Arch. Mech., 43, 6, pp. 743–751, Warszawa 1991

Numerical analysis of viscoelastic fluid flux through the orifice

J. POSPOLITA (OPOLE)

A MATHEMATICAL MODEL of the flow of the viscoelastic flow through a pipeline with the orifice has been formulated on the basis of the Phan–Thien–Tanner rheological equations. Equations of this model have been solved by the finite difference method. Numerical analysis of the influence of rheological parameters and of the Weissenberg number on the discharge coefficient of the orifice has been presented. Some conclusions, useful from the viewpoint of metrology, have been formulated. Directions for further investigations have been presented.

1. Introduction

MEASUREMENT of fluids with non-Newtonian rheological properties is a problem of metrology of flows. Polymer solutions, glues or various two-phase fluids which, under certain conditions, can be considered as homogeneous media having modified rheological properties, are examples of such fluids. The conventional flowmeters are designed and calibrated for measurements of Newtonian fluid flows and, if they are applied for non-Newtonian fluids, an additional measuring error will occur. Its value depends mainly on rheological properties of the fluid. In most cases non-Newtonian fluids are very viscous and this fact makes the measurements more difficult.

There are many journals and papers relating to non-Newtonian fluids but only a small number of papers concern measurements of their flows [1, 2, 7, 15] and the results presented in these papers do not enable us to formulate any reasonable conclusions of the metrological character. Theoretical considerations and experiments, the results of which were presented in [6, 13, 14], dealt with problems connected with application of orifices for measurements of power law fluids flow [16]. This rheorogical model has been assumed because it yields rheological properties of many non-Newtonian fluids with a good approximation. The assumed model is very simple; owing to that, determination of material constants and realization of numerical investigations become quite easy. From the results obtained certain important conclusions have been drawn. Since in many non-Newtonian fluids, such as polymer solutions with high molecular weight, certain effects resulting from differences of normal stresses can be observed (for example Weissenberg effect, [17]) numerical investigations should be based on a more complicated model, suitable for viscoelastic fluids. In paper [17] it has been observed that rheological properties of viscoelastic fluids are functions of their "histories". These properties cannot be expressed only by the relationship between shear stress and shearing rate. The rheological description of these fluids should include not only shear stress and shear rate but time derivatives of these quantities as well. Occurrence of effects of normal stresses is often understood as a criterion for viscoelastic properties of the fluid. It often happens that the difference in normal stresses may be, at certain shear rates, higher than the shear stress. Thus, during the determination of rheological properties of the considered fluids, not only shear stress but the difference of normal stresses versus shear rate should be known as well.

The numerical investigations presented in this paper were carried out in order to consider a possibility of application of orifices for measurements of viscoelastic fluid flows. Here one very important question arises: for which parameters characterizing the fluid, the flow, and the flowmeter can we use pipe orifices without taking into account differences in characteristics $\alpha = f(\mathrm{Re})$, resulting from viscoelastic properties of the medium. The criterion may be the value of the additional measuring error. Since many viscoelastic fluids are very viscous, the analysis should include special orifices [9, 11] used for flows with low values of Reynolds numbers.

2. Mathematical model of viscoelastic fluid flow through the orifice

The viscoelastic fluid flow through a pipeline with the orifice is considered (Fig. 1). It is assumed that the flow is axisymmetric and the velocity profile at the inlet section is fully developed. For description of rheological properties of the fluid the Phan-Thien-Tanner model was used [3]. This model allows us to obtain stable solutions for Weissenberg numbers (We $= \lambda v_m/D$) characterizing viscoelastic fluid flows which were higher than in case of other models, for example the MAXWELL or OLDROYD [3, 10].

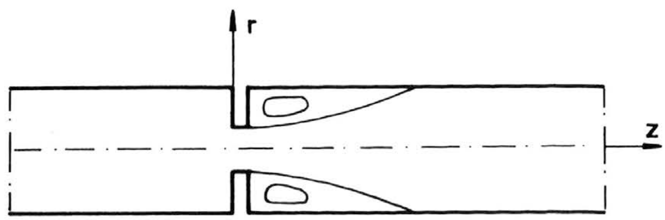

FIG. 1. The flow system

The mathematical model was based on equations of fluid motion

$$\varrho \frac{Dv}{Dt} = -\nabla p + \nabla \cdot \tau, \tag{2.1}$$

where

$$\tau = \tau^* + 2\eta_s D$$

equation of continuity

$$\nabla \cdot v = 0 \tag{2.2}$$

and Phan-Thien-Tanner rheological equation

$$\tau^* = -\lambda\left(\frac{D\tau^*}{Dt} - L\tau^* - \tau^* L^T\right) - \varepsilon\frac{\lambda}{\eta_m} tr(\tau^*)\tau^* - \lambda\xi(D\tau^* + \tau^* D^T) + 2\eta_m D. \tag{2.3}$$

