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A boundary method applied to elastic scattering problems

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A BOUNDARY method for solving some elastic scattering problems is presented. The scattered field is represented in terms of a linear combination of wave functions which are particular solutions of the governing equations. Green's functions with singularities or sources located outside the region of interest have been chosen. Coefficients of these sources are determined such that, for each given frequency of excitation, boundary conditions at the interface the scatterer and the rest of the region are satisfied in the least-squares sense. The scatterer can be a cavity or an inclusion. For elastic inclusions an interior problem arises and thus the refracted field is obtained using exterior sources. Results are presented for the problem of scattering and diffraction of harmonic *SH* waves by cavities or canyons in an elastic half-space. Comparison with known analytical and numerical solutions yields very good agreement.

Przedstawiono zastosowania metody brzegowej do analizy problemów rozpraszania sprężystego. Pole rozproszone przedstawia się za pomocą liniowej kombinacji funkcji falowych będących rozwiązaniami szczególnymi równań rozważanego problemu. Wykorzystano funkcje Greena z osobliwościami lub źródłami znajdującymi się poza rozpatrywanym obszarem. Intensywności tych źródeł ustalono w ten sposób, by dla każdej częstości wymuszenia warunki brzegowe na powierzchniach rozdziału między elementami rozpraszającymi a resztą obszaru spełnione były w sensie warunku najmniejszych kwadratów. Czynnikiem rozpraszającym może być pustka lub inkluzja. W przypadku inkluzji pojawia się problem wewnętrzny i odpowiednie pole fal załamanych otrzymać można za pomocą źródeł zewnętrznych. Rozwiązania przedstawiono dla zagadnienia rozproszenia i załamania harmonicznych fal ścinania *SH* na pustkach kulistych i cylindrycznych w półprzestrzeni sprężystej. Porównanie ze znanymi rozwiązaniami analitycznymi i numerycznymi pozwala stwierdzić bardzo dobrą zgodność wyników.

Представлены применения граничного метода для анализа задач упругого рассеяния. Рассеянное поле представляется при помощи линейной комбинации волновых функций, будучих частными решениями уравнений рассматриваемой задачи. Использованы функции Грина с особенностями или источниками, находящимися вне рассматриваемой области. Интенсивности этих источников установлены таким образом, чтобы для каждой частоты вынуждения граничные условия, на поверхностях раздела между рассеивающими элементами и остальной частью области, были в смысле условия наименьших квадратов. Рассеивающим фактором может быть пустота или включение. В случае включения появляется внутренная задача и соответствующее поле преломленных волн можна получить при помощи внешних источников. Решения представлены для задач рассеяния и преломления гармонических волн сдвига SH на сферических и цилиндрических пустотах в упругом полупространстве. Сравнение с известными аналитическими и численными решениями позволяет констатировать очень хорошее совпадение результатов.

1. Introduction

SOME problems in earthquake engineering and seismology can be formulated as problems of scattering and diffraction of elastic waves. It is of interest to know the surface motion at a given site due to incoming and scattered seismic waves in order to assess the potential ground motion. Local geology and topography should be taken into account [16, 29]. Other problems involve the dynamic stress concentrations at cavities or other scatterers.

Many problems of scattering and diffraction of elastic waves have been solved using suitable wave functions when the geometry permits use of separation of variables. The excellent monography by Mow and Pao on the subject presents many such solutions for infinite domains [26]. The same method of separation of variables has been applied to solve the scattering and diffraction of horizontally-polarized harmonic *SH* waves by semi-circular and semi-elliptical cylindrical canyons and alluvial deposits on the surface of an elastic half-space [42, 43, 45, 46].

For arbitrarily-shaped cavities and incidence of compressional *P* waves an integral formulation has been used to solve the steady state response [7]. The same approach has been used to calculate the scattering and diffraction of harmonic *SH* waves by canyons of arbitrary shape and has been appllied to study the effects of the topography on ground motion due to the San Fernando earthquake of 1971 [44].

A formulation in terms of a singular Freedholm integral equation of the second kind has been used to solve the problem of scattering of *SH*-waves by ridges or other surface irregularities in an elastic half-space [38].

