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Discussion on the effects of strain history for viscoplastic materials

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THE PAPER presents an attempt to describe theoretically the broad class of materials (fcc, bcc, hcp) in which it has been experimentally established that the dynamic stress-strain curve was strain-history-dependent. The internal parameters: α — inelastic deformations, $\varkappa$ — hardening and γ — viscosity are introduced to the description. It is shown that, depending on the evolution equations assumed, the experimentally-observed effects of finite and fading memory may be accounted for.

W niniejszej pracy przedstawiono próbę opisu teoretycznego szerokiej klasy materiałów (fcc, bcc i hcp), dla których doświadczalnie wykazano wpływ historii deformacji na krzywą dynamiczną: naprężenie-odkształcenie. Do opisu wprowadzono parametry wewnętrzne; α — deformację niesprężystą, $\varkappa$ — parametr wzmocnienia, γ — parametr lepkości. Wykazano, że zależnie od przyjętych równań ewolucji można opisać, zaobserwowany doświadczalnie, efekt skończonej i zanikającej pamięci.

В настоящей работе представлена попытка теоретического описания широкого класса материалов, для которых экспериментально показано влияние истории деформации на динамическую кривую (напряжение — деформация). Для описания введены внутренние параметры: α — неупругие деформации, $\varkappa$ — параметр упрочнения, γ — параметр вязкости. Показано, что в зависимости от принятых уравнений эволюции можно описать, наблюдаемый экспериментально, эффект конечной и исчезающей памяти.

1. Introduction

THE DYNAMIC stress-strain curve for viscoplastic materials depends not only on the strain rate but also on the strain history, what has been proved by numerous experiments performed in the recent years on various types of materials: fcc, bcc and hcp. By deforming a specimen at a constant strain rate $\dot{\gamma}_1$ and next, by suddenly changing the strain rate from $\dot{\gamma}_1$ to another constant value $\dot{\gamma}_2 > \dot{\gamma}_1$, a stress-strain curve is obtained which differs from that following from the experiment in which the specimen is deformed at a constant strain rate $\dot{\gamma}_2$ from the very beginning. The experiments suggest that such a behaviour of the material results from the strain-rate history, and the suggestion is confirmed by the experimentalists themselves. Theoretical investigations show, however, that by introducing the strain history into the constitutive relations a good description of the experimental results can be obtained. By using the internal parameters (inelastic strain, viscosity and hardening parameters) and the corresponding equations of evolution, we arrive at the conclusion that the stress at the constant strain rate $\dot{\gamma}$ depends both on the strain and the strain history. Depending on the equations of evolution assumed, the effects of finite or fading memory may be described.

Comparison with the experimental results makes it possible to select the proper evolution equations, and also to determine the strain rates at which the history effect is most pronounced, what is connected with the mechanisms of plastic flow.

2. Discussion of experimental results

Let us present in this section a brief review of experimental results which show the considerable influence of the history of deformation upon the behaviour of viscoplastic materials.

T. E. TIETZ and J. E. DORN [7] were the first to observe a very interesting phenomenon: in the process of loading of a specimen (under isothermal conditions) occurring first at a constant strain rate $\dot{\gamma}_1$ and then changing the strain rate to $\dot{\gamma}_2$ ($\dot{\gamma}_2 > \dot{\gamma}_1$), the stress-strain relationship (dynamic curve) obtained differs from that corresponding to the process of loading at the constant strain rate $\dot{\gamma}_2$(1). In order to investigate the effect of temperature, similar experiments were performed but with the temperature changing abruptly from ϑ_1 to ϑ_2 ($\vartheta_1 > \vartheta_2$ or $\vartheta_1 < \vartheta_2$), and under the constant strain rate $\dot{\gamma}$. The dynamic curve obtained in this manner and corresponding to the changed temperature ϑ_2 does not coincide with the dynamic curves resulting from the experiment in which the temperature was kept constant at ϑ_2.

Most of the experiments of this type were performed on the fcc type materials, e.g. copper or aluminum (R. A. FRANTZ, J. DUFFY [2], J. KLEPACZKO, R. A. FRANTZ, J. DUFFY [3]), few were concerned with the hcp or bcc materials. The differences in the behaviour of various types of materials were considerable. A comparison of experimental results obtained for the fcc (aluminum, copper) and hcp (magnesium, zinc) materials is presented in the paper by P. E. SENSENY, G. DUFFY and R. H. HAWLEY [6]. These results are shown for the strain rate changes from the statical value $\dot{\gamma}_1 = 2 \cdot 10^{-4}$ to $\dot{\gamma}_2 = 3 \cdot 10^2$ at temperatures 77, 148, 298, 523K (aluminum, copper, Figs. 1 and 2) and to $\dot{\gamma}_2 = 8 \cdot 10^2$ at tempera-

