Arch. Mech., 33, 5, pp. 703-715, Warszawa 1981

Viscous flow through a half-infinite channel with moving and porous walls

A. ZACHARA (WARSZAWA)

THE SUBJECT of this paper is a flow in the interior of the channel, considered as a slightly disturbed Poiseuille flow as well as a flow inside the wall treated as an anisotropic porous material, where the flow obeys the Darcy law. The flow is driven by the wall motion which generates the suction of a fluid from the environment into the channel. Similarity laws connected with this problem have been found and pressure, velocity and streamline distributions have been calculated for the wall and channel flow. The validity of the adopted approximation has been discussed.

Przedmiotem pracy jest przepływ wewnątrz kanału opisywany równaniami cieczy lepkiej w ramach przybliżenia słabo zaburzonego przepływu Poiseuille'a, a także przepływ wewnątrz ścianki kanału (traktowanej jako anizotropowy ośrodek filtracyjny) opisywany uproszczonymi równaniami Darcy'ego. Ruch cieczy wymuszony jest ruchem ścianki, który powoduje zasysanie cieczy z otoczenia przez ściankę do wnętrza kanału. Zbadano prawa podobieństwa związane z rozpatrywanym przepływem oraz wyznaczono rozkłady ciśnień, prędkości i linii prądu we wnętrzu kanału oraz w ściance. Przedyskutowano zakres stosowalności przyjętego modelu.

В статье представлено течение внутри канала, описываемое уравнениями вязкой жидкости, как слабо возмущенное течение Пуазейля, а также течение внутри стенок канала, которые рассматриваются как анизотропная пористая среда, описанное упрощенными уравнениями Дарси. Движение жидкости вызвано движением стенок, в результате которого жидкость всасывается сквозь стенку в канал. Рассмотрены законы подобия, связанные с описываемым течением, определено распределение давления, скорости и линии тока как в канале, так и в стенке. Обсуждены условия допустимости принятой апроксимации.

1. Introduction

THE THEORETICAL investigation of a laminar flow in a channel with porous walls was initiated in 1953 by A. BERMAN [1]. Since then a number of papers have appeared where this problem was considered in its various aspects and types of approximation. Some examples of these works are listed in [2]. The purpose of this paper is to consider a channel of a particular type, as shown in Fig. 1. The characteristic feature of our analysis is to study not only a flow in a channel but also detailed structure of a flow within a wall which, thus far, has not been investigated.

The channel of plane or axisymmetrical geometry, shown in Fig. 1, is closed at one end by an impermeable diaphragm while the other, infinitely distant from the diaphragm, is open. We introduce the Cartesian or cylindrical coordinate system in which the z-axis coincides with the axis of symmetry and the x-axis is posed at the closed end of the channel. The velocity components in the x- and z-directions are denoted by u and w, the symbols referring to the inner and outer dimension of a wall R_1 and R_2 are marked by the subscripts

1 and 2, respectively. Permeable walls move in the z-direction from the diaphragm with an increasing velocity $W(z)$, whose final value W_∞ is reached some distance from the plane $z = 0$. As a result of viscous effects, the motion of the wall induces the flow of a fluid which penetrates through the porous wall into the channel and is then transported in the z-direction.

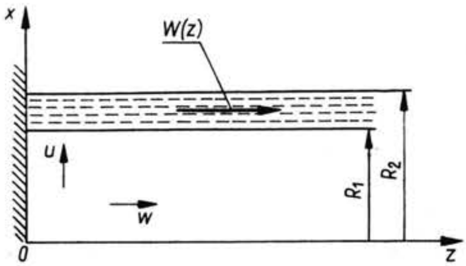


FIG. 1.

The problem set here-forth takes its origin in the field of man-made fibre technology and the picture described above presents one of the methods of fibre manufacturing. The moving wall of a channel corresponds to a bundle of fibres which is being extruded from a spinnerent fixed at $z = 0$, through a large nember of orifices uniformly distributed over an area of annular form. The case of plane geometry has here been included for the sake of completeness. Hydrodynamics of multifilament spinning has been subject of a series of studies reported in a survey paper [3] and in a monograph [4].

In this paper a channel as well as a wall flow will be treated with equal attention since the type of flow here considered results from the mutual interaction of both elements of the system. To calculate a flow in a wall, we shall apply the theoretical model introduced by A. SZANIAWSKI and the present author in [5]. We shall assume that the wall consists of a large number of identical, parallel fibres uniformly distributed over a cross-section of the wall and moving with the same velocity $W(z)$. The wall will be treated as an anisotropic porous material where the filtration velocity obeys the linear Darcy law. The number of fibers is large enough to consider the wall as a continuous porous medium; however, fibers are so thin that their volume is small compared with the total volume of the wall. The channel flow will be treated as a motion of a viscous fluid and described by the Navier–Stokes equations.

