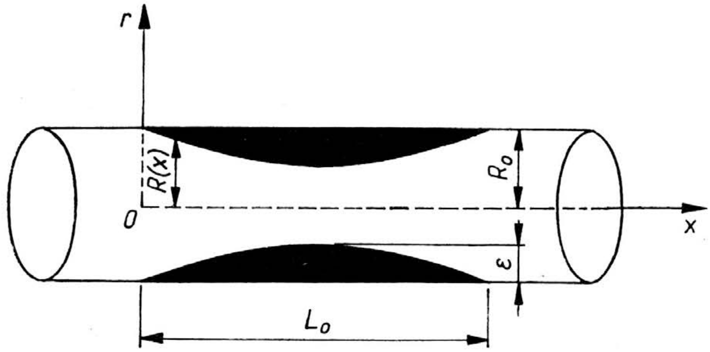

FIG. 1. Geometry of construction.

If the radial velocity component is assumed to be very small compared to the axial velocity component, then the approximate equation of motion which governs the flow in the tube, is

(2)
$$\frac{1}{\xi}\cdot\frac{d}{d\xi}(\xi\tau_{\xi x}) = -R_0^2\frac{dp}{dx},$$
$$\xi = r/R_0,$$

where p is the fluid pressure and $\tau_{\xi x}$ is the shear stress given by

(3)
$$\tau_{\xi x} = -\mu(\xi)\frac{du}{d\xi}.$$

Here $u(\xi)$ is the axial velocity component and $\mu(\xi)$ is the coefficient of viscosity of blood proposed by OLIVER and WARD [3] in the following form

(4)
$$\mu(\xi) = \mu_0/[1-\beta h(\xi)],$$

where μ_0 is the coefficient of plasma; β is a constant equal to 2.5 and $h(\xi)$ is the hematocrit. The relation (4) holds for a very dilute suspension of red cells which are supposed to be spherical rigid particles and a useful empirical formula describing the distribution of red cells is

(5)
$$h(\xi) = h_m[1-\xi^n].$$

Here h_m is the maximum hematocrit at the centre of the tube and n is a parameter determining the exact shape of the profile. Details of the relations (4) and (5) are given by LIH [2].

The boundary conditions are the following: (i) — the velocity must vanish at the wall of the tube, and (ii) — the flow is symmetrical about the tube axis. These conditions are mathematically expressed in the following forms:

(6)
$$u = 0 \quad \text{on} \quad \xi = R(x)/R_0,$$

(7)
$$du/d\xi = 0 \quad \text{on} \quad \xi = 0.$$

3. Solution of the problem

The equation (2) can be integrated with the boundary condition (7) and the expression for $\tau_{\xi x}$ is

$$\tau_{\xi x} = -\frac{R_0^2}{2}\frac{dp}{dx}\xi. \tag{8}$$

Combining (3) and (8) and then integrating with the help of boundary condition (6), the velocity component is found to be

$$u = -\frac{R_0^2}{4\mu_0}\cdot\frac{dp}{dx}\left[(1-k)\{(R/R_0)^2-\xi^2\}+\frac{2k}{n+2}\{(R/R_0)^{n+2}-\xi^{n+2}\}\right], \tag{9}$$

where $k = \beta h_m$.

The volumetric flow ratio Q of the fluid across any cross-section in the stenotic region of the tube is

$$Q = -\frac{\pi R_0^4}{8\mu_0}\cdot\frac{dp}{dx}\left[(1-k)(R/R_0)^4+\frac{4k}{n+4}(R/R_0)^{n+4}\right]. \tag{10}$$

If Q_0 is the flow rate of fluid in the tube in absence of stenosis, then

$$Q_0 = -\frac{\pi R_0^4}{8\mu_0}\cdot\left(\frac{dp}{dx}\right)_0\left(1-\frac{kn}{n+4}\right), \tag{11}$$

where $(dp/dx)_0$ is the pressure gradient of fluid in the unconstricted tube. If Q and Q_0 occur in the same system, then the ratio $Q/Q_0 = 1$ and we have the relative local pressure gradient from (10) and (11) as

$$\frac{dp/dx}{(dp/dx)_0} = \frac{(1-k)n+4}{(1-k)(n+4)(R/R_0)^4+4k(R/R_0)^{n+4}}. \tag{12}$$

The shearing stress at the wall can be defined by

$$\tau_R = \left[-\mu(r)\frac{du}{dr}\right]_{r=R} \tag{13}$$

which, on using Eq. (9), gives

$$\tau_R = -\frac{R_0}{2}\frac{dp}{dx}\cdot\frac{(1-k)(R/R_0)+k(R/R_0)^{n+1}}{1-k[1-(R/R_0)^n]}, \tag{14}$$

Then the value of τ_R at the throat of stenosis is

$$(\tau_R)_{x=L_0/2} = \frac{Q\mu_0}{\pi R_0^3}\cdot\frac{(1-k)(1-2b)+k(1-b)^{n+1}}{[1-k\{1-(1-2b)^n\}][f(1-2b)^4+g(1-2b)^{n+4}]}, \tag{15}$$

where

$$f = (1-k)/4, \quad g = k/(n+4), \quad 2b = \varepsilon/R_0. \tag{16}$$

If τ_N is the shear stress at the wall in absence of stenosis, then the non-dimensional form of (15) takes the form

$$\tau = \frac{(\tau_R)_{x=L_0/2}}{\tau_N} = \frac{(f+g)[(1-k)(1-2b)+(1-2b)^{n+1}}{[1-k\{1-(1-2nb)^n\}][f(1-2b)^4+g(1-2b)^{n+4}]}, \tag{17}$$

where

$$\tau_N = \frac{Q_0 \mu_0}{\pi R_0^3} \cdot \frac{1}{f+g}. \tag{18}$$

4. Numerical discussions

Most of the theoretical results have been obtained in this analysis and explained graphically for better understanding of the problem.