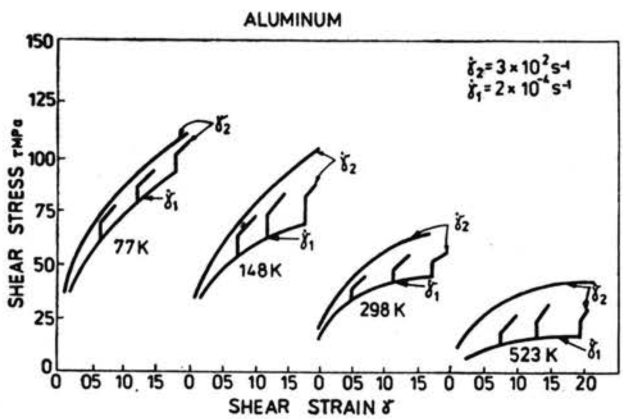

FIG. 1. Stress-strain curves from incremental and constant strain rate tests for 1100-0 aluminium. After P. E. SENSENY, J. DUFFY, R. H. HAWLEY [6].

(1) The same result is obtained when the strain rate decreases, i.e. $\dot{\gamma}_2 < \dot{\gamma}_1$.

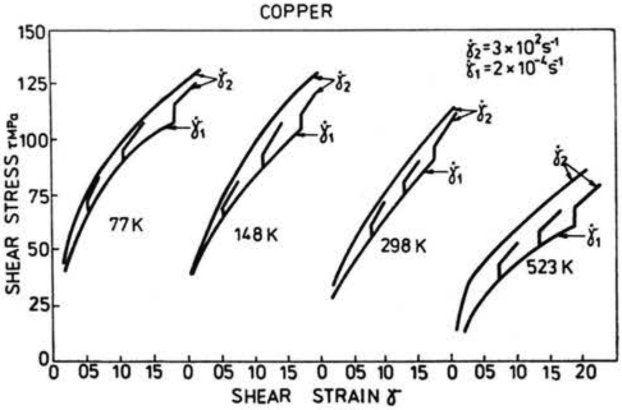

FIG. 2. Stress-strain curves from incremental and constant strain rate tests for OFHC copper. After P. E. SENSENY, J. DUFFY, R. H. HAWLEY [6].

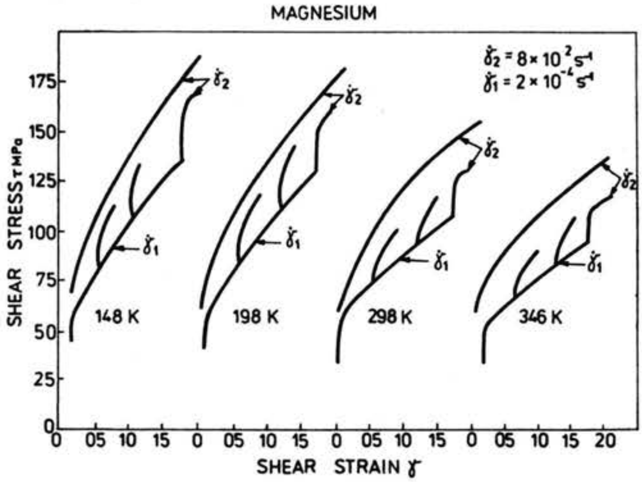

FIG. 3. Stress-strain curves from incremental and constant strain rate tests for Az31 B magnesium. After P. E. SENSENY, J. DUFFY, R. H. HAWLEY [6].

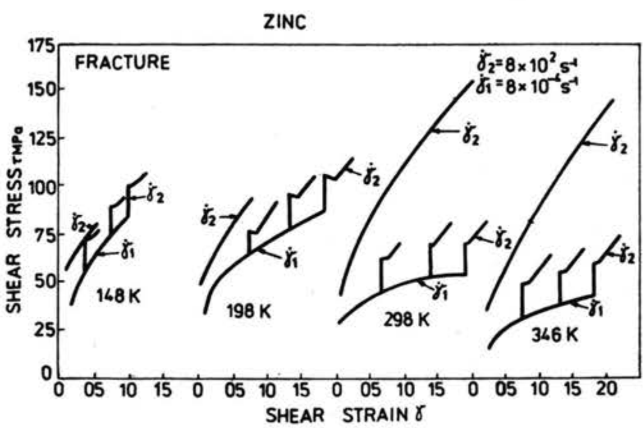

FIG. 4. Stress-strain curves from incremental and constant strain rate tests for commercially pure zinc. After P. E. SENSENY, J. DUFFY, R. H. HAWLEY [6].