For surface scatterers some approximate solutions have been obtained. Using asymptotic expansions, a solution for arbitrary shaped scatterers has been obtained [32]. This approximation is valid for long wavelengths. Other methods assume a scatterer with small slopes [20, 24] and/or periodic repetition of the scatterer [2, 9, 28]. Finite difference and finite element methods have been used [4, 5, 8, 21, 30] defining artificial boundaries which may introduce spurious waves. This difficulty may be overcome by using the so-called efficient-active boundaries [4].

In recent years boundary methods have gained increasing popularity [3, 6, 11, 12, 27]. This fact is due mainly to the availability of high speed computers. Moreover, in many problems the reduction of the dimensionality by one leads to considerable economy in numerical work.

A boundary method which employs multipole expansions of Hankel functions as base functions has recently been developed and applied to solve the scattering of *SH*-waves by surface cavities [15, 31] and alluvial valleys [30]. The coefficients of the expansions were obtained by least-squares treatment of boundary conditions.

Herrera in his recent work [17, 18] has developed a general theory of connectivity. This theory appears to be a powerful tool to construct base functions [19].

A recent survey [47] shows how boundary methods can be used in a finite element context. A method which is a combination of the finite element method and the boundary element method has been presented [6]. Numerical improvements have been found by means of the use of Galerkin's method.

In this paper a boundary method is presented for solving the scattering and diffraction of harmonic elastic waves by cylindrical scatterers of arbitrary shape. For infinite exterior domains the formulation is straightforward. This is also the case for antiplane disturbances or *SH* waves in an elastic half-space.

The method makes use of the superposition principle. The scattered field is represented in terms of linear combinations of wave functions which are in turn particular solutions of the governing equations. Green's functions with singularities or sources located outside the region of interest have been chosen. The idea is similar to COPLEY's [13] for the Weber equation and has been applied by De Mey [14] to the solution of Laplace's interior problem. The singularities which appear in the classical theory of singular integral equations [41] are avoided in this treatment. The coefficients of the sources are determined such that, for each given frequency of the excitation, boundary conditions at the interface between the scatterer and the rest of the region are satisfied in the least-squares sense.

Millar [25] has shown for a related problem that the least-squares criterion leads to a representation which converges uniformly in the mean to the solution of the problem, provided a complete set of functions is chosen.

The method has been applied to solve scattering and diffraction of *P*, *SV* and *SH* waves by canyons [33, 34, 35] as well as ground motion on alluvial valleys [37] and dynamic stress concentrations on underground cavities [36] for incident *SH* waves.

The method is presented in detail for the antiplane case, the formulation for the in-plane problem follows in parallel to that presented here.

2. Basic equations

In the propagation of elastic *SH* waves the antiplane displacement w in the z-axis direction satisfies the wave equation [1]

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\frac{1}{c_s^2}\frac{\partial^2 w}{\partial t^2}, \tag{2.1}$$

where $c_s=\sqrt{\mu/\varrho}$ — propagation velocity of S waves, μ — shear modulus, ϱ — medium density, and t — time.

For harmonic waves with time dependence given by $\exp(i\omega t)$, where ω — circular frequency, and $i=\sqrt{-1}$, Eq. (2.1) becomes the reduced wave or Helmholtz equation

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+k_s^2 w=0, \tag{2.2}$$

where $k_s=\omega/c_s$ — the shear wave number.

The non-zero components of the stress tensor are

$$\tau_{zx}=\mu\frac{\partial w}{\partial x},\qquad \tau_{zy}=\mu\frac{\partial w}{\partial y}. \tag{2.3}$$

The traction vector $\mathcal{T}$ in surfaces parallel with the z-axis has only the z-component which is given by

$$T_z=\mu\frac{\partial w}{\partial n}, \tag{2.4}$$

where $\mathbf{n}$ — vector normal to a surface parallel with the z-axis.

3. The excitation

For the sake of simplicity we will restrict our treatment to incident plane waves. Let an incident field of unitary amplitude be given by

$$w^{(i)} = \exp i\omega \left(t - \frac{x}{c_s} \sin\gamma + \frac{y}{c_s} \cos\gamma \right), \tag{3.1}$$

where γ — angle of incidence (Fig. 1).

