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# Propagation of generalized thermoelastic waves in cubic crystals

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THE PROPAGATION of plane harmonic waves in a thermally conducting cubic crystal has been studied. Four waves are found to exist. It is found that in case of one-dimensional waves, the transverse waves remain unaffected by thermal variations and vice-versa. Only longitudinal waves and thermal waves are coupled. In the two-dimensional case, the only unaffected wave due to thermal variation is the purely transverse (SH) wave and all other waves are dependent on each other. The approximations for the phase velocities and attenuation coefficients are obtained. The results obtained theoretically have been verified numerically for a single crystal of NaF.

Zbadano propagację płaskiej fali harmonicznej w krysztale kubicznym przewodzącym ciepło. Stwierdzono, że w przypadku fal jedmowymiarowych fale poprzeczne są niezakłócone przez efekty cieplne; sprzężone ze sobą są jedynie fale podłużne i termiczne. W przypadku dwuwymiarowym jedyną falą niezakłóconą przez efekty cieplne jest czysto poprzeczna fala (SH), a wszystkie pozostałe fale są ze sobą sprzężone. Wyznaczono przybliżone wartości prędkości fazowych i współczynników tłumienia. Wyniki teoretyczne zweryfikowano obliczeniami numerycznymi dla monokryształu NaF.

Исследовано распространение плоской гармонической волны в теплопроводящем кубическом кристалле. Констатировано, что в случае одномерных волн поперечные волны невозмущены термическими эффектами; сопряжены с собой только продольные и термические волны. В двумерном случае единственной невозмущенной термическими эффектами волной является чисто поперечная волна SH, а все остальные волны сопряжены с собой. Определены приближенные значения фазовых скоростей и коэффициентов затухания. Теоретические результаты проверены численными расчетами для случая монокристалла NaF.

## 1. Introduction

THE THEORY of elastic wave propagation in anisotropic solids is well known [1], CHADWICK and SEET [2], and CHADWICK [3] discussed propagation of plane harmonic waves in transversely isotropic and homogeneous anisotropic heat conducting solids, respectively, in the coupled theory of thermoelasticity. BANERJEE and PAO [4] and PAO and BANERJEE [5] investigated the propagation of plane harmonic waves in homogeneous anisotropic solids and dielectric crystals, respectively, by taking thermal relaxation times into consideration.

Recently, the generalized theory of thermoelasticity developed by LORD and SHULMAN [6] has been extended to homogeneous anisotropic media by DHALIWAL and SHERIEF [7]. SINGH and SHARMA [8] discussed the propagation of generalized thermoelastic waves in transversely isotropic media. SHARMA and SIDHU [9], and SHARMA and SINGH [10] studied the propagation of plane harmonic waves in homogeneous anisotropic solids in generalized theory of thermoelasticity [7]. SHARMA and SINGH [11] discussed propagation of one-dimensional generalized thermoelastic waves in crystals. The present article deals with the propagation of generalized thermoelastic waves in cubic crystals. The theory of algebraic functions is used to investigate the present problem.

## 2. The problem and derivation of frequency equations

We consider an unbounded, homogeneous cubic crystal undergoing small temperature variations. The displacement components $u_i$, $i = 1, 2, 3$ and the small temperature deviation $T$ from an equilibrium temperature $T_0$ are connected by the following differential equations [7]

$$
\begin{aligned}
&C_{11}u_{1,11}+C_{44}(u_{1,22}+u_{1,33})+(C_{44}+C_{12})(u_{2,12}+u_{3,13})-\beta T_{,1} = \varrho\ddot{u}_1,\\
&C_{11}u_{2,22}+C_{44}(u_{2,11}+u_{2,33})+(C_{44}+C_{12})(u_{1,12}+u_{3,23})-\beta T_{,2} = \varrho\ddot{u}_2,\\
&C_{11}u_{3,33}+C_{44}(u_{3,11}+u_{3,22})+(C_{44}+C_{12})(u_{1,13}+u_{2,23})-\beta T_{,3} = \varrho\ddot{u}_3,\\
&KT_{,jj}-\varrho C_e(\dot{T}+\tau_0\ddot{T}) = T_0\beta(\dot{u}_{i,i}+\tau_0\ddot{u}_{i,i}), \qquad i,j = 1,2,3,
\end{aligned}
\tag{2.1}
$$

where $\beta = (C_{11}+2C_{12})\alpha$, $C_{ij}$ are the isothermal elasticities (see [12]), $\varrho$, $C_e$ and $\tau_0$ are the density, specific heat at constant strain, and thermal relaxation time, respectively, $K$ is the thermal conductivity, $\alpha$ — the linear thermal expansion coefficient of the crystal. Comma notation is used for spatial derivatives and superposed dot represents differentiation with respect to time. It can be proved [13] thermodynamically that $K \geqslant 0$, $\tau_0 \geqslant 0$ and, of course, $\varrho > 0$, $T_0 > 0$. We assume in addition that $C_e > 0$ and that the isothermal linear elasticities are components of a positive definite fourth-order tensor. The necessary and sufficient conditions for the satisfaction of the latter requirement are

$$
C_{11} > 0, \quad C_{11} > C_{12}, \quad C_{11}^2 > C_{12}^2, \quad C_{44} > 0. \tag{2.2}
$$