Although governed by equations of a different kind, the flows in the wall and in the channel are coupled by a proper condition at their boundary $x = R_1$, thus obtaining a flow description of a continuous character in the whole channel-wall system.

2. Wall and channel flow

Since the problem posed in the previous section is very complex, there is a need for certain model simplifications which would take into account merely the most important effects and make the problem more tractable for effective theoretical analysis. In this way the wall flow will be considered not in frames of a full filtration model [5], but in terms of its simplified version [6] which we call a transversal model. In this version the only component of the filtration velocity taken into account is the component transversal to fibres while in the z-wise direction the fluid moves with the velocity of fibres. Although these assumptions are fully adequate some distance from the spinneret, they also give surprisingly good results within its vicinity, as it was given in [6].

Thus the components of the filtration velocity in a laboratory reference system are as follows:

$$u_f = -\frac{S}{\mu} F_{\perp}(\varphi) \frac{\partial p}{\partial x}, \quad w_f = (1-\varphi) W(z), \tag{2.1}$$

where S is the cross-section of a wall per one fibre, $F_{\perp}(\varphi)$ is a nondimensional coefficient of filtration perpendicular to the fibres

$$F_{\perp}(\varphi) = \frac{1}{8\pi}\left(\ln\frac{1}{\varphi} - 1.5\right), \tag{2.1'}$$

and $\varphi(z)$ is the volume fraction of fibres in the wall.

Inserting $u = u_f$ and $w = w_f$ from Eq. (2.1) into the continuity equation

$$\frac{1}{x^k}\frac{\partial}{\partial x}(x^k u) + \frac{\partial w}{\partial z} = 0, \quad k = \begin{cases} 0 & \text{plane case,} \\ 1 & \text{cylindrical case} \end{cases} \tag{2.2}$$

and taking into account the fact that $\varphi \ll 1$, we obtain the following differential equation containing pressure as an unknown function:

$$\frac{1}{x^k}\frac{\partial}{\partial x}\left(x^k \frac{\partial p}{\partial x}\right) = \frac{\mu}{S \cdot F_{\perp}} W'(z). \tag{2.3}$$

Making use of the two boundary conditions

$$p(R_1, z) = p_1(z) \qquad p(R_2, z) = p_2 = \text{const}, \tag{2.4}$$

we find the solution, which has the following final forms corresponding to the plane ($k = 0$)

$$\frac{p-p_2}{\mu} = P(z)\frac{\chi-\xi}{\chi-1} - \frac{W'(z)(R_2^2-R_1^2)}{2SF_{\perp}} \frac{(\xi-1)(\chi-\xi)}{\chi^2-1} \tag{2.5}$$

and the axisymmetrical ($k = 1$) case

$$\frac{p-p_2}{\mu} = P(z)\left(1-\frac{\ln\xi}{\ln\chi}\right) - \frac{W'(z)\cdot(R_2^2-R_1^2)}{4SF_{\perp}}\left[\frac{\ln\xi}{\ln\chi} - \frac{\xi^2-1}{\chi^2-1}\right], \tag{2.5'}$$

where

$$\chi = \frac{R_2}{R_1}, \quad \xi = \frac{x}{R_1}, \quad P(z) = \frac{p_1-p_2}{\mu}. \tag{2.6}$$

From Eqs. (2.1) and (2.5) we have the transversal velocity distribution for the plane and axisymmetrical case:

$$u_f = KP(z)\frac{1}{\xi^k} - \frac{W'(z)R_1}{2^k}\left(\xi - \frac{G(\chi)}{\xi^k}\right), \tag{2.7}$$

where

$$K = \frac{SF_{\perp}(\varphi)}{R_1}\begin{cases}\dfrac{1}{\chi-1}\\[2mm] \dfrac{1}{\ln\chi}\end{cases}, \quad G = \begin{cases}\dfrac{1}{2}(\chi+1), & k=0,\\[2mm] \dfrac{\chi^2-1}{2\ln\chi}, & k=1.\end{cases} \tag{2.7'}$$

For the channel flow we shall confine our attention to the case which may be considered as a slightly disturbed Poisseulle flow [7]. This approximation imposes certain requirements on the geometry of streamlines as well as on the permeability and the velocity distribution of the wall. This question will be discussed in Sect. 5. According to this approximation, velocity components may be presented as follows:

$$\begin{aligned} w &= w_p + \tilde{w},\\ u &= \tilde{u}, \end{aligned} \tag{2.8}$$

where

$$w_p = W(z) - \frac{P'(z)}{2^{k+1}} R_1^2(1-\xi^2) \tag{2.9}$$

is basic Poiseuille solution and $\tilde{w}$, $\tilde{u}$ are its small perturbations. Inserting Eqs. (2.8) and (2.9) into the continuity equation (2.2) and neglecting $\partial\tilde{w}/\partial z$ as small compared with $\partial w_p/\partial z$, we obtain the transversal velocity distribution in a channel

$$\tilde{u} = \frac{1}{2^k}\left[\frac{P''R_1^3}{2^k(k+3)}U(\xi) - W'R_1\xi\right], \tag{2.10}$$

where

$$U(\xi) = \frac{1}{2}\xi[(k+3)-(k+1)\xi^2].$$

A knowledge of the velocity field makes it possible to calculate the streamlines $Q = \text{const}$ from the equation

$$dQ = (2\pi x)^k(wdx - udz) \tag{2.11}$$

where Q denotes the volume flow rate.