tures 148, 198, 298, 346 K (magnesium, zinc, Figs. 3 and 4). The range of applied temperatures implies the viscoplastic deformation to be a result of thermally-activated processes. Lower curves shown in Figs. 1-4 correspond to the strain rates $\dot{\gamma}_1$, the upper ones to $\dot{\gamma}_2 > \dot{\gamma}_1$, and the ones lying between them to the dynamic curves resulting from variations of the strain rates from $\dot{\gamma}_1$ to $\dot{\gamma}_2$. This means that the stress does not solely depend on the actual value of the strain γ, strain rate $\dot{\gamma}$ and temperature ϑ. The authors mentioned above suggest that the additional variable determining the actual, uniquely determined value of stress is the strain rate history; in the case of the constants γ and $\dot{\gamma}$, and temperatures varying from ϑ_1 to ϑ_2, the history of temperature is the additional variable.

In this paper an attempt has been made to construct a theoretical description of the viscoplastic material taking into account that effect by using the theory with internal parameters proposed by P. PERZYNA [4, 5]. Introduction of the internal parameters is shown to account for the influence of the strain history (and not the strain rate) upon the stresses(²).

On the other hand, the experiments performed up to now do not provide a definite answer to the problem whether the actual value of stress is influenced by the strain history or the strain rate.

Let us confine our considerations to the one-dimensional case; Fig. 5 presents the idealization of experimental results for three various dynamic curves (T, E), T — stress,

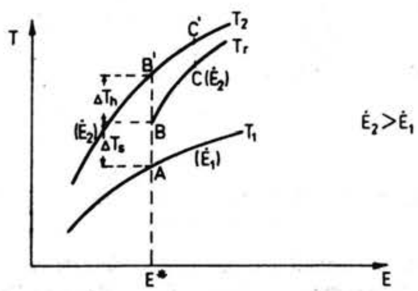

FIG. 5. Schematic stress-strain diagram showing changing of strain rate from $\dot{E}_1$ to $\dot{E}_1$

E — strain. The specimen is deformed with the strain rate $\dot{E}$ and at a certain value of strain (E^* in Fig. 5) the strain rate is changed instantaneously (without reducing the load) from $\dot{E}_1$ to $\dot{E}_2$. Behind point B the specimen is already deformed at the velocity $\dot{E}_2$. The stress jump ΔT_s is measured at the same value of E^* but for different rates $\dot{E}_1$ and $\dot{E}_2$. The question arises, what is the origin of the difference ΔT_h between the stresses at points B and B' since the strains and strain rates are equal. With a certain simplification we might say that ΔT_h is referred to two various microstructures due to different ways of access to the points B and B', and thus it is responsible for the influence of the history on the dynamic curve. In contrast, ΔT_s measures the real sensitivity of the material to the strain rate $\dot{E}$ produced by thermally-activated processes.

(²) A slightly different approach which makes use of the idea of material description by internal parameters was applied by R. S. BODNER and A. MERZER [1]; however, despite the authors' statement, the equations applied by them do not account for the strain rate history but for the strain history.

Properties of the BC curve, slope and the magnitudes ΔT_s and ΔT_h depend not only on the initial strain E^* and temperature, but — first of all — on the material itself. For the fcc type materials (Figs. 1 and 2) the value of ΔT_s increases with increasing strains independently of the temperature, and in both cases a definite yield point occurs (R. A. FRANZ and J. DUFFY [2] demonstrated the existence of the upper and lower points for aluminum). From the instant of yielding, behind B, the hardening is stronger than that occurring along the curve $B'C'$ (cf. P. E. SENSENY, J. DUFFY, R. H. HAWLEY [6], J. KLEPACZKO, R. A. FRANZ, J. DUFFY [3]). The essential question is whether the material remembers, at each instant of time, the processes it was subjected to in the past. Experimental evidence suggests that only the fcc materials have the property of fading memory (curve BC tends to the curve $B'C'$, Fig. 5). From Fig. 1 it follows that in the case of aluminum the fading memory was observed at higher temperatures, while in the case of copper — at all temperatures applied in the experiments (Fig. 2). The situation is different in the cases of magnesium and zinc: the intermediate dynamic curve suggests rather the finite memory property. The problem has not been sufficiently clarified until now since we still miss the experimental points which would enable us to draw the curve behind a certain point C.