As we are considering plane harmonic waves, therefore we may take

$$
u_k = U_k\exp\{iw(v^{-1}x_pn_p-t)\}, \qquad T = \Theta\exp\{iw(v^{-1}x_pn_p-t)\}, \tag{2.3}
$$

where $w$ is the angular frequency (assumed to be real), $v$ is the phase velocity (in general — complex) of the waves, and the unit vector $\mathbf{n} = (n_1, n_2, n_3)$ is the wave normal specifying the direction of propagation of waves. Substituting from Eqs. (2.3) in (2.1) and simplyfying, we obtain the frequency equation

$$
\begin{vmatrix}
(1-C_1)n_1^2+C_1-\zeta & C_2n_1n_2 & C_2n_1n_3 & n_1\\
C_2n_1n_2 & (1-C_1)n_2^2+C_1-\zeta & C_2n_2n_3 & n_2\\
C_2n_1n_3 & C_2n_2n_3 & (1-C_1)n_3^2+C_1-\zeta & n_3\\
\varepsilon\tau n_1\zeta & \varepsilon\tau n_2\zeta & \varepsilon\tau n_3\zeta & -\xi-\tau\zeta
\end{vmatrix}, \tag{2.4}
$$

where

$$
\zeta = \varrho v^2/C_{11}, \quad C_1 = C_{44}/C_{11}, \quad C_2 = (C_{44}+C_{12})/C_{11}, \quad \chi = w/w^*,
$$

$$
w^* = C_eC_{11}/K, \quad \varepsilon = T_0\beta^2/\varrho C_eC_{11}, \quad z = i\chi, \quad \tau = 1-\tau_0w^*z.
$$

The determination Eq. (2.4) is of fourth degree in $\zeta$ and hence has four roots. Thus, in general, there are four waves, namely: a quasi-longitudinal, two quasi-transverse, and a quasi-thermal wave ($T$-mode), which can propagate in these crystals. All the waves are coupled with each other.

## 3. Discussion of frequency equation

### Case I. Plane waves propagation along $x_1$-axis

In this case the wave normal $\mathbf{n} = (1, 0, 0)$. The secular equation (2.4) reduces to

$$(C_1 - \zeta)^2 = 0 \tag{3.1}$$

and

$$\tau\zeta^2 - a\zeta - z = 0,$$

where

$$a = (1+\varepsilon)\,\tau - z.$$

The Eq. (3.1) corresponds to two purely transverse waves, which get decoupled from the rest of the motion and vice-versa. These waves are not affected by thermal relaxation and propagate without dispersion or damping with velocity $(C_{44}/\varrho)^{1/2}$, independently of the thermodynamic conditions. The roots of equation $(3.1)_2$ are given by

$$\zeta_1, \zeta_2 = [a \pm (a^2 - 4z\tau)^{1/2}]/2\tau. \tag{3.2}$$

When $w \to 0$, i.e. $|z| \to 0$, the roots $\zeta_1 \to 1+\varepsilon$ and $\zeta_2 \to 0$. Because at lowfrequency, the conditions are isentropic, therefore $\zeta_1$ corresponds to the longitudinal wave and $\zeta_2$ corresponds to the thermal wave ($T$-mode). Thus the longitudinal waves in this limiting case travel with isentropic velocity $(\overline{C}_{11}/\varrho)^{1/2}$. $\overline{C}_{11} = C_{11}(1+\varepsilon)$ and the thermal wave does not exist at all.

Again when $w \to \infty$ i.e., $|z| \to \infty$ the roots $\zeta_1$ and $\zeta_2$ reduce to

$$\zeta_1^*, \zeta_2^* = \{1+\tau_0 w^* + \varepsilon\tau_0 w^* \pm [(1+\tau_0 w^* + \varepsilon\tau_0 w^*)^2 - 4\tau_0 w^*]^{1/2}\}/2\tau_0 w^*. \tag{3.3}$$

Now

$$(1+\tau_0 w^* + \varepsilon\tau_0 w^*)^2 - 4\tau_0 w^* = (1-\tau_0 w^* + \varepsilon\tau_0 w^*)^2 + 4\varepsilon\tau_0 w^* > 0,$$

hence $\zeta_1^*$, $\zeta_2^*$ are real. Thus there are two waves called longitudinal and thermal, corresponding to $\zeta_1^*$ and $\zeta_2^*$, respectively, which travel with real velocities given by $(C_{11}\zeta_1^*/\varrho)^{1/2}$, $i = 1, 2$ and depend upon the thermal relaxation time.

