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On the compressibility of a spherical solid body

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A CRYSTALLINE sphere is subject to hydrostatic pressure or tension and to a uniform temperature field. A nonlocal phenomenological model of mechanical properties of the sphere is proposed. The model makes it possible to account for nonlocal phenomena occurring in the crystalline body due to the long-range binary interactions.

W pracy rozważana jest krystaliczna kula poddawana wszechstronnemu ciśnieniu lub ciągnieniu oraz działaniu jednorodnych temperatur. Zaproponowano nielokalny fenomenologiczny model własności mechanicznych tej kuli. Pokazano, że model ten pozwala na uwzględnienie nielokalnych zjawisk w krystalicznym ciele wynikających z wpływu dalekich binarnych oddziaływań.

В работе рассматривается кристаллический шар, подвергнутый всехстороннему давлению или растяжению, а также действию однородных температур. Предложена нелокальная феноменологическая модель механических свойств этого шара. Показано, что эта модель позволяет учитывать нелокальные явления в кристаллическом теле, вытекающие из влияния далеких бинарных взаимодействий.

1. Introduction

INVESTIGATION of compressibility of crystalline solid bodies subject to hydrostatic pressure is of considerable importance for determining their elastic properties. This is connected with the fact that under conditions of hydrostatic pressure it is possible to determine experimentally the elastic response of the body to the deformation

$$\mathbf{F} = \lambda \mathbf{1}, \quad \lambda > 0 \tag{1.1}$$

without producing any plastic deformations due to the lattice defects (e.g. [1, 2]). As a result, the macroscopic elastic response of the body determined by measuring the volume variation due to pressure changes is more closely connected with the character of atomic interactions in ideal crystals than in other types of deformations. This statement is confirmed for example by a fairly good experimental verification of certain formulae (like that of Gillman) for the bulk modulus K [3]:

$$K = \frac{3e^2}{4\pi r_0^4}. \tag{1.2}$$

Here e — electron charge, r_0 — atomic radius.

In the case of hydrostatic pressure leading to three-dimensional compression of the body (deformation (1.1) with $\lambda < 1$), the relation between the external pressure and the actual volume may easily be investigated by experimental methods (e.g. [1, 2, 4]). In contrast, all-round tension leading to expansion of the body (deformation (1.1) with $\lambda > 1$) cannot be produced easily and hence the relevant experimental data are not available. In view of this situation, use will be made of the results of the paper [5]. In that paper the curve describing the relations between pressure or tension and deformation (1.1) was found by means of numerical calculations based on the analysis of bond energy changes in a cubic crystal lattice subject to such deformation; in the range of pressures the results of the paper [5] coincide with the type of curve known from experiments. In the present paper the crystalline sphere (or cube) subject to three-dimensional pressure or tension will be considered as a body with the size effect and liquid-like response [6, 7]. Such a model will be shown to comply with the results of the paper [5].

2. Size effect

Let $\mathfrak{B}_0$ be a homogeneous material sphere of radius $R_0 > 0$ subject to hydrostatic pressure or tension and to temperatures in the range enabling us to disregard the dissipative phenomena. Let us assume the sphere to exhibit, in the states considered, isotropic mechanical and thermal properties (like e.g. in the case of crystals with cubic lattices — [8]). Denote by R^+ the set of positive real numbers and by $I \subset R^+$ — a certain interval of absolute temperatures. Under such assumptions the sphere $\mathfrak{B}_0$ may be considered as a thermodynamic system with states determined by ordered pairs $(\lambda, \theta) \in R^+ \times I$, with $\lambda > 0$ denoting deformation (1.1) of the sphere $\mathfrak{B}_0$ and $\theta > 0$ being the uniform temperature of the deformed sphere. Then the physical properties of the sphere $\mathfrak{B}_0$ may be described by means of three scalar functions of class $C^1(R^+ \times I)$: Ψ — free energy, E — internal energy and S — entropy. The functions are related to each other by the formulae

$$\Psi = E - \theta S,$$

$$S = -\frac{\partial \Psi}{\partial \theta}. \tag{2.1}$$

The functions depend also on the solid $\mathfrak{B}_0$ (cf. [6]); due to the assumed isotropy the dependence is reduced to the single parameter $R_0 > 0$.