A completely different result obtained for mild steel (cf. M. L. WILSON, R. H. HAWLEY, J. DUFFY [8]) suggests that in certain materials (of the bcc type in our case) the effect

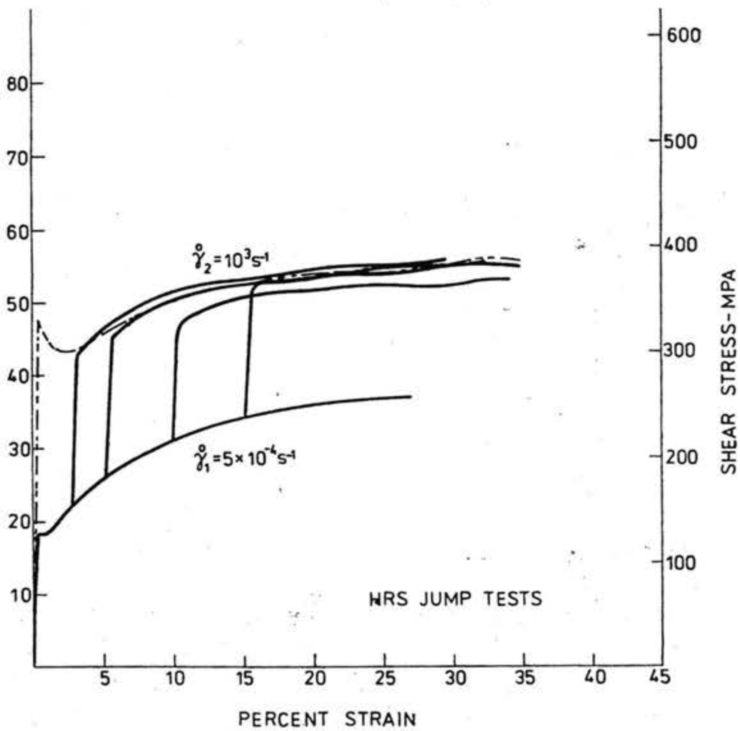

FIG. 6. Results of constant strain rate and incremental strain rate tests on 1020 hot rolled steel at room temperature. After M. L. WILSON, R. H. HAWLEY, J, DUFFY. [8]

of history is negligible (Fig. 6); the effect of ranges of the strain rates $\dot{E}_1$ and $\dot{E}_2$ upon the character of the memory remains open to further considerations.

In the following sections it will be shown that application of the necessary evolution equations for the internal parameters makes it possible to account for the various types of memory presented above.

3. Internal parameters

Let us consider the one-dimensional problem. The following internal parameters are used for describing the viscoplastic material: inelastic deformation α, hardening parameter $\varkappa$ and viscosity parameter γ. Within the strain rate range in which the thermally-activated processes occurs, the evolution equations for the parameters α, $\varkappa$, γ are assumed to be (P. PERZYNA [4]):

$$\dot{\alpha} = \gamma \Phi\left(\frac{T}{\varkappa} - 1\right), \tag{3.1}$$

$$\dot{\varkappa} = K(E, \alpha)\dot{\alpha}, \tag{3.2}$$

$$\dot{\gamma} = \Gamma(E, \alpha)\dot{\alpha}, \tag{3.3}$$

together with the initial conditions $\alpha_0 = \alpha(0)$, $\varkappa_0 = \varkappa(0)$, $\gamma_0 = \gamma(0)$.

In Eqs. (3.2) and (3.3) use can be made of Eq. (3.1),

$$\dot{\varkappa} = K(E, \alpha)\gamma \Phi\left(\frac{T}{\varkappa} - 1\right), \tag{3.4}$$

$$\dot{\gamma} = \Gamma(E, \alpha)\gamma \Phi\left(\frac{T}{\varkappa} - 1\right). \tag{3.5}$$

Proper selection of the functions Φ, K, Γ depends on the kind of material to be considered.

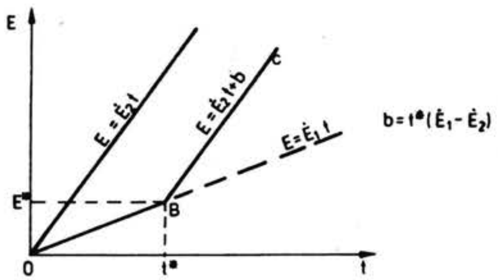

FIG. 7. Deformation as the linear function of time t for two different processes.

From the form of the evolution equations it follows that α, $\varkappa$, γ representing the solution of the initial problem depend on the history of deformation E. The course of all the three experiments performed at constant strain rates $\dot{E}_1$, $\dot{E}_2$ and abrupt variation from $\dot{E}_1$ to $\dot{E}_2$ may be expressed in terms of the time variation of the strain (as in Fig. 7). Time t^* corresponds to the strain E^* in Fig. 5. The straight line $E = \dot{E}_2 t$ corresponds to the curve $B'C'$, the broken line OBC — to the line ABC.