In general, roots $\zeta_1$, $\zeta_2$ given by (3.2) may be approximated as in [9].

i) Low-frequency approximations ($\chi \leqslant 1$)

$$\begin{aligned} \zeta_1 &= 1+\varepsilon+\varepsilon z/(1+\varepsilon)+\varepsilon z^2(1-\tau_0 w^*(1+\varepsilon)^2)/(1+\varepsilon)^3 + \ldots, \\ \zeta_2 &= z/(1+\varepsilon) - (\varepsilon+\tau_{0w^*})(1+\varepsilon)^2)z^2/(1+\varepsilon)^3 + \ldots \end{aligned} \tag{3.4}$$

and ii) High-frequency approximations ($\chi \geqslant 1$)

$$\begin{aligned} \zeta_1 &= 1+\varepsilon\hat{z} - \varepsilon(1+\varepsilon)\hat{z}^2 + \ldots, \\ \zeta_2 &= -\hat{z}^{-1} + \varepsilon - \varepsilon\hat{z} + \ldots, \end{aligned} \tag{3.5}$$

where

$$\hat{Z} = \hat{z} - \tau_0 w^*, \quad \hat{z} = z^{-1}.$$

If we write

$$v^{-1} = V^{-1} + iw^{-1}q, \tag{3.6}$$

where $V$ and $q$ are real, the exponents in the plane wave in Eq. (2.3) become

$$-qx_pn_p+iw(V^{-1}x_pn_p-t).$$

This shows that $V$ is the speed of propagation and $q$ is the attenuation coefficient of the wave. Using Eq. $(3.6)_1$ in Eq. (3.4) we obtain

$$(3.7)_1 \qquad V_i = V^*\sqrt{R_i}/\cos(\phi_i/2), \quad q_i = w\sin(\phi_i/2)/V^*\sqrt{R_i}, \quad i = 1, 2,$$

where

$$R_i = (A_i^2+B_i^2)^{1/2}, \quad \phi_i = \tan^{-1}(\pm|Bi/Ai|),$$
$$A_1 = 1+\varepsilon-\varepsilon\{1-\tau_0 w^*(1+\varepsilon)^2\}\chi^2/(1+\varepsilon)^3, \quad B_1 = \varepsilon\chi/(1+\varepsilon),$$
$$A_2 = \chi^2(\varepsilon+\tau_0 w^*(1+\varepsilon)^2)/(1+\varepsilon)^3, \quad B_2 = \chi/(1+\varepsilon), \quad V^* = (C_{11}/\varrho)^{1/2}.$$

The signs + or − in $\phi_i$ may be taken according to whether $x_pn_p > 0$ or $< 0$. Now using Eq. $(3.6)_1$ in Eq. (3.5) we get

$$(3.7)_2 \qquad V_i = c_i\sqrt{r_i}/\cos(\psi_i/2), \quad q_i = w\sin(\psi_i/2)/c_i\sqrt{r_i}, \quad i = 1, 2,$$

where

$$r_i = (a_i^2+b_i^2)^{1/2}, \quad \psi_i = \tan^{-1}(\pm|bi/ai|),$$
$$a_1 = 1-\varepsilon r\cos\psi-\varepsilon(1+\varepsilon)\,r^2\cos(2\psi),$$
$$b_1 = -r\varepsilon\sin\psi-\varepsilon(1+\varepsilon)\,r^2\sin(2\psi),$$
$$a_2 = -\cos\psi/r+\varepsilon r\cos\psi-r^2\varepsilon\cos 2\psi,$$
$$b_2 = \sin\psi/r+\varepsilon r\sin\psi-r^2\varepsilon\sin 2\psi,$$
$$r = (\chi^{-2}+\tau_0^2 w^{*2})^{1/2}, \quad \psi = \tan^{-1}(1/\tau_0 w), \quad c_i = (C_{11}\zeta_i^*/\varrho)^{1/2}.$$

Clearly, as $w \to \infty$, i.e. $|z| \to 0$, the roots $\zeta_1, \zeta_2$ in (3.5) reduce to

$$\zeta_1 = 1-\varepsilon\tau_0 w^*-\varepsilon(1+\varepsilon)\,\tau_0^2 w^{*2}-\ldots,$$
$$\zeta_2 = (\tau_0 w^*)^{-1}+\varepsilon+\varepsilon\tau_0 w^*+\ldots.$$