Inserting $w = w_f$, $u = u_f$ (2.1) for the wall flow and $w = w_p$ (2.9), $u = \tilde{u}$ (2.10) for the channel flow, after some rearrangements we obtain the following expressions in which the axes x and z of the coordinate system coincide with the streamline $q = 0$:

$$q = \frac{Q}{(\pi R_1)^k R_1} = \text{const} = W\xi^{k+1} - \frac{P'R_1^2}{(k+1)(k+3)}\begin{cases}\xi^k U(\xi), & 0 \leqslant \xi \leqslant 1.\\ 1, & 1 < \xi \leqslant \chi.\end{cases} \tag{2.12}$$

As it was pointed out in [7] the pressure in the x-wise direction may be taken as constant, which means that the pressure distribution in the channel flow is determined merely by the pressure value at the boundary of the wall $p(R_1, z) = p_1(z)$ or, in terms of our notation, by the function $P(z)$ (2.6). This quantity and its derivatives, unknown as yet, which appear in all expressions for the pressure, velocity and streamline distributions in the wall as well as in the channel flow (2.5), (2.7), (2.9), (2.10) and (2.12) may be found from the continuity condition for the transversal velocity at the boundary $x = R_1$

$$u_f(R_1, z) = \tilde{u}(R_1, z). \tag{2.13}$$

Putting $\xi = 1$ into Eqs. (2.7) and (2.10) and inserting them into Eq. (2.13), we obtain after some rearrangements the second-order ordinary differential equation for the pressure function

$$P''R_1^3 - 4^k(k+3)KP = 2^k(k+3)G(\chi)\,W'R_1. \tag{2.14}$$

With the aid of this equation we are able to transform Eq. (2.10) into the following form:

$$\tilde{u} = KPU(\xi) + W'R_1 \frac{1}{2^k}[G(\chi)U(\xi) - \xi]. \tag{2.15}$$

The transversal velocity in the channel is then a linear combination of two separate components determined by the permeability and the extensibility of the wall.

The axial velocity perturbation $\tilde{w}$ in the channel flow has been neglected in Eqs. (2.2) and (2.12) as small in comparison with the other terms. However, we shall use it to estimate the range of validity of the applied approximation. Putting Eq. (2.8) into the Navier–Stokes equations and linearizing them with respect to small perturbations, we obtain the equation [7]

$$w_p \frac{\partial w_p}{\partial z} + \tilde{u}\frac{\partial w_p}{\partial x} = \frac{\nu}{x^k}\frac{\partial}{\partial x}\left(x^k \frac{\partial \tilde{w}}{\partial x}\right).$$

With the use of Eqs. (2.9), (2.15) and the boundary conditions

$$\frac{\partial}{\partial x}\tilde{w}(0, z) = 0, \qquad \tilde{w}(R_1, z) = 0,$$

we find the solution

$$\tilde{w} = \frac{W'R_1^2}{\nu}\left[W\,W_1(\xi) - \frac{P'R_1^2}{2^{k+1}}W_2(\xi)\right] + \frac{KPR_1}{\nu}\left[W\,W_3(\xi) - \frac{P'R_1^2}{2^{k+1}}W_4(\xi)\right], \tag{2.16}$$

where

$$W_1(\xi) = \frac{1}{2^{k+1}}[G(\chi)\cdot 2\,W_3(\xi) - (1-\xi^2)],$$

$$W_2(\xi) = \frac{1}{2^{k+1}}\left[G(\chi)\cdot 2\,W_4(\xi) - (1-\xi^2) - \frac{1-k}{6}(1-\xi^4)\right],$$

$$W_3(\xi) = \frac{1-\xi^2}{8}[(k+5) - (k+1)\xi^2],$$

$$W_4(\xi) = \frac{1-\xi^2}{12(k+5)}[2(6k+23) - (15k-1)\xi^2 + (3k+1)\xi^4].$$

We can see that $\tilde{w}$ is a linear combination of the two Reynolds numbers based on both of the transversal velocity components resulting from the extensibility and the permeability of the wall. Thus the axial velocity perturbation $\tilde{w}$ is small in comparison with w_p $(2.8)_1$ if both Reynolds numbers are small or if the two components of the formula (2.16) are of an opposite sign and balance each other.

The flow in channel of permeable, infinitely thin walls reported in [7] may be treated as a limiting case of the problem considered in this paper. For $R_2 \to R_1$, or $\chi \to 1$ we obtain from Eq. (2.7')

$$\lim_{\chi \to 1} G(\chi) = 1. \tag{2.17}$$

Putting Eq. (2.17) into the relations (2.14), (2.15) and (2.16), we transform them to the corresponding formulae of the paper [7].