The stress T is calculated from Eq. (3.1),

$$T = \left[\Phi^{-1}\left(\frac{\dot{\alpha}}{\gamma}\right)+1\right]\varkappa. \tag{3.6}$$

At $\dot{E}$ = const the stress T depends on the strain and strain history. It is assumed in this paper that for $\dot{E}$ = const also $\dot{\alpha}$ = const.

Let us solve the evolution equation for $\varkappa$ and γ at $\dot{E}$ = const and $\dot{\alpha}$ = const, and substitute it into Eq. (3.6). Three cases must be considered (Fig. 7).

1) $\dot{E} = \dot{E}_1$ = const, $\dot{\alpha} = \dot{\alpha}_1$ = const, whence $E = \dot{E}_1 t$

$$\varkappa_1(t) = \varkappa_0 + \int_0^t K(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt, \tag{3.7}$$

$$\dot{\gamma}_1(t) = \gamma_0 + \int_0^t \Gamma(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt, \tag{3.8}$$

$$T_1(t) = \left[\Phi^{-1}\left(\dot{\alpha}_1\left(\gamma_0+\int_0^t \Gamma(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt\right)^{-1}\right)+1\right]\left[\varkappa_0+\int_0^t K(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt\right]. \tag{3.9}$$

2) $\dot{E} = \dot{E}_2$ = const, $\dot{\alpha} = \dot{\alpha}_2$ = const, whence $E = \dot{E}_2 t$, $\alpha = \dot{\alpha}_2 t$

$$\varkappa_2(t) = \varkappa_0 + \int_0^t K(\dot{E}_2 t, \dot{\alpha}_2 t)\dot{\alpha}_2 dt, \tag{3.10}$$

$$\dot{\gamma}_2(t) = \gamma_0 + \int_0^t \Gamma(\dot{E}_2 t, \dot{\alpha}_2 t)\dot{\alpha}_2 dt, \tag{3.11}$$

$$T_2(t) = \left[\Phi^{-1}\left(\dot{\alpha}_2\left(\gamma_0+\int_0^t \Gamma(\dot{E}_2 t, \dot{\alpha}_2 t)\dot{\alpha}_2 dt\right)^{-1}\right)+1\right]\left[\varkappa_0+\int_0^t K(\dot{E}_2 t, \dot{\alpha}_2 t)\dot{\alpha}_2 dt\right]. \tag{3.12}$$

3)

$$\dot{E} = \begin{cases}\dot{E}_1, & t < t^*,\\ \dot{E}_2, & t \geqslant t^*,\end{cases} \qquad \dot{\alpha} = \begin{cases}\dot{\alpha}_1, & t < t^*,\\ \dot{\alpha}_2, & t \geqslant t^*,\end{cases} \tag{3.13}$$

whence

$$E = \begin{cases}\dot{E}_1 t, & t < t^*,\\ \dot{E}_2 t + b, & t \geqslant t^*,\end{cases} \qquad \alpha = \begin{cases}\dot{\alpha}_1 t, & t < t^*,\\ \dot{\alpha}_2 t + a, & t \geqslant t^*,\end{cases} \tag{3.14}$$
$$b \equiv t^*(\dot{E}_1 - \dot{E}_2), \qquad a \equiv t^*(\dot{\alpha}_1 - \dot{\alpha}_2).$$

Solutions for $\varkappa$ and γ and for the stress T under variable rates (3.13) are denoted by the index r:

$$\varkappa_r(t) = \begin{cases}\varkappa_0 + \int_0^t K(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt, & t < t^*,\\ \varkappa_0 + \int_0^{t^*} K(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt + \int_{t^*}^t K(\dot{E}_2 t + b, \dot{\alpha}_2 t + a)\dot{\alpha}_2 dt, & t \geqslant t^*;\end{cases} \tag{3.15}$$

$$(3.16)\qquad \gamma_r(t) = \begin{cases} \gamma_0 + \int\limits_0^t \Gamma(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt, & t < t^*, \\ \gamma_0 + \int\limits_0^{t^*} \Gamma(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt + \int\limits_{t^*}^t \Gamma(\dot{E}_2 t + b, \dot{\alpha}_2 t + a)\dot{\alpha}_2 dt, & t \geqslant t^*; \end{cases}$$

$$(3.17)\qquad T_r(t) = \begin{cases} \left[\Phi^{-1}\left(\dot{\alpha}_1\left(\gamma_0 + \int\limits_0^t \Gamma(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt\right)^{-1}\right) + 1\right]\left[\varkappa_0 + \int\limits_0^t K(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt\right], \\ \hfill t < t^*, \\ \left[\Phi^{-1}\left(\dot{\alpha}_2\left(\gamma_0 + \int\limits_0^{t^*} \Gamma(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt + \int\limits_{t^*}^t \Gamma(\dot{E}_2 t + b, \dot{\alpha}_2 t + a)\dot{\alpha}_2 dt\right)^{-1}\right) + 1\right] \\ \quad \times \left[\varkappa_0 + \int\limits_0^{t^*} K(\dot{E}_1 t, \dot{\alpha}_1 t)\dot{\alpha}_1 dt + \int\limits_{t^*}^t K(\dot{E}_2 t + b, \dot{\alpha}_2 t + a)\dot{\alpha}_2 dt\right], \quad t \geqslant t^*. \end{cases}$$

The magnitudes of $\varkappa$ and γ are seen to remain continuous at $t = t^*$ what means that the hardening and viscosity do not change due to the velocity jump from the point A to B.