### Case II

We consider plane waves propagating in a principal plane perpendicular to the principal direction (0, 1, 0), i.e. the wave normal $\mathbf{n} = (n_1, 0, n_3)$ with $n_1^2+n_3^2 = 1$. The frequency equation (2.4) reduces to

$$(3.8)_1 \qquad (C_1-\zeta) = 0$$

and

$$(3.8)_2 \qquad (1-\tau_0 w^*z)\zeta(\zeta-\lambda_1^*)(\zeta-\lambda_2^*)+z(\zeta-\lambda_1)(\zeta-\lambda_2) = 0,$$

where

$$(3.9) \qquad \lambda_1, \lambda_2 = (a_1\pm(a_1^2-4a_2)^{1/2})/2, \quad \lambda_1^*, \lambda_2^* = [A_1\pm(A_1^2-4A_2)^{1/2}]/2,$$

$$a_1 = 1+C_1, \quad a_2 = C_1(1+2C_1)+(1+C_1^2-C_2^2-2C_1)\sin^2\theta\cos^2\theta,$$

(3.10)
$$A_1 = (1+\varepsilon)(1+\bar{C}_1), \quad A_2 = (1+\varepsilon)^2\{\bar{C}_1(1+2\bar{C}_1)+(1+\bar{C}_1-\bar{C}_2^2-2\bar{C}_1)\times\sin^2\theta\cos^2\theta,$$

$$\bar{C}_1 = C_1/(1+\varepsilon), \quad \bar{C}_2 = (C_2+\varepsilon)/(1+\varepsilon)$$

and $\theta$ is the inclination of the wave normal to the $x_3$-axis. Equation $(3.8)_1$ corresponds to the purely transverse (SH) wave, which is not affected by thermal variations, and vice-versa. This wave propagates without dispersion or damping, with speed $(C_{44}/\varrho)^{1/2}$ and gets decoupled — from the rest of the motion. Equation $(3.8)_2$ being cubic in $\zeta$ give three roots and, hence, yields three dispersive waves which are affected by thermal variations and relaxation time. In case of $w \to 0$ i.e. $|z| \to 0$, Eq. $(3.8)_2$ reduces to

$$\zeta(\zeta-\lambda_1^*)(\zeta-\lambda_2^*) = 0 \tag{3.11}$$

whose roots are

$$\zeta_1 = \lambda_1^*, \quad \zeta_2 = \lambda_2^*, \quad \zeta_3 = 0. \tag{3.12}$$

The first two roots correspond to the usual elastic waves at isentropic conditions, and the third one — to the thermal mode. Thus at low frequency the elastic waves propagate with real isentropic speeds $(C_{11}\lambda_i^*/\varrho)^{1/2}$, $i = 1, 2$ without dispersion, and the thermal wave does not exist at all. Again, when $w \to \infty$ i.e. $|z| \to \infty$, Eq. $(3.8)_2$ becomes

$$\zeta^{-1}(\zeta-\lambda_1)(\zeta-\lambda_2)-\tau_0 w^*(\zeta-\lambda_1^*)(\zeta-\lambda_2) = 0. \tag{3.13}$$

Its roots may be obtained as a special case from the high-frequency approximations given in the following analysis. However, if $\tau_0 \to 0$, then (3.13) reduces to

$$\zeta^{-1}(\zeta-\lambda_1)(\zeta-\lambda_2) = 0 \tag{3.14}$$

whose roots are

$$\zeta_1 = \lambda_1, \quad \zeta_2 = \lambda_2, \quad \zeta_3 = \infty. \tag{3.15}$$

Velocities of the first two modes associated with have real values $(C_{11}\lambda_i/\varrho)^{1/2}$. The third mode $(T)$ has infinite velocity of propagation and is thus diffusive in nature. Following the approach presented by SHARMA and SIDHU [9], we obtain the following approximation of the roots of equation $(3.8)_2$ combined with Eqs. (3.12) and (3.15) at low and high-frequencies, respectively:

i) Low-frequency approximations $(\chi \leqslant 1)$

(3.16)
$$\zeta_i(z) = \lambda_i^*\Big[1+\sum_1^\infty C_n^{(i)}(-z)^n\Big],$$

$$\zeta_3(z) = \sum_1^\infty d_n(-z)^n,$$

where the first coefficients in these series are given by

(3.17)
$$c_1^{(i)} = g(\lambda_i^*)/\lambda_i^* f'(\lambda_i^*), \quad d_1 = g(0)/f'(0),$$

$$c_2^{(i)} = c_1^{(i)}[g'(\lambda_i^*)-\tau_0 w^* f'(\lambda_i^*)-c_1^{(i)}\lambda_i^* f''(\lambda_i^*)/2]/f'(\lambda_i^*),$$

$$d_2 = d_1[g'(0)-\tau_0 w^* f'(0)-d_1 f''(0)/2]/f'(0),$$

(3.18) $$f(\zeta) = \zeta(\zeta-\lambda_1^*)(\zeta-\lambda_2^*), \quad g(\zeta) = (\zeta-\lambda_1)(\zeta-\lambda_2).$$