3. Calculation of the pressure function $P(z)$

The pressure function $P(z)$ can be calculated from the differential equation (2.14) which contains two functions $K(z)$ and $W(z)$, not specified as yet.

Throughout the present analysis it will be assumed that the permeability of the wall does not vary with z,

$$K(z) = \text{const.} \tag{3.1}$$

while $W(z)$ increases from 0 to W_∞, according to the relation

$$W(z) = W_\infty(1-e^{-z/\Lambda}), \tag{3.2}$$

where Λ is a length scale (see [6]).

The assumptions (3.1) and (3.2) are justified as the first approximation of real conditions in man-made fibre manufacturing. In fact, bundle permeability (2.7′) does vary with the distance z since fibre volume fraction $\varphi(z)$ decreases with z owing to the extensibility of fibres, but this variation is of a logarithmic character (2.1′) and does not have a significant influence on the final results of the calculations. The expression (3.2) is also not quite adequate since the initial velocity of the fibres $W(0)$ cannot be equal to zero; however, being much less than the final velocity W_∞, it may be neglected in Eq. (3.2).

Inserting Eqs. (3.1) and (3,2) into Eq. (2.14) and introducing the nondimensional parameters

$$\Phi = \frac{K}{R_1} \quad \text{and} \quad \beta = \frac{R_1}{\Lambda}, \tag{3.3}$$

we obtain

$$P''R_1^3 - 4^k(k+3)\Phi R_1 P = 2^k(k+3)G(\chi)\beta e^{-\beta\frac{z}{R_1}} W_\infty. \tag{3.4}$$

This equation contains three dimensionless parameters Φ, β, χ which determine the effects of permeability, extensibility and the thickness of the wall on the pressure function $P(z)$ and thereby on the flow in a wall-channel system. To reduce the number of parameters we shall transform Eq. (3.4) into nondimensional form

$$\Pi'' - \Pi = \gamma e^{-\gamma\zeta} \tag{3.5}$$

where the nondimensional pressure function

$$\Pi(\zeta) = \frac{P}{\overline{P}}, \quad \overline{P} = \sqrt{k+3}\,\frac{G(\chi)}{\sqrt{\Phi}}\,\frac{W_\infty}{R_1}, \tag{3.5′}$$

depends on the following coordinate.

$$\zeta = \alpha\frac{z}{R_1}, \quad \alpha = 2^k\sqrt{k+3}\,\sqrt{\Phi}, \tag{3.5″}$$

and a single free parameter:

$$\gamma = \frac{\beta}{\alpha} = \frac{\beta}{2^k\sqrt{k+3}\sqrt{\Phi}}, \tag{3.5'''}$$

which comprises the influence of extensibility and permeability of the wall. The other effect of finite wall thickness is included in the reference pressure $\bar{P}$ (3.5). The parameter γ takes values from the range $\langle 0, \infty \rangle$. The lower bound $\dot{\gamma} = 0$ corresponds to the case of rest ($W(z) = 0$, that is when $\beta = 0$ or $\Lambda \to \infty$) or to the case of infinite permeability ($\Phi \to \infty$), while the upper bound ($\gamma \to \infty$) may be interpreted as wall motion at constant speed $W(z) = W_\infty$. However, the latter case exceeds the frames of the model since it implies nonzero flow rate through the diaphragm at $z = 0$ (see Eq. (2.9)).

It should be noticed that the form of Eq. (3.5) is common to both plane and axisymmetrical cases. To find its solution we introduce two boundary conditions:

$$\Pi'(0) = 0 \qquad \lim_{\zeta \to \infty} \Pi(\zeta) = 0. \tag{3.6}$$

The first results from the impermeability of a plane $z = 0$ (see Eqs. (2.9,) (3.2) and (3.5')) and the second from the requirement that the pressure function ought to be finite at infinity. With the use of these boundary conditions the solution of Eq. (3.5) has the form

$$\Pi(\zeta) = \begin{cases} -\dfrac{\gamma}{\gamma^2-1}(\gamma e^{-\zeta} - e^{-\gamma\zeta}), & \gamma \neq 1, \\ -\dfrac{1}{2}(1+\zeta)e^{-\zeta}, & \gamma = 1. \end{cases} \tag{3.7}$$

The distribution of the nondimensional pressure function $\Pi(\zeta)$ has been calculated from Eq. (3.7) and presented in Fig. 2. It can be seen that the pressure function is always negative and increases monotonically up to zero at infinity, where the pressure on both sides of the wall equalize. Thus the greatest sucking effect takes place at the initial section of the channel, $z = 0$. This will be the subject of our detailed examination.