In order to investigate the mutual behaviour of curves T_2 and T_r (Fig. 5), let us express the stress in Eqs. (3.12) and (3.17) in terms of the strain. The difference $(T_2 - T_r)(E)$ true for $E \geqslant E^*$ is written as

$$(3.18)\qquad \begin{aligned} T_2(E) - T_r(E) = {} & \left[\Phi^{-1}\left(\dot{\alpha}_2\left(\gamma_0 + \int\limits_0^E \Gamma(E, A_2 E)A_2 dE\right)^{-1}\right) + 1\right] \\ & \times \left[\varkappa_0 + \int\limits_0^E K(E, A_2 E)A_2 dE\right] - \left[\Phi^{-1}\left(\dot{\alpha}_2\left(\gamma_0 + \int\limits_0^{E^*} \Gamma(E, A_1 E)A_1 dE\right.\right.\right. \\ & \left.\left.\left. + \int\limits_{E^*}^E \Gamma\big(E, (E-b)A_2 + a\big)A_2 dE\right)^{-1}\right) + 1\right]\left[\varkappa_0 + \int\limits_0^{E^*} K(E, A_1 E)A_1 dE\right. \\ & \left. + \int\limits_{E^*}^E K\big(E, (E-b)A_2 + a\big)A_2 dE\right], \end{aligned}$$

$$A_1 \equiv \frac{\dot{\alpha}_1}{\dot{E}_1}, \quad A_2 \equiv \frac{\dot{\alpha}_2}{\dot{E}_2}.$$

The limiting value of $(T_2 - T_r)(E)$ at $E \to \infty$ allows for the evaluation of the influence of the past history on the behaviour of the material and, namely, if $\lim\limits_{E\to\infty} (T_2 - T_r)(E) = 0$, the memory is fading, while for $\lim\limits_{E\to\infty} (T_2 - T_r)(E) = \text{const}$ the memory is finite. Functions Γ, K, Φ are such that $(T_2 - T_r) > 0$.

In the expression (3.18) three material functions Φ, K and Γ appear. How do the functions influence the difference $(T_2 - T_r)(E)$? Two extreme possibilities are considered to answer the question posed: a) either α and $\varkappa$ are the internal parameters ($\gamma = \text{const}$), and $K = K(E)$; or b) α and γ are the internal parameters ($\varkappa = \text{const}$), and $\Gamma = \Gamma(E)$. Equation (3.12) in the case a) has the form

$$(3.19)\qquad T_2(E)-T_r(E) = \left[\Phi^{-1}\left(\frac{\dot{\alpha}_2}{\gamma}\right)+1\right]\left(A_2\int_0^E K(E)dE - A_1\int_0^{E^*} K(E)dE - \int_{E^*}^{E} K(E)A_2dE\right)$$

$$= \left[\Phi^{-1}\left(\frac{\dot{\alpha}_2}{\gamma}\right)+1\right](A_2-A_1)\int_0^{E^*} K(E)dE = \text{const}.$$

Independently of Φ, the difference T_2-T_r is constant for arbitrary $E > E^*$.

Equation (3.18) in the case b) is reduced to

$$(3.20)\qquad T_2(E)-T_r(E)$$

$$= \varkappa\left[\Phi^{-1}\left(\dot{\alpha}_2\left(\gamma_0+\int_0^E \Gamma(E)A_2dE\right)^{-1}\right)-\Phi^{-1}\left(\dot{\alpha}_2\left(\gamma_0+\int_0^{E^*}\Gamma(E)A_1dE+\int_{E^*}^{E}\Gamma(E)A_2dE\right)^{-1}\right)\right].$$

Assume the function Φ^{-1} to satisfy the Lipschitz condition with the constant $\mathscr{K}$; then

$$(3.21)\qquad T_2(E)-T_r(E)$$

$$\leqslant \varkappa\dot{\alpha}_2\mathscr{K}\left|[(A_1-A_2)\big(H(E^*)-H(0)\big)]\,[(\gamma_0-A_2H(0)+A_2H(E))\big(\gamma_0+(A_1-A_2)H(E^*)-A_1H(0)+A_2H(E)\big)]^{-1}\right|,$$

where $H(E) = \int \Gamma(E)dE$. The right-hand side expression of the inequality (3.21) tends to zero at $E\to\infty$ provided $\lim\limits_{E\to\infty} H(E) = \infty$.