In those equations v is a vector of velocity with components v_z, v_r, v_θ where v_z is a component in direction of the z — axis, v_r is a radial component and v_θ is an angular component which is, in the considered case, equal to zero, τ is a stress tensor, L — a velocity gradient tensor, D — a deformation rate tensor. The stress tensor τ consists of the term $2\eta_s D$, expressing the contribution of stresses from the Newtonian fluid (solvent) with viscosity η_s, and an additional tensor τ^*, expressing viscoelastic properties of the fluid. Components of the tensor τ^* are determined in the rheological equation (2.3), where λ,

η_m, ε and ξ are physical parameters of the considered viscoelastic fluid. λ is relaxation time, η_m expresses viscosity of the medium (this parameter is treated as constant, $\eta_m > \eta_s$) ε and ξ define particitation of particular terms in the rheological equation. The parameter ε expresses viscoelastic fluid behaviour during elongational flows [4, 17]; ξ represents the fluid properties duringsimple shearing.

In the assumed cylindrical coordinate system, Eqs. (2.1), (2.2) and (2.3) of the mathematical model take the following forms: equation of motion for the axial component v_z of the velocity vector

$$\varrho \frac{Dv}{Dt} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial \tau_{zz}}{\partial z}, \tag{2.4}$$

where the derivative Dv_z/Dt is expressed by

$$\frac{Dv_z}{Dt} = \frac{\partial}{\partial z}(v_z^2) + \frac{1}{r}\frac{\partial}{\partial r}(r v_r v_z);$$

equation of motion for the radial component v_r of the velocity vector

$$\varrho \frac{Dv_r}{Dt} = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}, \tag{2.5}$$

where

$$\frac{Dv_r}{Dt} = \frac{\partial}{\partial z}(v_z v_r) + \frac{1}{r}\frac{\partial}{\partial r}(r v_r^2);$$

equation of continuity

$$\frac{\partial v_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r v_r) = 0. \tag{2.6}$$

Taking into account axial symmetry of the flow, $v_\theta = 0$, $\partial\phi/\partial\theta = 0$, we obtain the following equations components of the stress tensor

$$\begin{aligned}\tau^*_{rr} = {}& -\lambda\left(v_r\frac{\partial \tau^*_{rr}}{\partial r} + v_z\frac{\partial \tau^*_{rr}}{\partial z} - 2\frac{\partial v_r}{\partial r}\tau^*_{rr} - 2\frac{\partial v_r}{\partial z}\tau^*_{rz}\right)\\ &-\varepsilon\frac{\lambda}{\eta_m}(\tau^*_{rr} + \tau^*_{zz} + \tau^*_{\theta\theta})\tau^*_{rr} - \lambda\xi\left(2\frac{\partial v_r}{\partial r}\tau^*_{rr} + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)\tau^*_{rz}\right) + 2\eta_m\frac{\partial v_r}{\partial r},\end{aligned} \tag{2.7}$$

$$\begin{aligned}\tau^*_{zz} = {}& -\lambda\left(v_r\frac{\partial \tau^*_{zz}}{\partial r} + v_z\frac{\partial \tau^*_{zz}}{\partial z} - 2\frac{\partial v_z}{\partial z}\tau^*_{zz} - 2\frac{\partial v_z}{\partial r}\tau^*_{rz}\right)\\ &-\varepsilon\frac{\lambda}{\eta_m}\left(\tau^*_{rr} + \tau^*_{zz} + \tau^*_{\theta\theta}\right)\tau^*_{zz} - \lambda\xi\left(2\frac{\partial v_z}{\partial z}\tau^*_{zz} + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)\tau^*_{rz}\right) + 2\eta_m\frac{\partial v_z}{\partial z},\end{aligned} \tag{2.8}$$

$$\begin{aligned}\tau^*_{rz} = {}& -\lambda\left(v_r\frac{\partial \tau^*_{rz}}{\partial r} + v_z\frac{\partial \tau^*_{rz}}{\partial z} - 2\frac{\partial v_r}{\partial r}\tau^*_{rz} - \frac{\partial v_r}{\partial z}\tau^*_{zz} - \frac{\partial v_z}{\partial r}\tau^*_{rr} - \frac{\partial v_z}{\partial z}\tau^*_{rz}\right)\\ &-\varepsilon\frac{\lambda}{\eta_m}(\tau^*_{rr} + \tau^*_{zz} + \tau^*_{\theta\theta})\tau^*_{rz} - \lambda\xi\left(2\frac{\partial v_r}{\partial r}\tau^*_{rz} + \frac{1}{2}\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)\tau^*_{zz}\right.\\ &\left.+\frac{1}{2}\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)\tau^*_{rr} + \frac{\partial v_z}{\partial z}\tau^*_{rz}\right) + \eta_m\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right),\end{aligned} \tag{2.9}$$