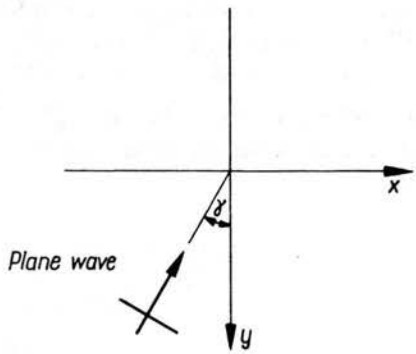

FIG. 1. Incident plane wave, free-field solution.

In an infinite medium the incident wave-field will be termed as the free-field solution; that is, the solution in the absence of the scatterer. For half-space the free-field can be constructed superposing the reflected plane waves in such a way that the plane free surface will have zero traction.

4. The scattered field

Let us consider a scatterer occupying the region R as shown in Fig. 2. Let E be the exterior domain, S the common boundary, and $\mathbf{n}$ the inward normal unit vector. The scatterer can be a cavity or an elastic or rigid inclusion. We will present below the appropriate boundary conditions.

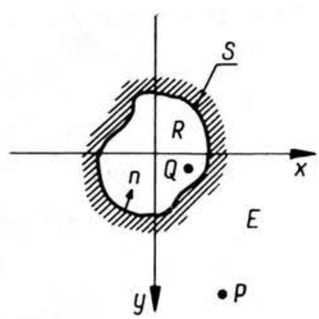

FIG. 2. Definition of regions E and R.

The exterior field can be written as

$$w = w^{(0)} + w^{(s)}, \tag{4.1}$$

where $w^{(0)}$ — free-field solution ($w^{(0)} = w^{(i)}$ for infinite domains), and $w^{(s)}$ — scattered field. The displacement $w^{(s)}$ must be a solution of Eq. (2.2) in E, fulfilling SOMMERFELD's radiation condition [40]

$$\lim_{r \to \infty} r^{1/2} \left(\frac{\partial w^{(s)}}{\partial r} + ik_s w^{(s)} \right) = 0. \tag{4.2}$$

This condition means that the energy which is radiated from the sources must scatter to infinity; no energy may be radiated from infinity into the prescribed singularities of the field [40].

Let $w^{(s)}$ be of the form

$$w^{(s)}(P) = \sum_{m=1}^{M} \alpha_m G(P, Q_m), \tag{4.3}$$

where M — number of sources of SH-waves, α_m — complex constants to be determined from boundary conditions, and $G(P, Q_m)$ is the Green's function, i.e. it holds that

$$\nabla^2 G(P, Q_m) + k_s^2 G(P, Q_m) = -\delta(\overline{PQ_m}), \tag{4.4}$$

where $\delta(\cdot)$ is the Dirac delta, $\overline{PQ_m}$— distance between the points P and Q_m, P is a point in E or S, and Q_m is a point in R. The solution of Eq. (4.4) which satisfies the radiation condition is given by

$$G(P, Q_m) = \frac{i}{4} H_0^{(2)}(k_s r_m) e^{i\omega t}, \tag{4.5}$$

where $H_0^{(2)}(\cdot)$ — Hankel function of the second kind and order zero, and $r_m = \overline{PQ_m}$. Equation (4.5) represents cylindrical SH-waves propagating radially from Q_m with velocity c_s. This fact becomes evident if we use the asymptotic representation of the Hankel function. Thus, for large values of $k_s r_m$ we have

$$G(P, Q_m) \sim \frac{i}{4} \sqrt{\frac{2}{\pi k_s r_m}} \, e^{-i(k_s r_m - \omega t - \pi/4)}. \tag{4.6}$$

For computations the points Q_m will be located equally spaced on curves which should have approximately the same shape as the boundary. This choice appears to be reasonable for smooth geometries.

The Green's function for a half-space is immediately obtained by applying the method of images [1]. Thus the scattered field satisfies also the free-boundary condition on the half-space's surface and the solution procedure needs to consider only the interface between the scatterer and the rest of the half-space. Surface and buried scatterers are shown in Fig. 3. Figure 4 shows how the method of images is used to construct the scattered field.