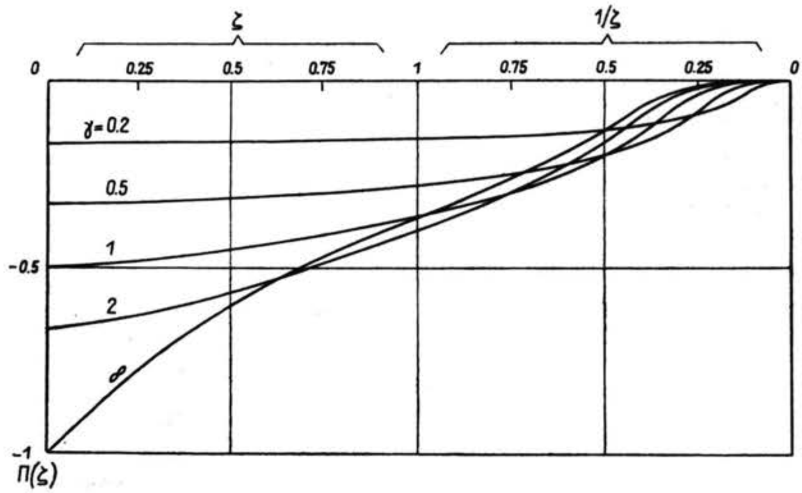


FIG. 2.

By means od Eqs. (3.5) and (3.7) we can express this pressure drop at $z \equiv 0$ in the following nondimensional form:

$$P(0)\frac{R_1}{W_\infty} = -G(\chi)\sqrt{\frac{k+3}{\Phi}}\,\frac{\beta}{\beta+2^k\sqrt{(k+3)\Phi}} \tag{3.8}$$

which displays the influence of the three factors: permeability, extensibility and thickness of the wall for both plane and axisymmetrical geometry. The relation (3.8) has been plotted in Fig. 3 for a wide range of the three parameters β, Φ, χ. It is seen that walls of

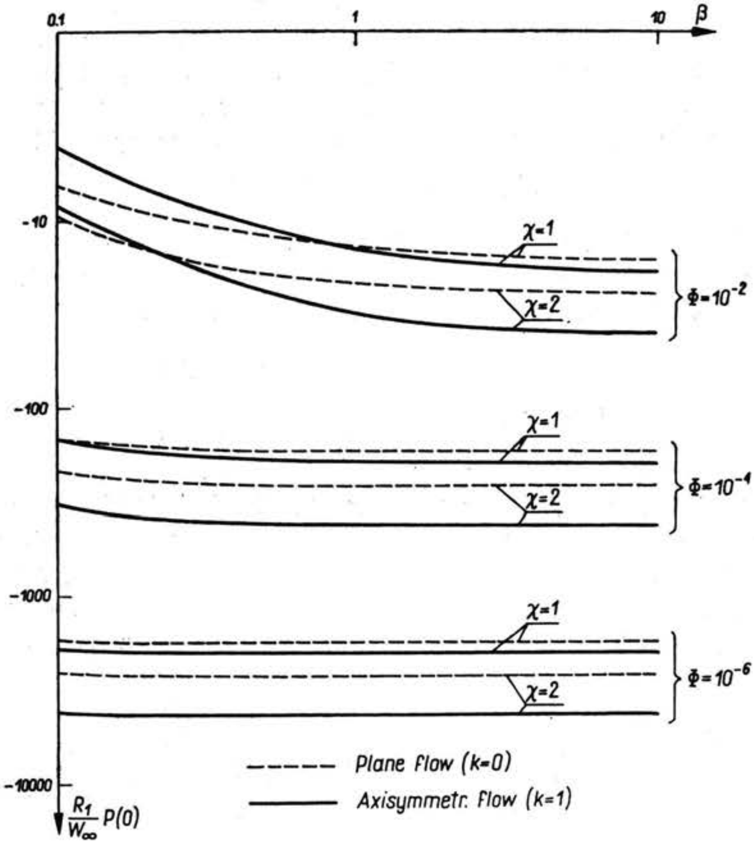


FIG. 3.

a high hydrodynamic resistance (large thickness χ or small permeability Φ) increase the pressure drop between the environment and the interior of the channel. The extensibility parameter β has a similar influence, which is better seen for walls of large permeability (e.g. $\Phi = 10^{-2}$, Fig. 3).

4. Pressure and streamlines distributions

Once the pressure function $P(z)$ has been found, all characteristics of the flow given in Sect. 2 can be calculated. Thus the pressure distribution within the wall, given by

Eq. (2.5), may be expressed in a nondimensional form for the plane and axisymmetrical geometry, respectively,

$$\frac{p-p_2}{\mu\bar{p}} = \Pi(\zeta)\frac{\chi-\xi}{\chi-1} - \frac{1}{2}\psi'(\zeta)\frac{(\xi-1)(\chi-\xi)}{\chi^2-1}, \tag{4.1}$$

$$\frac{p-p_2}{\mu\bar{p}} = \Pi(\zeta)\left(1-\frac{\ln\xi}{\ln\chi}\right) - \frac{1}{2}\psi'(\zeta)\left(\frac{\ln\xi}{\ln\chi} - \frac{\xi^2-1}{\chi^2-1}\right), \tag{4.2}$$

where $\psi(\zeta)$, derived from Eqs. (3.2), (3.3′) and (3.5‴),

$$\psi = 1-e^{-\gamma\zeta} \tag{4.3}$$

is the nondimensional wall velocity.