$$\tau^*_{\theta\theta} = -\lambda\left(v_r\frac{\partial \tau^*_{\theta\theta}}{\partial r} + v_z\frac{\partial \tau^*_{\theta\theta}}{\partial z}\right) - \varepsilon\frac{\lambda}{\eta_m}(\tau^*_{rr} + \tau^*_{zz} + \tau^*_{\theta\theta})\tau^*_{\theta\theta} - 2\lambda\xi\frac{v_r}{r}\tau^*_{\theta\theta} + \eta_m\frac{2v_r}{r}. \tag{2.10}$$

The equations of motion and the equation of continuity can be expressed in a general form

$$(2.11)\quad \varrho\left(\frac{\partial}{\partial z}(v_z\phi)+\frac{1}{r}\frac{\partial}{\partial r}(rv_r\phi)\right)=\frac{\partial}{\partial z}\left(\Gamma_\phi\frac{\partial\phi}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi\frac{\partial\phi}{\partial r}\right)+S_\phi+S_{\phi_D},$$

where S_{ϕ_D} is an additional term expressing the contribution of the additional tensor τ^*.

The values of Γ_ϕ, S_ϕ and S_{ϕ_D} for particular equations are shown in Table 1.

Table 1. Coefficients of Eq. (2.11)

ϕ	Γ_ϕ	S_ϕ	S_{ϕ_D}
v_z	η	$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\eta\frac{\partial v_r}{\partial z})+\frac{\partial}{\partial z}\eta\frac{\partial v_z}{\partial z}$	$\frac{\partial\tau^*_{zz}}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\tau^*_{rz})$
v_r	η	$-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}(r\eta\frac{\partial v_r}{\partial r})+\frac{\partial}{\partial z}(\eta\frac{\partial v_z}{\partial r})-2\eta\frac{v_r}{r^2}$	$\frac{1}{r}\frac{\partial}{\partial r}(r\tau^*_{rr})-\frac{\tau^*_{\theta\theta}}{r}+\frac{\partial\tau^*_{rz}}{\partial z}$
1	0	0	0

The stresses τ^*_{rr}, τ^*_{zz}, τ^*_{rz}, $\tau^*_{\theta\theta}$ represent this part of components of the tensor τ which express viscoelastic properties of the fluid. Without the term S_{ϕ_D}, equations of motion (2.11) are the mathematical model for the Newtonian fluid flow through the orifice. The system of equations (2.4), (2.5) and (2.6)–(2.10) is, in the considered flow area, completed with suitable boundary conditions for velocity vector components and for the stress tensor components.

It was assumed that at the inlet section (several diameters before the orifice) the velocity profile and values of the stress tensor components correspond to a fully developed flow of the fluid with the given rheological parameters assumed for the calculations. The velocity profile and values of components of the tensor τ^* are a condition of Dirichlet's type in the considered boundary value problem. This profile and values of stress tensor components were obtained by numerical solution of the problem of forming a fully developed fluid flow (having the given rheological parameters) through a pipe. The profile was assumed as fully developed when relative changes of the component v_z and components of the tensor τ^* in further sections of the pipeline did not exceed 1%; it occured at the distance of 15–20 diameters from the initial section.

At the outlet, for the radial component of the velocity vector $v_r = 0$ and for the other variables $\partial\phi/\partial z = 0$ are assumed. At the symmetry axis it has been assumed that $v_r = 0$ and $\partial\phi/\partial r = 0$. Thus at the symmetry axis components of the stress tensor are

$$\tau^*_{rr} = r^*_{rz} = r^*_{\theta\theta} = 0,$$

and

$$\tau^*_{zz} = -\lambda\left(v_z\frac{\partial\tau^*_{zz}}{\partial z}-2\frac{\partial v_z}{\partial z}\tau^*_{zz}\right)-\varepsilon\frac{\lambda}{\eta_m}(\tau^*_{rr}+\tau^*_{zz}+\tau^*_{\theta\theta})\tau^*_{rr}-2\lambda\xi\frac{\partial v_z}{\partial z}\tau^*_{zz}+2\eta_m\frac{\partial v_z}{\partial z}.$$

At the pipeline walls and the orifice it is assumed that $v_r = 0$ and $v_z = 0$. For simplicity it is also assumed that $\partial v_z/\partial z = 0$ and $\partial v_r/\partial r = 0$. Equations expressing τ^*_{rr}, τ^*_{zz}, τ^*_{rz}, $\tau^*_{\theta\theta}$ in the immediate vicinity of the walls result from Eqs. (2.7)–(2.10) under the above assumptions for components of the velocity vector.