Obtaining solutions for sources of P or SV waves in a half-space is a difficult task which requires solving complicated integrals [22, 23]. Their numerical handling requires

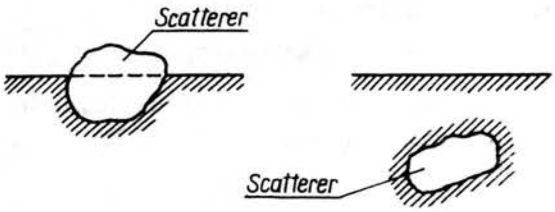

FIG. 3. Surface and buried scatterers in a half-space.

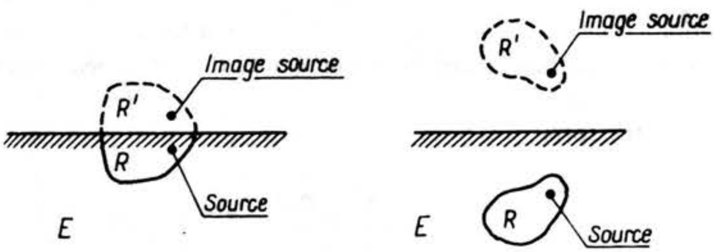

FIG. 4. Application of the method of images to construct the scattered field in a half-space. The image source is located in an image domain R'.

considerable computation. Approximate results can be obtained for such a class of diffraction problems using, in addition to sources, homogeneous and inhomogeneous plane waves in order to satisfy the boundary conditions on the surface of the half-space [10, 33].

5. Boundary conditions

Let $\mathcal{T}$ be the traction vector associated to a point and to a plane with unit normal $\mathbf{n}$, and $\boldsymbol{\delta}$ the displacement at the same point. In general we can write

(5.1) $$\mathcal{T} = \mathcal{T}^{(0)} + \mathcal{T}^{(s)}$$

and

(5.2) $$\boldsymbol{\delta} = \boldsymbol{\delta}^{(0)} + \boldsymbol{\delta}^{(s)}.$$

It is clear that the first term in Eqs. (5.1) and (5.2) stands for the free-field solution while the second represents the scattered field.

If the scatterer is a cavity, we have

(5.3) $$\mathcal{T} = \mathcal{T}^{(0)} + \mathcal{T}^{(s)} = 0, \quad \text{at} \quad S.$$

For an elastic inclusion, conditions that ought to be satisfied are

(5.4) $$(\mathcal{T})_E = (\mathcal{T})_I, \quad \text{at} \quad S$$

and

(5.5) $$(\boldsymbol{\delta})_E = (\boldsymbol{\delta})_I, \quad \text{at} \quad S,$$

to have continuity of stresses and displacements. The subscripts E and I mean the exterior and interior solutions, respectively.

For rigid inclusions, rigid body displacements should be imposed as conditions on the boundary. If the mass of the scatterer is considered, additonal equilibrium considerations are needed [26].

6. The interior problem

Let us restrict ourselves to treat the case of an elastic inclusion.

The interior solution or refracted field can be constructed in a manner analogous to the exterior one.

Consider the refracted displacement with the form

$$w^{(r)}(P) = \sum_{n=1}^{N} \beta_n G(P, Q_n^+), \tag{6.1}$$

where β_n — coefficients of the *exterior* sources which are given by

$$G(P, Q_n^+) = \frac{i}{4} H_0^{(2)}(k_{s_I} r_n^+) e^{i\omega t}, \tag{6.2}$$

where k_{s_I} is the shear wave-number in the interior domain, and r_n^+ is the distance between the exterior point Q_n^+ and the interior point P as can be seen in Fig. 5.

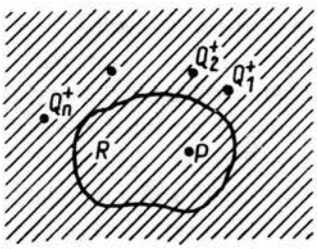

FIG. 5. Location of exterior sources to construct the solution of the interior problem.