The pressure distribution in the channel is given by the function $\Pi(\zeta)$ itself since, according to our approximation, the pressure is uniform throughout the channel cross-section. Introducing the nondimensional flow rate

$$q = \frac{Q}{\bar{Q}}, \tag{4.4}$$

where $\bar{Q} = (\pi R_1)^k R_1 W_\infty$ is an asymptotic flow rate far from the plane $z = 0$, we can transform the expressions (2.12) into the following form:

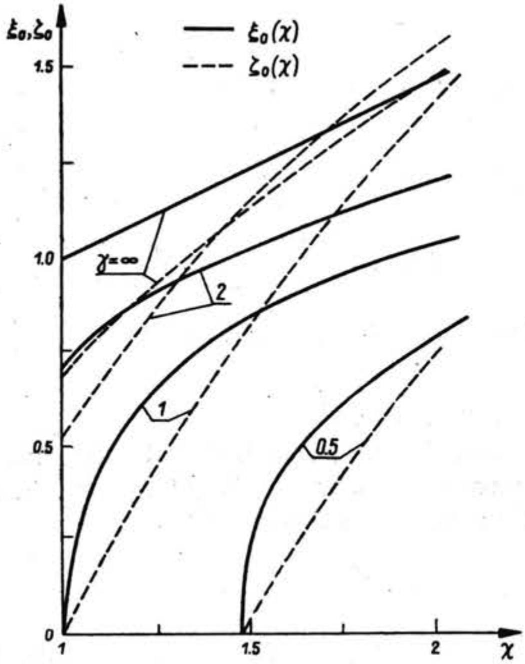
$$q(\xi, \zeta) = \psi(\zeta)\xi^{k+1} - G(\chi)\Pi'(\zeta)\begin{cases}\xi^k U(\xi), & 0 \leqslant \xi \leqslant 1,\\ 1, & 1 < \xi \leqslant \chi.\end{cases} \tag{4.5}$$

The analysis of Eq. (4.5) displays some peculiar features of the flow pattern. According to our definition, the streamline $q = 0$ coincides with the axes of the coordinate system. However, under certain conditions a supplementary streamline $q = 0$ is also possible. It extends from the axis ξ to the axis ζ, thus by passing the origin of the coordinate system (Fig. 7b). Below this streamline we can observe eddy which circulates with a negative flow rate forming a kind of separation bubble. It may be confined to the interior of the channel or may also penetrate the wall. The interesection points of the supplementary streamline with the axes of the coordinate system are denoted by $(\xi_0, 0)$ and $(0, \zeta_0)$. The coordinate ξ_0 may be calculated from Eq. (4.5) by putting $q = 0$,

$$\xi_0 = \begin{cases}\sqrt{\dfrac{1}{2^k}\left[(k+3) - \dfrac{2(\gamma+1)}{\gamma G(\chi)}\right]}, & 0 \leqslant \xi \leqslant 1,\\[2ex] \left[G(\chi)\dfrac{\gamma}{\gamma+1}\right]^{\frac{1}{k+1}}, & 1 < \xi \leqslant \chi,\end{cases} \tag{4.6}$$

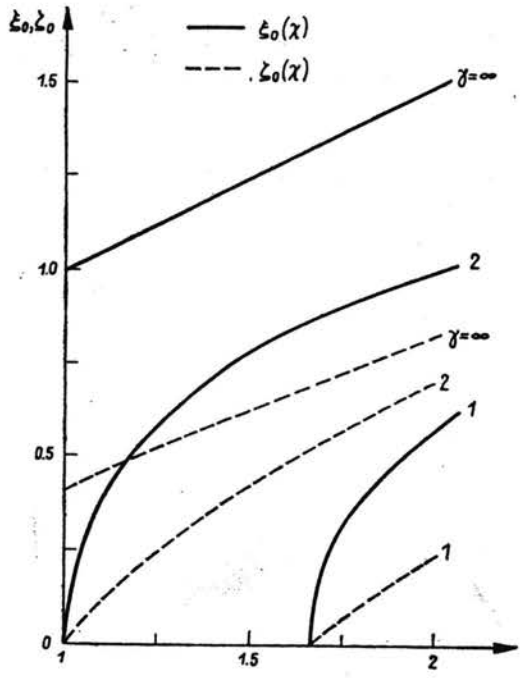
whereas the abscissa ζ_0 is determined by a transcendental equation $q(0, \zeta_0) = 0$ derived from Eq. (4.5). Both coordinates have been plotted against χ in Figs. 4 and 5 for the plane and axisymmetrical cases.