Dashes denote derivatives with respect to the argument. Using Eq. (3.6) in Eqs. (3.16), we obtain the values for $V$ and $q$ for different modes

(3.19) $$V_i = V_i^*\sqrt{R_i}/\cos(\phi_i/2), \quad q_i = w\sin(\phi_i/2)/V_i^*\sqrt{R_i},$$

where

(3.20) $$R_i = (A_i^2+B_i^2)^{1/2}, \quad \phi_i = \tan^{-1}(\pm/B_i/A_i|), \quad i = 1, 2, 3,$$
$$A_i = 1-c_2^{(i)}\chi^2, \quad B_i = c_1^{(i)}\chi, \quad V_i^* = (C_{11}\lambda_i^*/\varrho)^{1/2}$$

for elastic waves,

$$A_3 = -\chi^2 d_2, \quad B_3 = d_1\chi, \quad V_3^* = (C_{11}/\varrho)^{1/2}$$

for thermal wave.

The sign + or − in the determination of $\phi_i$ is taken according to whether $x_p n_p > 0$ or $< 0$ in Eq. (3.6).

(ii) High-frequency approximations ($\chi \gg 1$)

(3.21) $$\zeta_i(\hat{Z}) = \lambda_i\Big[1+\sum_1^\infty c_n^{(i)}(-\hat{Z})^n\Big],$$
$$\zeta_3(\hat{Z}) = \eta(\hat{Z})/\hat{Z}, \quad \eta(\hat{z}) = \sum_0^n d_n(-\hat{Z})^n,$$

where $\hat{Z} = \hat{z}-\tau_0 w^*$ and the first two coefficients in these series are given by

(3.22) $$c_1^{(i)} = f(\lambda_i)/\lambda_i g'(\lambda_i), \quad c_2^{(i)} = c_1^{(i)}[f'(\lambda_i)-\lambda_i c_i^{(i)}g''(\lambda_i)/2]/g'(\lambda_i),$$
$$d_0 = -1, \quad d_1 = \sum_1^2 (\lambda_i^*-\lambda_i), \quad d_2 = 2[\lambda_1^*\lambda_2^*-\lambda_1\lambda_2$$
$$+d_1\{2(\lambda_1^*+\lambda_2^*)-\lambda_1-\lambda_2+2d_1\}],$$

$f(\zeta)$ and $g(\zeta)$ are defined by Eq. (3.18).

Using Eq. (3.6), we obtain

(3.23) $$V_i = c_i\sqrt{r_i}/\cos(\psi_i/2), \quad q_i = w\sin(\psi_i/2)/c_i\sqrt{r_i}, \quad i = 1, 2, 3,$$

where

(3.24) $$r_i = (a_i^2+b_i^2)^{1/2}, \quad \psi_i = \tan^{-1}(\pm|b_i/a_i|),$$

(3.25) $$a_i = 1-r\cos\psi c_1^{(i)}+r^2\cos 2\psi c_2^{(i)}, \quad b_i = -r\sin\psi c_1^{(i)}+r^2\sin 2\psi c_2^{(i)},$$
$$c_i = (C_{11}\lambda_i/\varrho)^{1/2}$$

for elastic waves,

(3.26) $$a_3 = -\cos(\psi)/r+rd_1\cos\psi+r^2 d_2\cos 2\psi,$$
$$b_3 = \sin(\psi)/r+rd_1\sin\psi+r^2 d_2\sin 2\psi,$$
$$c_3 = (C_{11}/\varrho)^{1/2}$$

for thermal waves,

$$r = (\chi^{-2}+\tau_0^2 w^{*2})^{1/2}, \qquad \psi = \tan^{-1}(1/\tau_0 w). \tag{3.27}$$

The + or − signs in the determination of $\psi_i$ are to be taken according to whether $x_p n_p > 0$ or $< 0$. The approximate values of the roots of Eq. (3.13) can be obtained from Eq. (3.21) on letting $\chi \to \infty$. We obtain

$$\zeta_i = \lambda_i[1+\tau_0 w^* c_1^{(i)}+\tau_0^2 w^{*2} c_2^{(i)}+ \ldots], \qquad i = 1, 2,$$
$$\zeta_3 = (\tau_0 w^*)^{-1}+d_1-d_2 \tau_0 w^* + \ldots .$$

The real values of propagation speeds follow from the above expansions directly. We get

$$V_i = c_i\{1+\tau_0 w^* c_1^{(i)}+\tau_0^2 w^{*2} c_2^{(i)}+ \ldots\} \quad \text{for elastic waves,}$$
$$V_3 = c_3\{(\tau_0 w^*)^{-1}+d_1-\tau_0 w^* d_2+ \ldots .\} \quad \text{for thermal wave.}$$

This last result shows that the $T$-mode has now a finite velocity of propagation, whereas in the coupled thermoelasticity ($\tau_0 \to 0$) this mode is evidently diffusive.