7. Numerical solution

For the sake of illustration we will consider only the cavity case. Thus, Eq. (5.3) becomes

$$T_z^{(0)} + T_z^{(s)} = 0, \quad \text{at} \quad S, \tag{7.1}$$

where $T_z = \mu \dfrac{\partial w}{\partial n}$. Equation (7.1) can be written as

$$\frac{\partial w^{(0)}}{\partial n} + \frac{\partial w^{(s)}}{\partial n} = 0, \quad \text{at} \quad S. \tag{7.2}$$

Substituting Eq. (4.3) into Eq. (7.2), we have

$$\sum_{m=1}^{M} \alpha_m \frac{\partial G(P, Q_m)}{\partial n} = -\frac{\partial w^{(0)}}{\partial n}. \tag{7.3}$$

The set of coefficients α_m, $m = 1, 2, \ldots, M$ will be obtained in the least-squares sense. That is, a set for which the mean-square error on S is a minimum. Let the mean square error be defined as

$$\int_S \left| \frac{\partial w^{(0)}}{\partial n} + \frac{\partial w^{(s)}}{\partial n} \right|^2 dS. \tag{7.4}$$

A straightforward derivation leads us to a system of linear equations given in matrix form by

$$[F_{lm}]\{\alpha_m\} = \{W_l\}, \tag{7.5}$$

where

$$F_{lm} = \int_S \frac{\partial G^*(P, Q_l)}{\partial n} \frac{\partial G(P, Q_m)}{\partial n} dS, \tag{7.6}$$

$$W_l = -\int_S \frac{\partial G^*(P, Q_l)}{\partial n} \frac{\partial w^{(0)}(P)}{\partial n} dS \tag{7.7}$$

and $\partial G^*(P, Q_l)/\partial n$ is the complex conjugate of $\partial G(P, Q_l)/\partial n$. The system in Eq. (7.5) is of order $M \times M$ and the matrix $[F_{lm}]$ is hermitian. Integrals in Eqs. (7.6) and (7.7) are evaluated by numerical integration.

Once the system is solved, Eqs. (4.1) and (4.3) allow us to compute the displacement w at any point of the region E and its boundary.

For a related problem MILLAR [25] has shown that when the mean-square error is minimized on the boundary, the obtained representation converges uniformly in the mean to the unique solution of the problem provided a complete set of functions is chosen. The completeness of the functions $H_0^{(2)}(kr_m(s))$ defined on the boundary and its normal derivatives can be established. The proof is closely related to that given by MILLAR [25] for multipole expansions. Of course, this argument supports the method only for the anti-plane case. For the inplane problem the support comes from the results obtained for geometries which allow separation of variables [26], and from heuristic considerations. The complete proof of completeness needs to be developed.

8. Examples

In order to test the accuracy of the method, displacements have been computed at points on the surface of a semi-circular canyon under incident plane SH-waves (Fig. 6) for several incidence angles and normalized frequencies. Let the normalized frequency η be given by

$$\eta = \frac{k_s a}{\pi} = \frac{2a}{\Lambda},$$

where Λ — incident wave length, a — radius of the canyon, and thus η is the canyon width-to-wave-length ratio.

Values of real and imaginary parts of w at some points are presented in Tables 1 and 2 for $\eta = 0.5$, 1.0 and $\gamma = 30°$, 60°, 90° using several values of M (the number of sources) for the computation. Comparison is provided with the values obtained from TRIFUNAC's

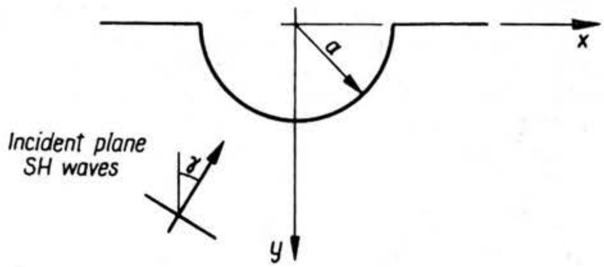

FIG. 6. Semi-circular canyon under incident plane SH waves, γ = angle of incidence.