The eddy intensity q_0 can be measured by the value of a flow rate in the eddy centre where fluid velocity drops to zero. The intensity q_0 has been calculated and plotted in Fig. 6 against χ with γ as a shape parameter. We can see that the intensity q_0 is usually very small so that the separation bubble may be considered as a fluid at rest, except in



ξ_0, ζ_0
$\xi_0(\chi)$
$\zeta_0(\chi)$
1.5
1.0
0.5
0
1
1.5
2
χ
$\gamma=\infty$
2
1
0.5

FIG. 4.



ξ_0, ζ_0
$\xi_0(\chi)$
$\zeta_0(\chi)$
1.5
1.0
0.5
0
1
1.5
2
$\gamma=\infty$
2
$\gamma=\infty$
2
1
1

FIG. 5.

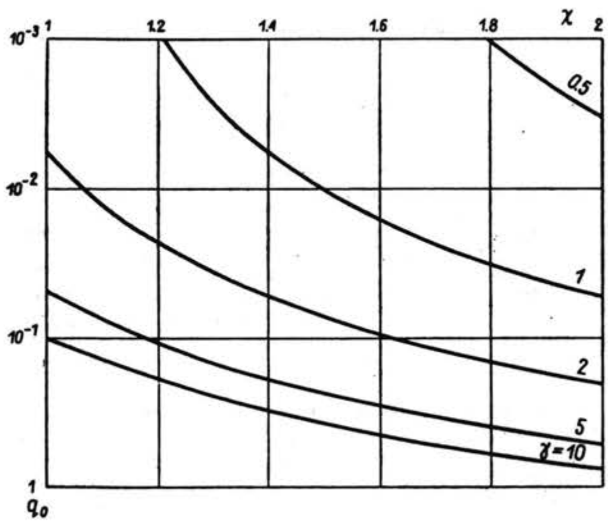


FIG. 6.

the cases of higher values γ and χ where the intensity is comparable with the asymptotic flow rate at infinity.

The eddy appears if the following condition is satisfied:

$$\gamma > \frac{2}{(k+3)G(\chi)-2} \tag{4.7}$$

following from Eq. (4.6) for $\xi_0^2 > 0$.

As an example, detailed calculations of isobars and streamlines have been carried out for the axisymmetrical channel in which the geometry is characterized by the parameter $\chi = 1.5$. The values $\gamma = 0.25$ and $\gamma = 1$ have been chosen so as to illustrate the non-separated and separated flow pattern (Figs. 7a, b).

5. Validity of the model

The model introduced in this paper enables calculation of the pressure and velocity distributions in the wall as well as in the channel using simple formulae. This has been made possible owing to simplifying assumptions which, however, limit the generality of the model. In this section we shall briefly discuss the range of its applicability in the case of axisymmetrical geometry. The results for plane geometry are of a similar character with some quantitative discrepancies.

The main assumption of the model is the adopted approximation of the slightly disturbed Poiseuille flow, which can be expressed by the conditions

$$\frac{\tilde{u}}{w_p} \ll 1, \quad \frac{\tilde{w}}{w_p} \ll 1. \tag{5.1}$$

The first is generally satisfied except for the region near the closed end of the channel ($z = 0$) where it locally fails as it can be seen in Figs. 7a and b.

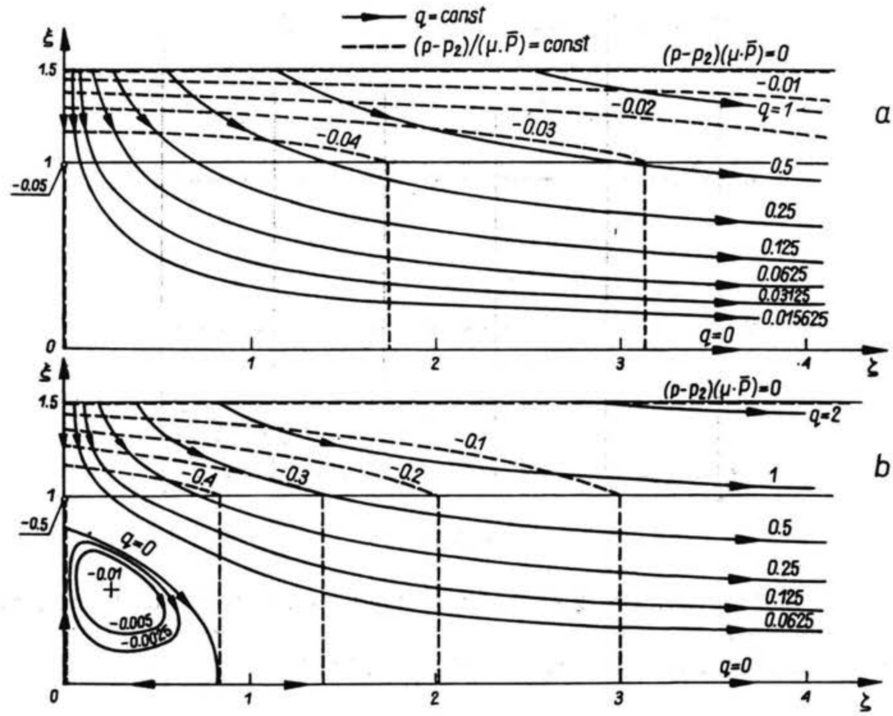


FIG. 7.