The generalized thermodynamic force N acting on the sphere $\mathfrak{B}_0$ has the form (cf. [6])

$$N = N(R_0; \lambda, \theta) = -\frac{\partial \Psi}{\partial \lambda}(R_0; \lambda, \theta) = -V(\lambda) t(R_0; \lambda, \theta) \lambda^{-1} \tag{2.2}$$

with the notations

$$t(R_0; \lambda, \theta) = \frac{1}{V(\lambda)} \frac{\partial \Psi}{\partial \lambda}(R_0; \lambda, \theta) \lambda,$$

$$V(\lambda) = V_0 \lambda^3, \qquad V_0 = 4/3\pi R_0^3. \tag{2.3}$$

The tensor $\mathbf{T}(R_0; \mathbf{F}, \theta) = \frac{1}{3} t\,(R_0; \lambda, \theta)\,\mathbf{1}$ for $\mathbf{F} = \lambda\,\mathbf{1}$ is the Cauchy stress tensor in the actual configuration $\mathfrak{B}$ (being another sphere of radius $R = \lambda R_0$) of sphere $\mathfrak{B}_0$. In the language of the paper [6] the tensor describes "the isotropic ideal thermoelastic body $\mathfrak{B}_0$" with the *size effect* in the form of a relation between the mechanical properties of the body and its radius (cf. Eq. (2.3)).

The equation of temperature of the deforming body has the form (cf. [6])

$$K_\lambda(R_0;\theta)\dot{\theta} = V(\lambda)\theta\frac{\partial t}{\partial\theta}(R_0;\lambda,\theta)d+Q(R_0;t),$$
$$V(\lambda) = V_0\lambda^3, \quad d = \dot{\lambda}\lambda^{-1}, \tag{2.4}$$

$K_\lambda(R_0;\theta)$ being the heat capacity at a constant deformation, i.e. (cf. [6, 8])

$$K_\lambda = \frac{\partial E}{\partial\theta} = \theta\frac{\partial S}{\partial\theta} = -\theta\frac{\partial^2\psi}{\partial\theta^2} \tag{2.5}$$

and $Q(R_0;t)$ is a heating. The thermodynamic processes considered here are reversible, and so $Q = \theta\dot{S}$. The functions K_λ and Q are assumed to be known from elsewhere.

If a homogeneous sphere $\mathfrak{B}_0$ of mass $m > 0$ is acted on by a time-dependent external pressure or tension $\hat{p} = \hat{p}(t)$ and the generalized thermodynamic force $N(R_0;\lambda,\theta)$ then, disregarding the body forces, the dynamics of the process of deformation of the sphere is characterized by the equation (cf. [6])

$$J_0\ddot{\lambda} = \frac{1}{3}N(R_0;\lambda,\theta)+V_0\hat{p}(t),$$
$$J_0 = \frac{1}{5}mR_0^2, \quad \lambda(0) = 1. \tag{2.6}$$

Here $3J_0$ denotes the moment of inertia of the sphere $\mathfrak{B}_0$ with respect to its mass center.

Equations (2.2), (2.4) and (2.6) constitute a closed system describing the time variation of deformation and temperature of $\mathfrak{B}_0$ with the size effect.

3. Spherical body

A homogeneous thermodynamical system with states determined by ordered pairs (R,θ), $R > 0$ being the radius of a homogeneous material sphere of fixed mass $m > 0$, and θ — its uniform absolute temperature, will be called a *spherical body* of mass m. The sphere $\mathfrak{B}$ of radius R will be called the *actual configuration* of a spherical body. A distinguished sphere of radius R_0 will be called the *reference configuration* of a spherical body. The reference configuration $\mathfrak{B}_0$ of a spherical body may be considered as a thermodynamical system the states of which are determined by pairs $(\lambda,\theta)\in R^+\times I$, where $\lambda = R/R_0$ is the deformation (1.1) due to passing from configuration $\mathfrak{B}_0$ to the actual configuration $\mathfrak{B}$ of the spherical body. From the definition of spherical bodies it follows that if $\Phi = \Phi(R,\theta)$ is the free energy function of a spherical body and $\Psi = \Psi(R_0;\lambda,\theta)$ is the free energy function of its reference configuration considered as a thermodynamical system, then

$$\forall R_0 > 0, \quad \forall(\lambda,\theta)\in R^+\times I, \quad \Psi(R_0;\lambda,\theta) = \Phi(R_0\lambda,\theta). \tag{3.1}$$

An analogous condition is satisfied by the functions $E = E(R_0;\lambda,\theta)$ and $S = S(R_0;\lambda,\theta)$ (cf. Eq. (2.1)).