Table 1. Comparison of results to exact solution, semi-cylindrical canyon, $\eta = 0.50$

x/a	$M = 10$		15		20		EXACT	
	$\eta = 0.50$	$\gamma = 30°$						
−1.50	1.58050	2.34651	1.58499	2.34411	1.58567	2.34345	1.58597	2.34345
−1.00	1.78212	2.50086	1.79853	2.51549	1.80367	2.52048	1.80367	2.52048
−0.50	0.26967	1.18741	0.27035	1.18490	0.27065	1.18443	0.27064	1.18443
0.00	−0.37513	1.32332	−0.37752	1.33098	−0.37746	1.33235	−0.37746	1.33235
0.50	0.11779	1.13952	0.11346	1.15365	0.11203	1.15642	0.11202	1.15643
1.00	1.33354	−1.04309	1.32919	−1.12375	1.32788	−1.14951	1.32787	−1.14951
1.50	1.17565	−1.51192	1.16390	−1.52688	1.16050	−1.52934	1.16050	−1.52934
	$\eta = 0.50$	$\gamma = 60°$						
x/a	$M = 10$		15		20		EXACT	
−1.50	−0.35111	2.76297	−0.36179	2.77166	−0.36363	2.77404	−0.36363	2.77404
−1.00	0.05580	3.42918	0.00487	3.43468	−0.01118	3.43636	−0.01119	3.43634
−0.50	1.34090	1.89831	1.35122	1.89879	1.35326	1.89902	1.35327	1.89901
0.00	1.55088	0.68743	1.56362	0.68570	1.56577	0.68548	1.56577	0.68549
0.50	0.97520	−0.20820	0.98511	−0.20577	0.98698	−0.20502	0.98699	−0.20502
1.00	−0.66929	−0.76096	−0.72590	−0.76897	−0.74373	−0.77164	−0.74373	−0.77164
1.50	−0.94689	−0.60061	−0.96035	−0.59339	−0.96288	−0.59116	−0.96288	−0.59116
	$\eta = 0.50$	$\gamma = 90°$						
x/a	$M = 10$		15		20		EXACT	
−1.50	−1.00549	2.57714	−1.02199	2.58653	−1.02516	2.58821	−1.02516	2.58821
−1.00	−0.69753	3.53787	−0.76532	3.52327	−0.78662	3.51835	−0.78662	3.51835
−0.50	1.79185	2.51528	1.80165	2.51953	1.80362	2.52052	1.80365	2.52050
0.00	2.69832	0.36766	2.71540	0.36239	2.71774	0.36164	2.71774	0.36164
0.50	1.31194	−1.14324	1.32482	−1.14875	1.32782	−1.14953	1.32785	−1.14954
1.00	−1.50072	−0.25680	−1.56074	−0.21942	−1.57964	−0.20761	−1.57965	−0.20761
1.50	−1.62852	0.25712	−1.63677	0.27342	−1.63765	0.27722	−1.63765	0.27722

Table 2. Comparison of results to exact solution, semi-cylindrical canyon, $\eta = 1.00$

			$\eta = 1.00$	$\gamma = 30°$				
x/a	$M = 10$		15		20		EXACT	
−1.50	−0.63367	1.93452	−0.57841	1.99894	−0.56649	2.00745	−0.56649	2.00745
−1.00	−0.27270	2.90126	−0.42421	3.17473	−0.47423	3.25607	−0.47425	3.75604
−0.50	−2.17119	−1.09776	−2.18913	−1.07495	−2.19088	−1.07481	−2.19089	−1.07480
0.00	−2.12876	0.57789	−2.19232	0.65095	−2.20223	0.66019	−2.20222	0.66020
0.50	0.27455	1.93222	0.27639	1.95458	0.27938	1.95399	0.27932	1.95397
1.00	−0.20900	−1.91136	−0.49837	−1.75527	−0.59183	−1.71278	−0.59190	−1.71278
1.50	−1.51268	−1.51789	−1.48657	−1.44253	−1.47878	−1.42857	−1.47877	−1.42856
			$\eta = 1.00$	$\gamma = 60°$				
x/a	$M = 10$		15		20		EXACT	
−1.50	−1.14891	−0.38026	−1.11820	−0.33940	−1.11216	−0.33234	−1.11216	−0.33234
−1.00	−3.42344	0.88232	−3.55026	0.99850	−3.59159	1.03019	−3.59159	1.03019
−0.50	−0.45780	0.64924	−0.45267	0.67753	−0.45104	0.67898	−0.45104	0.67898
0.00	−0.47612	0.69691	−0.52651	0.76313	−0.53307	0.77163	−0.53307	0.77163
0.50	0.94607	0.30089	0.93891	0.31092	0.93835	0.30893	0.93833	0.30895
1.00	−0.56992	−0.07240	−0.75248	0.20648	−0.81286	0.29022	−0.81285	0.29021
1 50	−0.63866	0.75647	−0.58768	0.81934	−0.57536	0.82815	−0.57536	0.82815
			$\eta = 1.00$	$\gamma = 90°$				
x/a	$M = 10$		15		20		EXACT	
−1.50	−0.30442	−0.66994	−0.27899	−0.65685	−0.27456	−0.65505	−0.27456	−0.65505
−1.00	−3.75639	−0.54895	−3.78081	−0.46217	−3.78995	−0.43683	−3.78994	−0.43682
−0.50	−0.47112	3.23936	−0.47485	3.25487	−0.47436	3.25603	−0.47430	3.25604
0.00	2.69676	0.28322	2.69189	0.32237	2.68953	0.32773	2.68952	0.32773
0.50	−0.58142	−1.72149	−0.59171	−1.71117	−0.59201	−1.71276	−0.59197	−1.71278
1.00	1.12446	0.44336	1.05657	0.60338	1.03279	0.65191	1.03282	0.65190
1.50	1.29966	−0.13729	1.33270	−0.11293	1.33999	−0.10967	1.33999	−0.10967