The second is of a dynamical character as the velocity perturbation $\tilde{w}$, expressed by the relation (2.16), takes partly into account the so-far neglected inertial terms of the Navier–Stokes equations. This condition requires more detailed examination. It is always satisfied at the wall ($x = R_1$) and at infinity ($z \to \infty$), where $\tilde{w} = 0$; however, in the interior of the channel including its initial cross-section $z = 0$ the fulfilment of this condition depends on flow properties. We shall examine the condition $(5.1)_2$ in the centre of the closing wall of the channel ($x = 0, z = 0$) where the fulfilment of this condition is most difficult. Inserting Eqs. (2.16) and (2.9) into Eq. $(5.1)_2$ we obtain after some rearrangements

$$\varepsilon = \frac{\tilde{w}(0,0)}{w_p(0,0)} \approx \beta \cdot \mathrm{Re}\frac{1}{4}\left[G(\chi)\cdot\frac{3}{2}\,\frac{\gamma}{1+\gamma}-1\right] \ll 1, \tag{5.2}$$

where

$$\mathrm{Re} = \frac{W_\infty R_1}{\nu}.$$

The quantity $\varepsilon/(\beta \cdot \mathrm{Re})$ has been plotted against γ for $\chi = 1, 2$ in Fig. 8. It should be noticed that ε is not proportional to the Reynolds number itself but to the product $\beta \cdot \mathrm{Re}$, which means that the low Reynolds number is not necessarily required to satisfy Eq. (5.2). This relation has the same character as in the case of a viscous flow through a slightly divergent channel, where neglecting inertial terms is justified if $\delta \cdot \mathrm{Re} \ll 1$, δ being the angle of divergence. The latter case may also be considered as a slightly disturbed Poiseuille flow [7]. It is worth noting that for a given parameter χ there exists such a value γ that $\varepsilon = 0$, apart from a magnitude of the product $\beta \cdot \mathrm{Re}$.

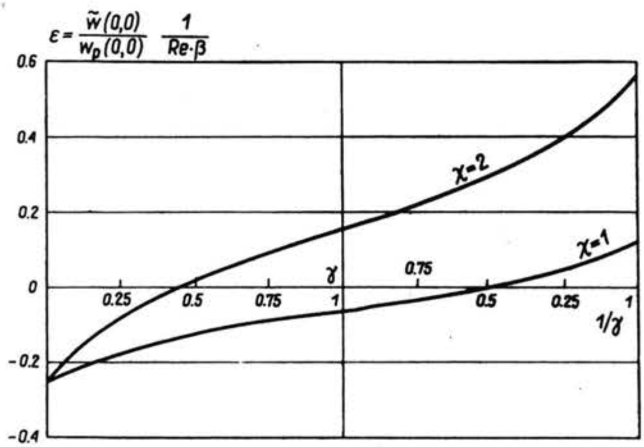


FIG. 8.

It results from our analysis that the validity of the model presented in this paper is limited to the case where the parameters determining the flow properties satisfy the condition (5.2). A more general model of a wider range of applicability should abandon the Poiseuille approximation and take into account inertial terms of the Navier–Stokes equations.

References

1. A. BERMAN, *Laminar flow in channels with porous walls*, J. Appl. Phys. **24**, 9, 1953.
2. B. K. GUPTA, E. K. LEVY, *Symmetrical laminary channel flow with wall suction*, J. Fluid Eng. (Trans. ASME), Ser. I, 3, 1976.
3. A. SZANIAWSKI, A. ZACHARA, *Hydrodynamic problems of multifilament spinning*, Chemické Vlákná, **26**, 1–2, 1976.
4. A. ZIABICKI, *Fundamentals of fibre formation*, John Wiley, 1976.
5. A. SZANIAWSKI, A. ZACHARA, *Penetracja kąpieli do strefy formowania wiązki włókien sztucznych*, Polimery, **19**, 3–4, 1974.
6. A. SZANIAWSKI, *Względny przepływ filtracyjny w poprzek symetrycznych wiązek włókien*, Polimery, **22**, 2, 1975.
7. A. SZANIAWSKI, A. ZACHARA, *Przepływ laminarny w kanale o zmiennym przekroju z ruchomymi i porowatymi ściankami*, Mech. Teoret. i Stos., **16**, 3, 1978.
8. G. K. BATCHELOR, *An introduction of fluid dynamics*, Cambridge, 1970.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received December 11, 1980.