From the examples a) and b) it follows that the finite memory effect is influenced by hardening, and the fading memory — by viscosity. This property may serve as a hint for a proper selection of internal parameters for viscoelastic materials; it might be useful in constructing simple mathematical models.

It should be stressed that the above conclusion has been drawn with respect to the functions K and Γ which depend solely on the total strain E.

Let us now consider, for instance, a function of evolution depending also on the viscoplastic strain α and leave the parameter γ constant. Then the difference (3.18) is simplified to (cf. Eq. (3.19)):

$$(3.22)\qquad T_2(E)-T_r(E) = \left[\Phi^{-1}\left(\frac{\dot{\alpha}_2}{\gamma}\right)+1\right]\left[\int_0^{E^*}\big(K(E,A_2E)A_2-K(E,A_1E)A_1\big)dE\right.$$

$$\left.+A_2\int_{E^*}^{E}\big(K(E,A_2E)-K(E,A_2E+h)\big)dE\right],$$

$$h \equiv bA_2+a = \text{const}.$$

Observe that the limit of Eq. (3.22) at $E\to\infty$ depends on the improper integral

$$(3.23)\qquad \int_{E^*}^{\infty}\big(K(E,A_2E)-K(E,A_2E+h)\big)dE.$$

The limit will be finite and different from zero; this indicates the finite memory provided the improper integral (3.23) converges to the value different from the constant

$$-\int_0^{E^*}\big(K(E,A_2E)A_2-K(E,A_1E)A_1\big)dE.$$

4. Particular forms of function

The functions Φ which are most frequently used in practice are the power and exponential functions; they yield the inverse functions

$$\Phi^{-1}\left(\frac{\dot{\alpha}_2}{\gamma}\right) = B\frac{\dot{\alpha}_2}{\gamma}, \quad B = \text{constant}, \tag{4.1}$$

and

$$\Phi^{-1}\left(\frac{\dot{\alpha}_2}{\gamma}\right) = \sqrt[n]{\frac{\dot{\alpha}_2}{\gamma}}. \tag{4.2}$$

Let us write the difference (3.18) for the function (4.1) and take into account both parameters $\varkappa$ and γ under the assumption that $K = K(E)$ and $\Gamma = \Gamma(E)$,

$$\begin{aligned} (4.3)\quad & T_2(E) - T_r(E) \\ &= B\dot{\alpha}_2\Big[\Big(\varkappa_0 + \int_0^E K(E)A_2 dE\Big)\Big(\gamma_0 + \int_0^E \Gamma(E)A_2 dE\Big)^{-1} - \Big(\varkappa_0 + \int_0^{E^*} K(E)A_1 dE + \int_{E^*}^E K(E)A_2 dE\Big) \\ &\quad \times \Big(\gamma_0 + \int_0^{E^*} \Gamma(E)A_1 dE + \int_{E^*}^E \Gamma(E)A_2 dE\Big)^{-1}\Big] + (A_2 - A_1)\int_0^{E^*} K(E) dE. \end{aligned}$$

The appearence of the constant $D = (A_2 - A_1)\int_0^{E^*} K(E) dE$ indicates that the parameter of hardening by the functions K yields the result that the curve $T_r(E)$ does not tend to the curve $T_2(E)$ (Fig. 5)([3]). It is clearly visible for

$$K(E) = 2\bar{K}E, \tag{4.4}$$

$$\Gamma(E) = 2\bar{\Gamma}(E) \quad \bar{K}, \bar{\Gamma} \text{ — constants} \tag{4.5}$$

since

$$\begin{aligned} (4.6)\quad & T_2(E) - T_r(E) \\ &= B\dot{\alpha}_2[\varkappa_0\bar{\Gamma}(E^*)^2(A_1 - A_2) + \gamma_0\bar{K}(E^*)^2(A_2 - A_1)]\big[(\gamma_0 + A_2\bar{\Gamma}E^2)\big(\gamma_0 + A_1\bar{\Gamma}(E^*)^2 \\ &\quad + A_2\bar{\Gamma}(E^2 - (E^*)^2)\big)\big]^{-1} + (A_2 - A_1)\int_0^{E^*} K(E) dE \end{aligned}$$

and

$$\lim_{E\to\infty}[T_2(E) - T_r(E)] = (A_2 - A_1)\int_0^{E^*} K(E) dE = (A_2 - A_1)\bar{K}E^* \tag{4.7}$$

([3]) Provided the expression in brackets (4.3) is different from $(-D)$ at $E \to \infty$.

the difference (3.6) increasing with increasing initial strain E^*, what agrees with the experimental results of KLEPACZKO [3], SENSENY, DUFFY, HAWLEY [6]. The graph of $T_r(E)$ is shown in Fig. 8.

