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# BRIEF NOTES

## Application of modified BGK-equations to the calculation of the shock wave structure in Xe—He mixtures

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THE MODIFIED set of the BKG-model kinetic equations has been solved by the generalised Mott-Smith assumption, with boundary conditions corresponding to propagation of the shock wave in mixtures of gases without internal degrees of freedom. The results obtained are in qualitative agreement with the experiments.

AS IT HAS BEEN noticed recently in [1, 2], the BGK-equations, when applied to mixtures of disparate-mass gases, must be modified to treat properly the different relaxation scales in such mixtures. In [3] the set of BGK-equations proposed in [1] was applied to the investigation of the shock wave structure in He—Xe mixtures. The results, although different from the classical shock wave structures, didn't correspond even qualitatively to the experimental results [4]. In this note we propose and solve a sligthly modified BGK-set of equations with different rescaling of relaxation times. This will lead to results qualitatively compatible with the experiments. The set of BGK-equations (in the one-dimensional steady case) can be written as follows:

$$v_x \frac{\partial F_1}{\partial x} = \tau_{11}^{-1}[M_{11}-F_1]+\tau_{12}^{-1}[M_{12}-F_1],$$

$$v_x \frac{\partial F_2}{\partial x} = \tau_{21}^{-1}[M_{21}-F_2]+\tau_{22}^{-1}[M_{22}-F_2],$$

where $F_1 = F_1(x, v)$, $F_2 = F_2(x, v)$ are one-particle distribution functions of gases 1 and 2, where 1 relates to the heavier gas (i.e. in our notation $m = \frac{m_2}{m_1} \ll 1$, where $m_i$ — atomic mass of $i$-th gas). $M_{ij}$ are Maxwellian type local distribution functions corresponding to different types of collisions. The unknown functions in $M_{ij}$ are calculated by comparing the velocity and temperature relaxation equations resulting from our model with those from the full Boltzmann equations. The procedure is standard and can be found, for example, in [1, 2, 6].

The relaxation times $\tau_{ij}$ are of primary importance in the model equations under consideration. The so-called self-collision relaxation times $\tau_{11}$ and $\tau_{22}$ are commonly admitted to be proportional to the inverse of the self-collision frequencies $\nu_{11}$ and $\nu_{22}$, so

one has $\frac{\tau_{22}}{\tau_{11}} \approx m^{1/2}$. As in disparate-gas mixtures the relaxation scales resulting from cross-collisions should be longer than those resulting from self-collisions, we propose the following assumptions for cross-collision relaxation times: $\tau_{21} \simeq m^{-1}\tau_{22}$ and $\tau_{12} \cong \cong m^{-1}\tau_{11}$. In that way we obtain the following reordering of the relaxation times:

$$\tau_{22} < \tau_{11} < \tau_{21} < \tau_{12},$$

where the ratio of the preceding to the following relaxation time is of the order $m^{1/2}$. It should be noted that this reordering is different from that proposed by GREEN [1]. The set of BGK-equations is then solved for the boundary conditions corresponding to the shock wave. To solve the model we use the modified Mott-Smith assumption for the distribution functions $F_1$, $F_2$ [8]. To close the set of equations we use two nonvanishing moments: $m_1 v_x^2$ and $m_2 v_x^2$. The choice of moments is arbitrary. By algebraic manipulations we arrive at the set of two ordinary differential equations with singular points corresponding to downstream and upstream equilibrium. The nature of singular points has been investigated. It was shown that it depends on the molecular mass and number density ratios of composite gases and changes from node to saddle point for a certain density ratio of the fixed molecules. It corresponds physically to a change of the character of the flow from supersonic for both gases to subsonic for the lighter gas (and supersonic for the heavier one). The details are described in [3 ]. The obtained set of differential equations was solved numerically using the standard Runge–Kutta's procedure. The hydrodynamical characteristics of the flow were then calculated by integrations. Some of the results are shown below.

Recent experiments [4] in Xe—He mixtures have shown nonclassical structure of the density of helium, i.e. a hump of the density of helium in the shock wave structure. In

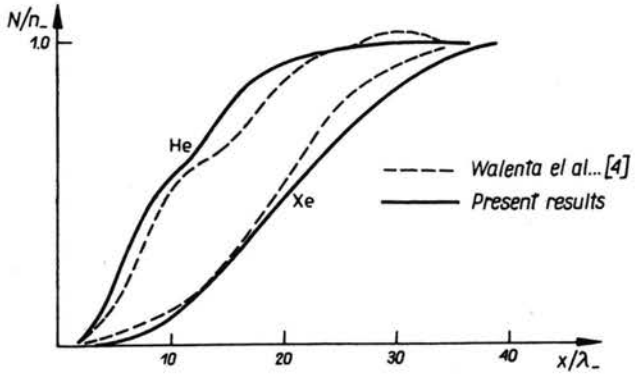


FIG. 1. Density structure in Xe—He for 6% Xe, $M = 4.4$.

Fig. 1 we compare our results with [4]. The qualitative agreement of theoretical and experimental results can be seen. It should be noted that recently the same qualitative behaviour of the helium density in the shock wave has been found by a different method based on a variational principle for the full Boltzmann equation [9]. It should be also noted (see Fig. 2) that the thickness of the shock wave, measured for the Xe—He mixture

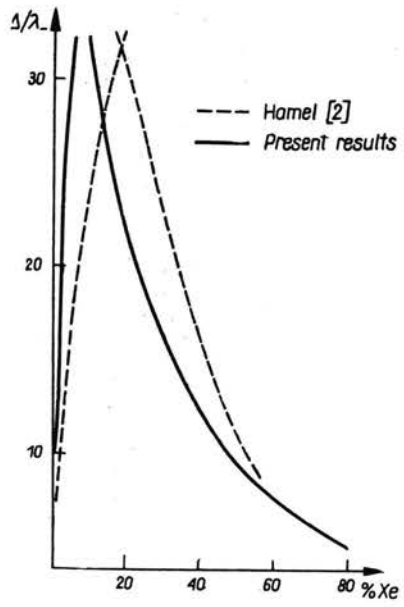


FIG. 2. Shock wave thickness $\Delta$ for Xe—He mixture, $M = 2.0$.

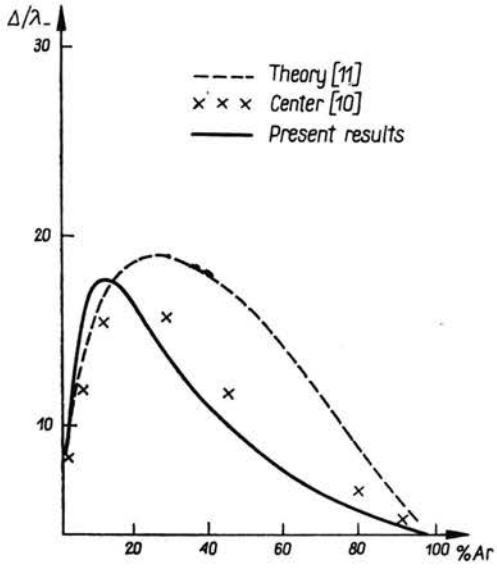


FIG. 3. Shock wave thickness $\Delta$ for Ar—He mixture, $M = 2.0$.

for different compositions shows the same qualitative behaviour as that resulting from the asymptotic theory of HAMEL [2]. Similar computations made for the Ar—He mixture are better suited to the experimental data of CENTER [10] than the results of [11], as can be seen from Fig. 3.

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