Figure 2 shows the variations of axial velocity profile at the throat of stenosis for different values of hematocrit. At zero percentage of hematocrit the velocity profile attains a parabolic shape. But the profile shows some departure from the parabolic form at the

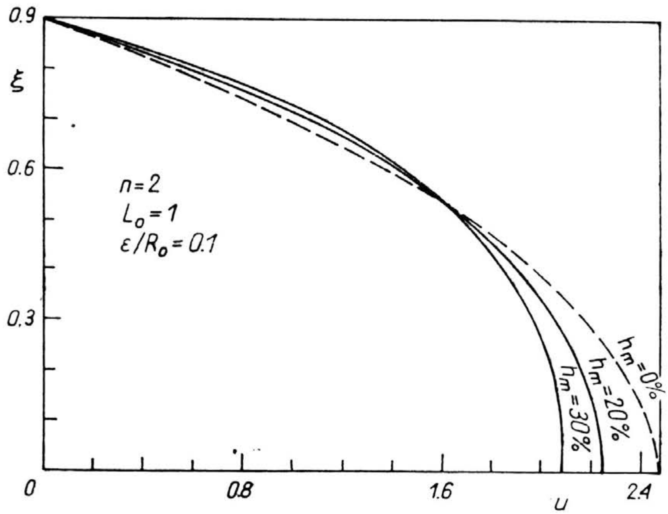

FIG. 2. Velocity distribution.

centre of the tube for low hematocrit percentage pointing out a slight flattening there. This flattening of the profile has become much more pronounced at higher percentage of hematocrit indicating a high shear near the wall. This means that the shear increases near the wall increasing with percentage of hematocrit (Fig. 3).

The variations of relative local pressure gradient are shown along the length of stenosis for different values of hematocrit (Fig. 4). At the beginning the variation of pressure gradient is very slow and then it increases rapidly up to a position just ahead of the throat of stenosis where its maximum value is attained. After attaining the maximum, the rapidity of downfall of pressure gradient is significant and reaches the end of stenosis with very slow variation. It is also observed that for a fixed value of x the pressure gradient increases with increasing hematocrit indicating the fall of pressure.

From the above discussions the following conclusions can be drawn:

(i) The wall shear stress increases as the hematocrit increases.

(ii) The lowest pressure occurs at the throat of stenosis due to the rise of hematocrit.

(iii) The flattening of the velocity profile has become much more pronounced at the tube centre for high hematocrit.

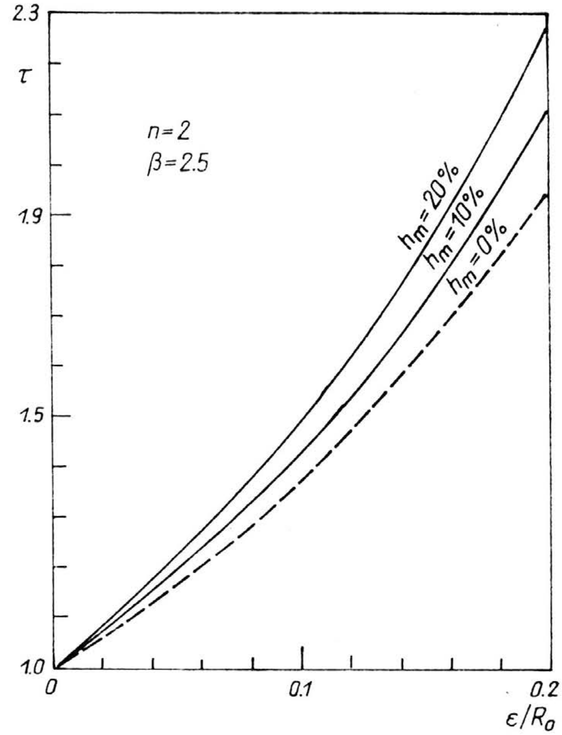

FIG. 3. Variation of τ with ε/R_0 for different h_m.

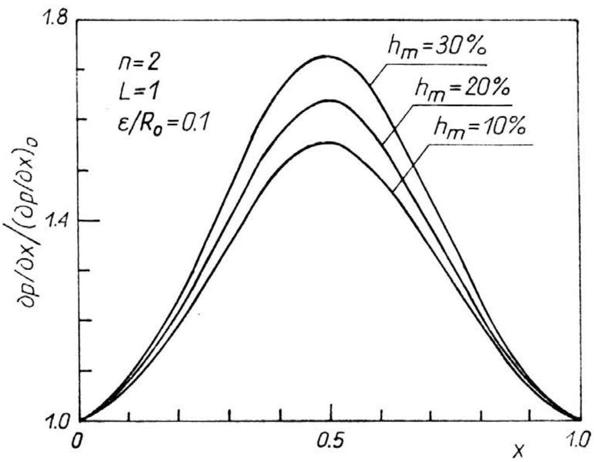

FIG. 4. Distribution of relative pressure gradient.

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