Let $\mathfrak{B}_0$ be the reference configuration of a spherical body and $d\varkappa$ — the relative change of actual volume $V = V(\lambda)$ of that body (cf. Eq. (2.3)),

$$d\varkappa = \frac{dV}{V}, \quad V(1) = V_0, \tag{3.2}$$

then

$$\varkappa = \ln\frac{V}{V_0} = 3\varepsilon, \quad \varepsilon = \ln\lambda. \tag{3.3}$$

Let us denote

$$\begin{aligned} &3\sigma(R_0; \varkappa, \theta) = t(R_0; e^{1/3\varkappa}, \theta), \\ &K_\theta(R_0; \lambda) = \frac{\partial\sigma}{\partial\varkappa}(R_0; 3\ln\lambda, \theta), \quad \theta = \text{const}. \end{aligned} \tag{3.4}$$

The function $K_\theta = K_\theta(R_0; \lambda)$, $\lambda \in R^+$ depending on the parameters R_0 and θ will be called the *isothermal compressibility of configuration* $\mathfrak{B}_0$ of a spherical body. Consider the case when, in a certain interval of temperatures $I \subset R^+$, the condition is satisfied:

$$\forall\theta \in I, \quad \overset{1}{\exists} R_0(\theta) > 0, \quad t(R_0(\theta); 1, \theta) = 0. \tag{3.5}$$

Configuration $\mathfrak{B}_\theta$ of radius $R_0(\theta)$ satisfying the condition (3.5) will be called the *natural configuration* of a spherical body at temperature θ. Observations of the behaviour of solids and liquids indicate that the existence of stress-free configurations is a characteristic feature of solids, while liquids do not assume such configurations (cf. [10]). Once the condition (3.5) is satisfied, the spherical body may be considered as a model describing the properties of a certain solid body. Let us denote

$$\begin{aligned} &3\sigma_\theta(\mu) = t(R_0(\theta); \mu, \theta), \\ &\nu(R_0; \theta) = \frac{R_0(\theta)}{R_0}. \end{aligned} \tag{3.6}$$

From Eqs. (2.3) and (3.1) it follows that

$$3\sigma_\theta(\mu) = t(R_0; \mu\nu(R_0; \theta), \theta). \tag{3.7}$$

Equation (3.7) enables us to reduce the analysis of mechanical properties of a spherical body to the analysis of stress functions for the natural configurations of that body. For instance, observations of the behaviour of real material bodies make it possible to assume the following postulate (cf. [10]):

Postulate of compressibility: In the process of deformation of a natural configuration, increasing volume requires tensile stresses, and decreasing volume — compressive stresses.

The condition (3.5) and the compressibility postulate may be written in the form of a condition for $\sigma_\theta = \sigma_\theta(\mu)$,

$$\forall\theta \in I \quad (\sigma_\theta(1) = 0, \quad \forall o < \mu \neq 1 \quad \sigma_\theta(\mu)(\mu - 1) > 0). \tag{3.8}$$

In order to investigate the consequences of the conditions (3.5) and (3.8), let us consider the small relative change Δ of the volume of the natural configuration $\mathfrak{B}_\theta$, that is (cf. Eq.(3.3))

$$\varkappa = \Delta + o(\Delta^2), \quad |\Delta| \ll 1,$$

(3.9)
$$\Delta = \frac{V(\mu) - V_0(\theta)}{V_0(\theta)}, \quad V(\mu) = V_0(\theta)\mu^3, \quad V_0(\theta) = 4/3\pi R_0(\theta)^3.$$

Then

(3.10)
$$\sigma_\theta(\mu) = K(\theta)\Delta + o(\Delta^2),$$
$$\mu = 1 + \frac{1}{3}\Delta + o(\Delta^2),$$

with the notations

(3.11)
$$K(\theta) = K_\theta(R_0(\theta); 1).$$

From the conditions (3.5), (3.8) and formula (3.10) it follows that $K(\theta) > 0$ and is (at the given temperature $\theta \in I$) a uniquely determined quantity. This means that $K(\theta)$ may be considered as a physical characteristics of compressibility of a spherical body, the so-called *isothermal bulk modulus* (cf. [1, 4, 5]). The function

(3.12)
$$K_\theta(\mu) = K_\theta(R_0(\theta); \mu), \quad K_\theta'(1) > 0$$

will be termed *isothermal compressibility* of a spherical body at temperature $\theta \in I$.