## 4. Numerical results and discussions

Numerical calculations have been done for the assigned frequency waves in a single crystal of NaF, the basic physical data for which are $T_0 = 17.3°K$, $C_{11} = 10.85\times10^{11}$

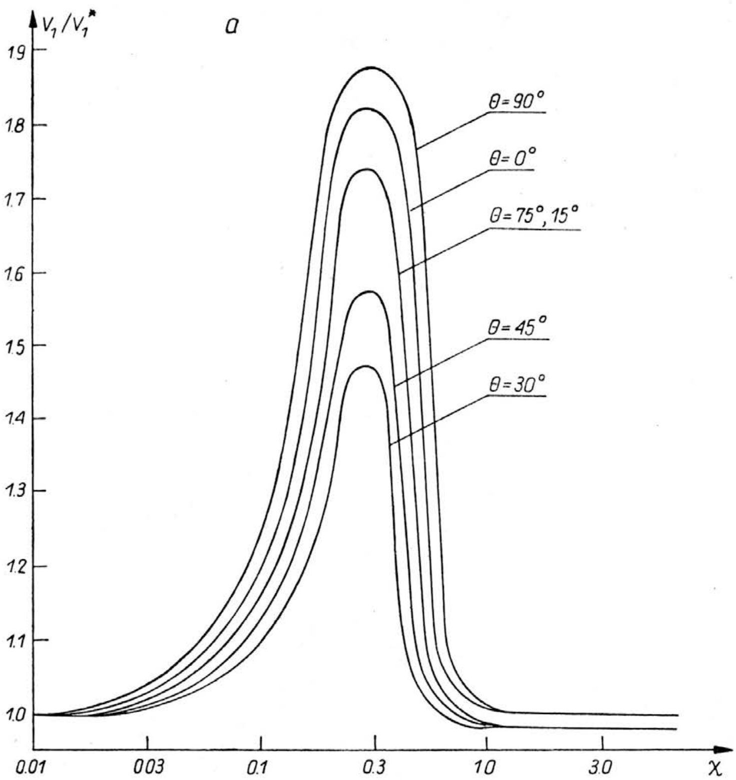


FIG. 1a. Variation of the phase velocity of the QL-wave in NaF-crystal with frequency and direction of propagation.

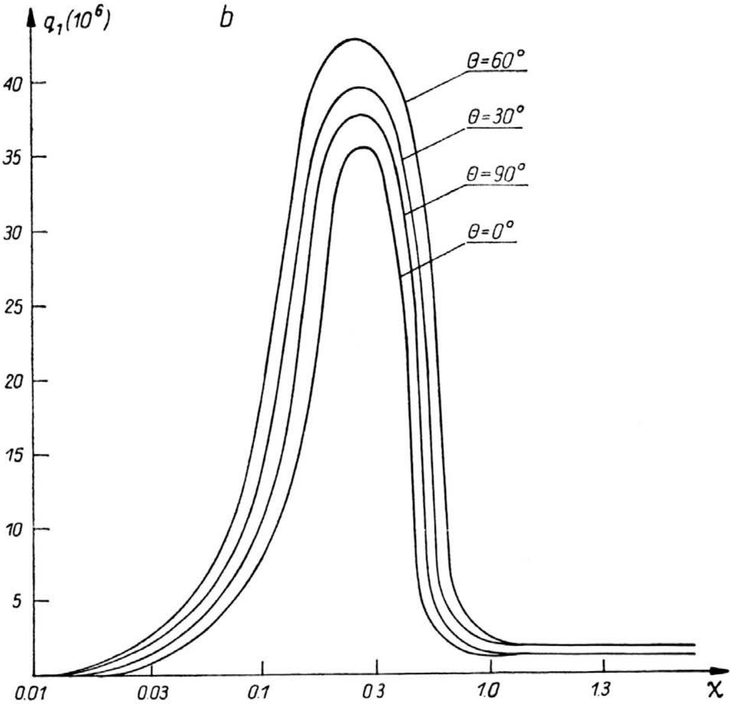


**FIG. 1b.** Variation of the attenuation coefficient of QL-wave in NaF-crystal with frequency and direction of propagation.