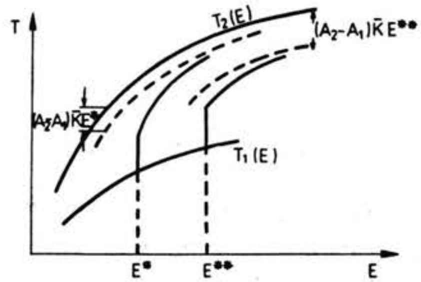

FIG. 8. Stress-strain curves for linear function Φ.

For comparison let us calculate $(T_r-T_1)(E)$:

(4.8)
$$T_r(E)-T_1(E)$$
$$= B\Big\{\dot{\alpha}_2\Big(\varkappa_0+\int\limits_0^{E^*} K(E)A_1dE+\int\limits_{E^*}^{E} K(E)A_2dE\Big)\Big(\gamma_0+\int\limits_0^{E^*} \Gamma(E)A_1dE+\int\limits_{E^*}^{E} \Gamma(E)A_2dE\Big)^{-1}$$
$$-\dot{\alpha}_1\Big(\varkappa_0+\int\limits_0^{E} K(E)A_1dE\Big)\Big(\gamma_0+\int\limits_0^{E} \Gamma(E)A_1dE\Big)^{-1}\Big\}+(A_2-A_1)\int\limits_{E^*}^{E} K(E)dE.$$

Evidently, for linear $K(E)$ and $\Gamma(E)$, Eqs. (4.4), (4.5) and the curve $T_r(E)$ will move away from $T_1(E)$.

For a power law Φ (4.2) we obtain

(4.9)
$$T_2(E)-T_r(E) = B\sqrt[n]{\dot{\alpha}_2}\,\Big\{\Big(\varkappa_0+\int\limits_0^{E} K(E)A_2dE\Big)\Big(\gamma_0+\int\limits_0^{E} \Gamma(E)A_2dE\Big)^{-1/n}$$
$$-\Big(\varkappa_0+\int\limits_0^{E^*} K(E)A_1dE+\int\limits_{E^*}^{E} K(E)A_2dE\Big)\Big(\gamma_0+\int\limits_0^{E^*} \Gamma(E)A_1dE+\int\limits_{E^*}^{E} \Gamma(E)A_2dE\Big)^{-1/n}\Big\}$$
$$+(A_2-A_1)\int\limits_0^{E^*} K(E)dE.$$

If $(A_2-A_1) < 0$ and the term in brackets is positive, then, independently of n,

$$\lim_{E\to\infty}(T_2(E)-T_r(E)) = (A_2-A_1)\int\limits_0^{E^*} K(E)dE.$$

The behaviour of the expression (4.9) at infinity is governed mainly by the first term; in order to describe properly the properties of the material, the functions K and Γ must be suitably selected, and the following expression should by analyzed:

(4.10)
$$(c_1+f_\infty)(d_2+g_\infty)^{-1/n}-(c_2+f_\infty)(d_2+g_\infty).^{-1/n}$$

Here

$$c_1 \equiv \varkappa_0 + A_2 \int_0^{E^*} K(E)dE, \qquad c_2 \equiv \varkappa_0 + A_1 \int_0^{E^*} K(E)dE,$$

$$d_1 \equiv \gamma_0 + A_2 \int_0^{E^*} \Gamma(E)dE, \qquad d_2 \equiv \gamma_0 + A_1 \int_0^{E^*} \Gamma(E)dE,$$

$$f_\infty \equiv A_2 \int_{E^*}^{\infty} K(E)dE, \qquad g_\infty \equiv A_2 \int_{E^*}^{\infty} \Gamma(E)dE.$$

For instance, the limit of the difference (4.9) is finite if the functions $K(E)$ and $T(E)$ are such that the integrals f_∞, g_∞ are divergent under the condition $f_\infty/(g_\infty)^n = \text{const} \neq 0$, since then the expression (4.10) tends to zero at $E \to \infty$.

5. Conclusions

The results presented in the paper indicate that the description of viscoplastic materials by means of internal parameters yields results which are in good agreement with the experiments. Owing to the arbitrary choice of material functions in the constitutive relations, the considerations presented are not confined to any particular material but they enable us to take into account the effect of strain history on dynamic curves of a broad class of materials. On the other hand, experimental results supply valuable information which is useful in identifying material functions and in selecting suitable internal parameter which determine the character of the memory.

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