Let $\mathfrak{B}_0$ be an arbitrary reference configuration of the spherical body. Thermodynamical stability of the configuration $\mathfrak{B}_0$ is defined by the following criteria (cf. [9, 11]):

(3.13)
$$\forall\theta \in I \quad K_\lambda(R_0; \theta) > 0 \quad \text{(thermal stability at } \lambda = \text{const)}$$

and

(3.14)
$$\forall\lambda \in R^+ \quad K_\theta(R_0; \lambda) > 0 \quad \text{(mechanical stability at } \theta = \text{const).}$$

Here $K_\lambda(R_0; \theta)$ is the heat capacity (Eq. (2.5) and $K_\theta(R_0; \lambda)$ — the isothermal compressibility (Eq. (3.4)) of configuration $\mathfrak{B}_0$. A spherical body will be called *stable* if its arbitrary reference configuration is thermally and mechanically stable, and *mechanically unstable* if its arbitrary reference configuration is thermally stable and is not mechanically stable at any temperature $\theta \in I$.

4. Liquid-like response of a spherical body

The spherical body will be called to possess the liquid-like response (using the results of [7] in the form of a postulate), if its free energy function Φ is of the form

(4.1)
$$\Phi(R, \theta) = a(\theta)V(R) + b(\theta)F(R) + c(\theta)M(R) + d(\theta),$$
$$V(R) = \frac{4}{3}\pi R^3, \quad F(R) = 4\pi R^2, \quad M(R) = 4\pi R.$$

The symbols $V(R)$, $F(R)$ and $M(R)$ denote, respectively: volume of the sphere of radius R, area of the spherical surface and the total mean curvature of the surface (cf. [7, 13]). The stress function of a spherical body with a liquid-like response is given by the formula

$\mathbf{T}(R_0; \lambda, \theta) = \frac{1}{3}\, t\,(R_0; \lambda, \theta)\,\mathbf{1}$, where (cf. formula (2.3) and (3.1))

$$t(R_0; \lambda, \theta) = 3a(\theta) + \frac{6b(\theta)}{R_0}\lambda^{-1} + \frac{3c(\theta)}{R_0^2}\lambda^{-2}. \tag{4.2}$$

The isothermal compressibility of configuration $\mathfrak{B}_0$ has the form (cf. Eq. (3.4))

$$K_\theta(R_0; \lambda) = -\frac{2}{3\lambda R_0^2}[b(\theta)R_0 + c(\theta)\lambda^{-1}]. \tag{4.3}$$

From (4.2) and the condition (3.8) it follows that

$$\forall \theta \in I: \; a(\theta) \geqslant 0, \quad c(\theta) \leqslant 0. \tag{4.4}$$

From (4.4) it follows that at an arbitrary temperature $\theta \in I$ there exists at most one reference configuration, and its radius $R_0(\theta)$ is determined by one of the following formulae:

$$R_0(\theta) = \frac{1}{a(\theta)}[-b(\theta) + \sqrt{b^2(\theta) - a(\theta)c(\theta)}] \quad \text{if} \quad a(\theta) > 0, \quad c(\theta) < 0,$$

$$R_0(\theta) = -\frac{1}{2}\frac{c(\theta)}{b(\theta)} \quad \text{if} \quad a(\theta) = 0, \quad b(\theta) > 0, \quad c(\theta) < 0, \tag{4.5}$$

$$R_0(\theta) = -2\frac{b(\theta)}{a(\theta)} \quad \text{if} \quad c(\theta) = 0, \quad b(\theta) < 0, \quad a(\theta) > 0.$$

In the remaining particular cases of the condition (4.4) the natural configuration does not exist.