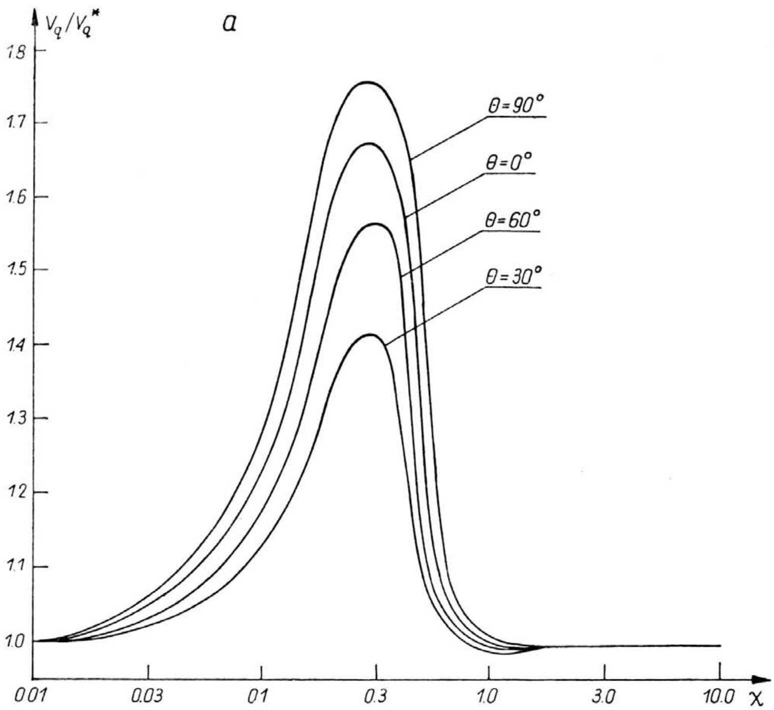


**FIG. 2a.** Variation of the phase velocity of the QT-wave in NaF crystal with frequency and direction of propagation.

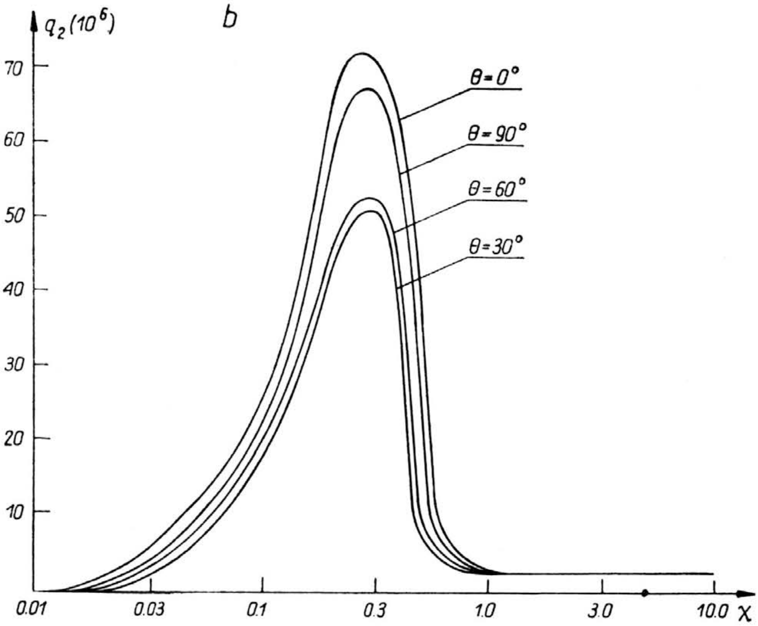


**Fig. 2b.** Variation of the attenuation coefficient of QT-wave in NaF crystal with frequency and direction of propagation.