exact solution [43]. Sources were located equally spaced along a semi-circumference with radius 0.8 a. For integration the trapezoidal rule with 99 points at the boundary was used.

The method has also been applied to a semi-elliptical canyon [34]; agreement with the published exact solution [45] for the incident plane SH-waves is excellent.

Amplitudes of surface displacements in a triangular canyon with 45° slopes under vertically incoming plane SH waves are given in Fig. 7 for two normalized frequencies $\eta = 0.25$, 0.5. Sources were placed along lines parallel to the slopes and separated from them a distance 0.07 a. At the boundary, 99 points were taken. The vertex of the canyon was smoothed with a segment of circumference tangent to the slopes. The same figure shows results obtained with the finite element method using the efficient-active boundaries [4]. The relative size of the discretized domain is also shown. Agreement between solutions is very good. Differences are probably due to the different discretization of the vertex.

The accuracy of the method depends on the number and location of sources; this number, for a given accuracy, is an increasing function of the frequency. Location of the

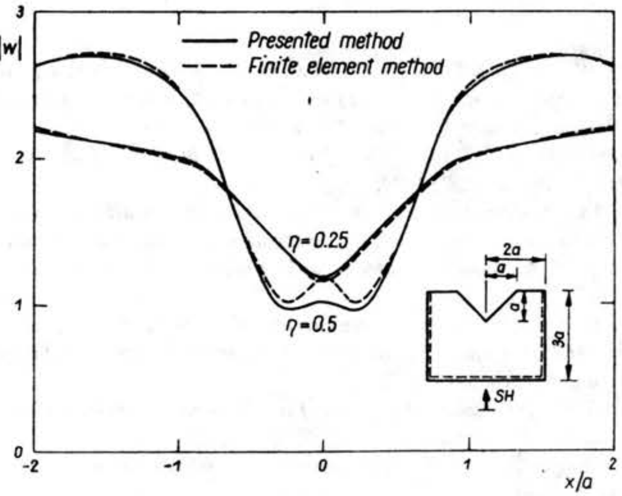

FIG. 7. Amplitudes of displacements for vertically incident SH waves $\eta = 0.25, 0.5$. Comparison of results with those obtained with the finite element method with efficient-active boundaries (3).

sources on a curve which follows the boundary appears to be reasonable. The definition of the best average distance between sources and the boundary requires further scrutiny.

9. Conclusions

A boundary method has been presented for solving some elastic scattering problems. The basis of the method is to construct the scattered wave fields in terms of linear combinations of solutions with singularities located outside the region of interest. The least-squares criterion simplifies numerical treatment and leads to a representation which converges uniformly in the mean to the unique solution of the problem provided that a complete set of functions is chosen.

The obtained results for diffraction of SH-waves were compared with known analytical and numerical solutions yielding very good agreement.

The method appears to be a useful tool in solving some problems of scattering of elastic waves. It could be applied in other fields to deal with sound, electromagnetic, or water waves.

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