Formulae (3.14) and (4.3) yield the conclusion that a spherical body with a liquid-like response is mechanically stable (in arbitrary temperatures $\theta \in I$ and for arbitrary reference configuration $\mathfrak{B}_0$) if and only if

$$\forall \theta \in I: \; b(\theta) \leqslant 0, \quad c(\theta) \leqslant 0, \quad b(\theta) + c(\theta) \neq 0. \tag{4.6}$$

From (4.4)–(4.6) it follows that for a stable spherical body with a liquid-like response, the following conditions should be satisfied:

$$\forall \theta \in I: \; a(\theta) > 0, \quad b(\theta) \leqslant 0, \quad c(\theta) \leqslant 0, \quad b(\theta) + c(\theta) \neq 0. \tag{4.7}$$

It also follows from the conditions (4.4)–(4.6) that in the case of a mechanically unstable spherical body with a liquid-like response, the following conditions should hold true:

$$\forall \theta \in I: \; a(\theta) \geqslant 0, \quad b(\theta) > 0, \quad c(\theta) < 0. \tag{4.8}$$

The stress function $\frac{1}{3} t\,(R_0; \lambda, \theta)$ of stable spherical bodies with a liquid-like response is a monotonously increasing function of the argument $\lambda \in R^+$, and

$$\lim_{\lambda \to \infty} t(R_0; \lambda, \theta) = 3a(\theta) > 0, \quad \lim_{\lambda \to 0} t(R_0; \lambda, \theta) = -\infty \tag{4.9}$$

what means that (at a given temperature $\theta \in I$) an arbitrary configuration of the body is able to carry unbounded pressures, but only a bounded hydrostatic tension. Figure 1 presents the graphs of stresses $\sigma_\theta(\mu)$ (cf. (3.6), (4.2), (4.5)) and isothermal compressibility $K_\theta(\mu)$ (cf. (3.12), (4.3), (4.5)) of a stable spherical body with a liquid-like response (cf. [12]).

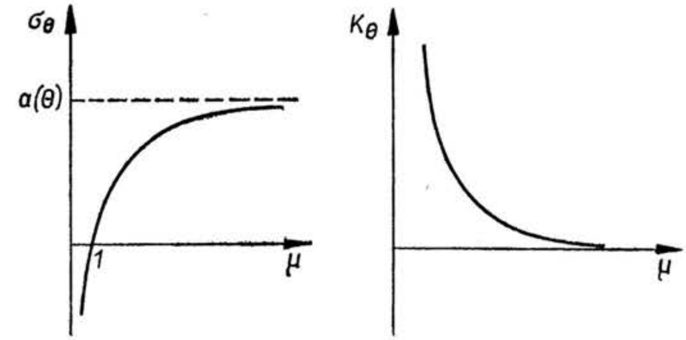

FIG. 1.

5. Mechanically unstable model of a crystalline sphere

Let us denote by $t_\theta(R_0; \lambda) = \dfrac{1}{3} t\,(R_0; \lambda, \theta)$ the function of isothermal stresses of a mechanically unstable spherical body with liquid-like response (formula (4.2) with the conditions (4.8)). Its graph has two characteristic points: a maximum (λ_p, t_p) and a saddle point (λ_r, t_r), with (cf. [12])

$$\text{(5.1)}\qquad \begin{aligned} \lambda_p &= \lambda_p(R_0; \theta) = -\frac{1}{R_0}\frac{c(\theta)}{b(\theta)}, \\ t_p &= t_p(\theta) = a(\theta) - \frac{b^2(\theta)}{c(\theta)}, \end{aligned}$$

$$\text{(5.2)}\qquad \begin{aligned} \lambda_r &= \lambda_r(R_0; \theta) = \frac{3}{2}\lambda_p(R_0; \theta), \\ t_r &= t_r(\theta) = a(\theta) - \frac{8}{9}\frac{b^2(\theta)}{c(\theta)}. \end{aligned}$$

If $R_0 = R_0(\theta)$ is the radius of the natural reference configuration at temperature $\theta \in I$, then the function $\sigma_\theta(\mu) = t_\theta[R_0(\theta); \mu]$ (cf. the formulae (3.6), (4.2), (4.5), (4.8)) posseses the characteristic points (μ_p, t_p) and (μ_r, t_r) where

$$\text{(5.3)}\qquad \begin{gathered} \mu_p = \mu_p(\theta) = \lambda_p(R_0(\theta); \theta), \quad \mu_r = \mu_r(\theta) = \lambda_r(R_0(\theta); \theta), \\ \mu_r(\theta) > \mu_p(\theta) > 1. \end{gathered}$$

In the interval $(0, \mu_p(\theta))$ the function of isothermal stresses $\sigma_\theta = \sigma_\theta(\mu)$ monotonously increases, while the function of isothermal compressibility $K_\theta = K_\theta(\mu)$ decreases. For $\mu = \mu_p(\theta)$ the stress attains its maximum value, and compressibility vanishes. Graphs of these functions are given in Fig. 2 (cf. [12]).