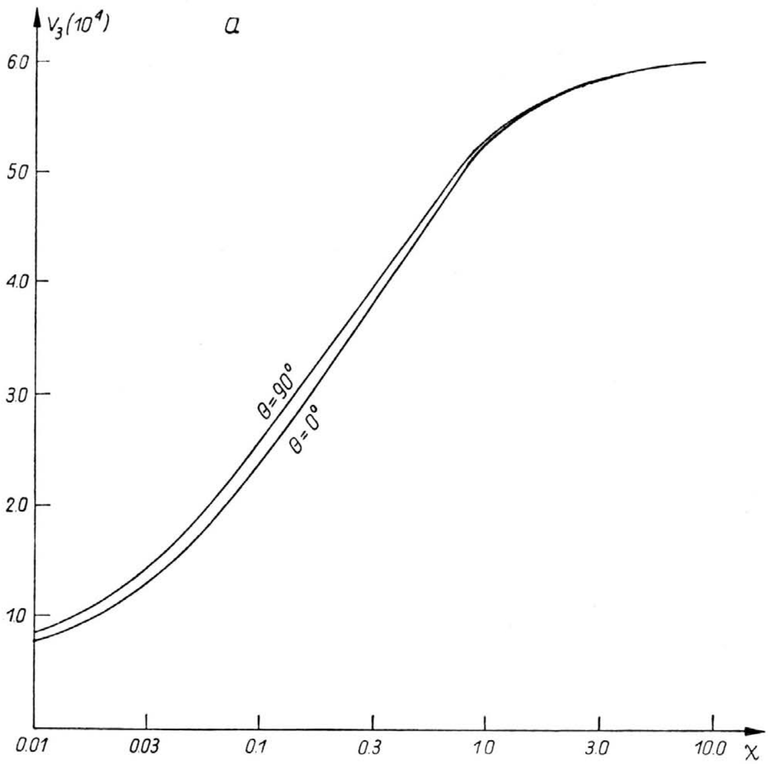


**Fig. 3a.** Variation of the phase velocity of the T-mode in NaF crystal with frequency and direction of pıopagation.

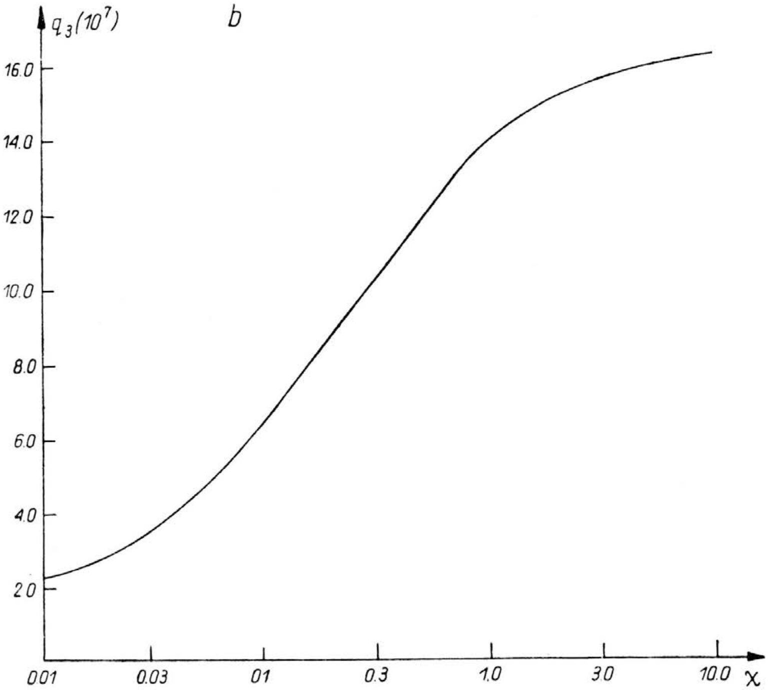


FIG. 3b. Variation of the attenuation coefficient of T-mode in NaF-crystal with frequency and direction of propagation.

dyne/cm², $C_{12} = 2.29 \times 19^{11}$ dyne/cm², $C_{44} = 2.899 \times 10^{11}$ dyne/cm², $\varrho = 2.851$ g/cm³, $\varrho C_e = 1.195 \times 10^5$ ergs/cm², $\alpha = 8.5 \times 10^{-7}$/°K. $\beta = 235$ watt/cm°K, $\tau_0 = 5.6 \times 10^{-14}$ s. The velocities of various waves corresponding to Eqs. (3.12), (3.15), (3.19) and (3.24) have been computed for various directions of propagations. The variations of velocity ratios for elastic waves have been plotted on log-linear scales in Figs. 1 and 2 for various directions of propagation with respect to reduced frequency $\chi$. The phase velocity $V_3$ of thermal wave is represented in Fig. 3a. The attenuation coefficients $q_1$, $q_2$ and $q_3$ of elastic and thermal waves have also been computed and plotted in Figs. 1b, 2b and 3b, respectively. It is observed that the velocity ratios and attenuation coefficients of elastic waves increase from their isentropic values for $0 \leqslant \chi \leqslant 3$, and then decrease to become closer to the isothermal values for $0.3 \leqslant \chi \leqslant 1.0$, and remain constant afterwards. The phase velocity and attenuation coefficient of the thermal wave are found to be weakly dependent upon the directional variations. The waves propagating in the neighbourhood of the direction making an angle of $\pi/6$ with the $x_3$-axis are less affected than those propagating in other directions. The longitudinal elastic waves are subjected to strong modifications as compared to transverse elastic waves.

## 5. Conclusions

In general, there are four waves: quasi-longitudinal, two quasi-transverse, and quasi-thermal wave, which can propagate in a cubic crystal. When plane waves are propagating along the axis of cubic crystal, then only longitudinal wave and thermal wave are coupled, whereas the transverse waves get decoupled from the rest of the motion components and hence, remain purely transverse waves. The coupled waves are dispersive in character and their velocities depend upon the thermal relaxation time. For plane waves propagating in one of the plane of the crystal, only SH-wave remains purely transverse and get decoupled from the rest of the motion, and vice-versa. The other three waves, namely: quasi-longitudinal (QL), quasi-transverse (QT), and quasi-thermal (T-mode) are coupled and, hence, depend upon thermal variations and relaxation time. These are dispersive in character. Numerical results reveal that the thermal waves are weakly dependent on directional variations, and the elastic waves travelling along the direction making an angle of $\pi/6$ with $x_3$-axis are less affected than those propagating in other direction in a NaF crystal.

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