From formulae $(5.1)_1$ and (5.3) it follows that

$$\text{(5.4)}\qquad R_0(\theta) = -\frac{1}{\mu_p(\theta)}\frac{c(\theta)}{b(\theta)}.$$

and the formulae (3.11), (4.3), (5.1) and (5.4) yield the results

(5.5)
$$3K(\theta) = -2\frac{b^2(\theta)}{c(\theta)}\mu_p(\theta)(\mu_p(\theta)-1) > 0,$$
$$\frac{b^2(\theta)}{c(\theta)} = a(\theta)-t_p(\theta) < 0.$$

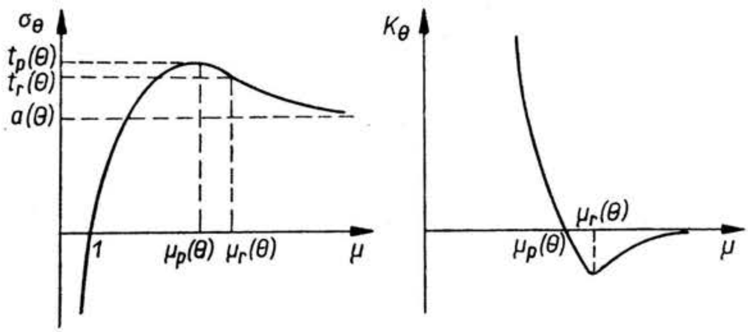

FIG. 2.

If $\mathfrak{B}_0$ is an arbitrary reference configuration, then, taking into account that $t_\theta(R_0; \lambda) = \sigma_\theta[\lambda\nu^{-1}(R_0;\theta)]$ (cf. (3.7)) and making use of formulae (5.4) and (5.5), we obtain $\mathbf{T}(R_0; \lambda, \theta) = t_\theta(R_0; \lambda)\,\mathbf{1}$ where

(5.6)
$$t_\theta(R_0;\lambda) = t_p(\theta) - \frac{3K(\theta)}{2\mu_p(\theta)(\mu_p(\theta)-1)}\left[1-\mu_p(\theta)\nu(R_0;\theta)\frac{1}{\lambda}\right]^2.$$

The formula (5.6) and the conditions (3.5) and $t_p(\theta) > 0$ yield

(5.7)
$$t_p(\theta) = \frac{3}{2}\left(1-\frac{1}{\mu_p(\theta)}\right)K(\theta).$$

In the isothermal processes in which the temperatures of the process and of the natural configurations coincide, the stress has the form

(5.8)
$$\sigma_\theta(\lambda) = t_\theta(R_0(\theta);\lambda) = t_p(\theta) - \frac{3K(\theta)}{2\mu_p(\theta)(\mu_p(\theta)-1)}\left[1-\mu_p(\theta)\frac{1}{\lambda}\right]^2.$$

The paper mentioned in the introduction [5] dealt with the deformation (1.1) of a cube (basic cell of crystal lattices), and not that of a sphere as here. The difference is, however, not of a considerable importance since the functions of stress and isothermal compressibility for a cube may be reduced to those for a sphere in the following manner: let us denote by $V(l)$, $F(l)$ and $M(l)$ the volume of a cube with edges l, the area of its lateral surface, and the total mean curvature of the surface, respectively, i.e. (cf. [13])

(5.9)
$$V(l) = l^3, \quad F(l) = 6l^2, \; M(l) = 3\pi l.$$

The free energy function $\Phi(l, \theta)$ describing of the liquid-like response of a "cubical body" has the form (cf. [7])

(5.10)
$$\Phi(l,\theta) = \alpha(\theta)V(l)+\beta(\theta)F(l)+\gamma(\theta)M(l)+\delta(\theta).$$

By putting $\alpha = a$, $\beta = 1/2b$, $\gamma = 1/\pi$, $\delta = d$ in (5.10), the description of the stress response and compressibility may be reduced, in the case of reference configuration having the form of a cube with edge l_0 and material coefficients $\alpha(\theta)$, $\beta(\theta)$, $\gamma(\theta)$, $\delta(\theta)$, to the description of the reference configuration in the spherical form with the radius $R_0 = l_0$ and the material coefficients $a(\theta)$, $b(\theta)$, $c(\theta)$, $d(\theta)$.

In paper [5] three types of cubic crystal lattices have been considered: primitive, body-centered and face-centered. Central binary interactions with Morse type potential (and natural reference configuration temperature 0°K) were assumed. It was shown that if the crystal lattice is subject to deformation (1.1), then the graphs of isothermal stress functions and of isothermal compressibility are, for $\mu \in (0, \mu_0)$, $\mu_p < \mu_0 < \mu_r$, of the type shown in Fig. 2. No results are given for the interval (μ_0, ∞). It is also shown that if the potential describing central binary interactions is of the arbitrary type but takes into account the interactions of immediate neighbours only, then the deformation μ_p of the change of sign of the isothermal compressibility function is the same for all types of lattices considered; if the potential is of the Morse type, the common value of μ_p is $\mu_p = 1.09120$ (at temperature 0°K). Taking into account the effects of all nodes of the lattice and using the Morse potential, the value of μ_p for the types of lattices considered here are close to $\mu_p = 1.09120$: $\mu_p = 1.09167$, 1.09171 and 1.09199 in the face-centered, body-centered and primitive lattices, respectively.

The results obtained make it possible to identify the deformation μ_p as a material parameter characterizing, within the range of central binary interactions, the response to hydrostatic tension of a crystalline body of the cubic lattice. The results of the paper [5] enable us to propose the mechanically unstable spherical body with a liquid-like response as a nonlocal phenomenological model of mechanical properties of a crystalline sphere of cubic lattice subject to the deformation (1.1). The model takes into account the nonlocal phenomena occurring in crystalline bodies as a result of long-range binary interactions. From formulae (5.7) and (5.8) it follows that if the reference configuration is stress-free, than, under isothermal conditions, the mechanical properties of a crystalline sphere (or cube) of a cubical lattice are determined by two (isothermal) material constants: the bulk modulus $K(\theta)$ and the stability loss deformation $\mu_p(\theta)$. The bulk modulus $K(\theta)$ is a parameter which has been repeatedly determined both experimentally and theoretically (e.g. (1.2)). Paper [5] suggests the possibility of theoretical evaluation of $\mu_p(\theta)$. Once the values of $K(\theta)$ and $\mu_p(\theta)$ are known, formulae (5.7) and (5.8) make it possible to verify the model experimentally, for instance on the basis of voluminal methods of measurement of compressibility of a sphere loaded by hydrostatic pressure under isothermal conditions.

References

1. A. H. COTTRELL, *The mechanical properties of matter*, John Wiley and Sons, New York 1964.
2. И. И. ГОЛЬДЕНБЛАТТ, *Нелинейные проблемы теории упругости*, Наука, Москва 1969.
3. R. ŻUCHOWSKI, *Influence of various agents on the values of elasticity moduli* [in Polish], Prace Nauk. Pol. Warsz., 24, 1975.

4. П. М. Огибалов, И. А. Кийко, *Очерки по механике высоких параметров*, Изд. Моск. Унив., Москва 1966.
5. F. M. Milstein, R. Hill, *Theoretical properties of cubic crystals at arbitrary pressure, I*, Mech. Phys. Solids, **25**, 1977.
6. A. Trzęsowski, *Size effect, Part I and II*, Bull. Acad. Polon. Sci., Série Sci. techn., 27, 5/6, 1979.
7. A. Trzęsowski, *On the liquid-like response of the size effect body*, Bull. Acad. Polon. Sci., Série Sci. techn., **28**, 2, 1980.
8. G. Shulze, *Metallphysik*, Akademie-Verlag, Berlin 1967.
9. A. C. Компаниец, *Курс теоретической физики*, т. II, Просвещение, Москва 1975.
10. C. Truesdell, *A first course in rational continuum mechanics*, Hopkins Univ., Baltimore 1972.
11. P. Glansdorff, I. Prigogine, *Thermodynamic theory of structure, stability and fluctuations*, Wiley — Interscience [Russian transl.].
12. I. N. Bronštein, K. A. Semendyaev, *Mathematics*. Encyclopedic Adviser [in Polish, transl. from Russian], PWN, Warszawa 1956.
13. H. Hadwiger, *Altes und Neues über konwex Körper*, Birkhäser Verlag, Basel 1955.

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