3. Algorithm of calculations

The presented equations of the mathematical model were solved by the finite difference method. Structure of the differential grid and approximation of particular derivatives are similar to those for digital simulation of Newtonian or power-law fluid flows [5, 6]. Equations for particular components of the stress tensor were approximated in a similar way; for example, differences down-stream oriented were used in the schemes of difference convection terms. The basic part of the algorithm is the same as for Newtonian fluids. The difference consists in solution of equations for components of the additional stress tensor during the basic iteration cycle. These equations form additional terms in equations of motion (2.4) and (2.5). The assumed algorithm consists in cyclic repeating the following steps:

1) determination of initial distribution of all the variables, together with pressure p and components of the stress tensor τ;

2) solving Eqs. (2.4) and (2.5) for components v_z and v_r of the velocity vector;

3) calculation of pressure p and correction of values v_z and v_r basing on the equation of continuity of flow;

4) solving, by the implicit method, Eqs. (2.7)–(2.10) for the stress tensor components;

5) the solutions obtained should be understood as approximate and now it is necessary to return to step 1.

In 4) an internal iteration loop was realized for all components of the tensor τ^* (3–10 iterations in each external cycle). In solving the differential equations of the model, relaxation of all variables was applied on the assumption that the relaxation parameter was 0.3–0.5 for components of the velocity vector and 0.1–0.3 for components of the stress tensor. The residual criterion was assumed as the criterion of convergence; as a result, difference equations can be satisfied with assumed accuracy in all points of the grid for each variable. The values of all variables obtained from the solved equations for lower values of We were assumed as initial. Such procedure, however, has not raised the boundary value of the Weissenberg number, below which stable solutions were obtained.

4. The results of calculations

In this section the results of calculations made on the basic fluid have been presented. The assumed mathematical model made it possible to obtain the stable solutions for flows for which Weissenberg numbers were close to one. It should be, however, noticed that the solutions obtained, based on the Phan-Thien-Tanner rheological model for higher We numbers, concerned mainly flows for which the Reynolds numbers were less than 1 (see also [12]). Such decrease of the orifice modulus made it difficult to obtain stable solutions, considering increase of the gradients of all variables. For the same reasons the investigations were conducted only for the flows for which We numbers were contained between $0 \div 0.1$, and for parameters ε and ξ from within the ranges 0–0.2 and 0–1, respectively, i.e. the same ranges as those for the known viscoelastic fluids. Although the values of We were relatively low, it was found how the discharge coefficients of the orifices depended on rheological parameters of the viscoelastic fluids. Figures 2–6 show the dependence of the discharge coefficient α for the orifice upon the parameters characterizing the flow and the viscoelastic fluid. The value of α was related to the discharge coefficient for the

Newtonian fluid at the same fluid flux and the Reynolds number defined by

$$\mathrm{Re} = \frac{V_m D \rho}{\eta},$$

where

$$\eta = \eta_m + \eta_s.$$

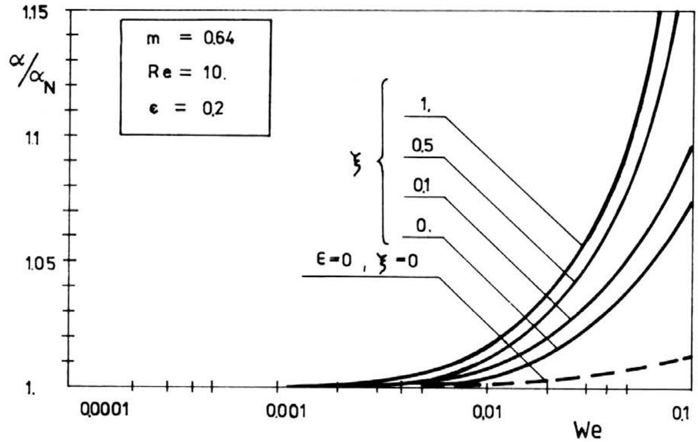

FIG. 2. Dependence of the discharge coefficient for the orifice on the Weissenberg number and parameter ξ.

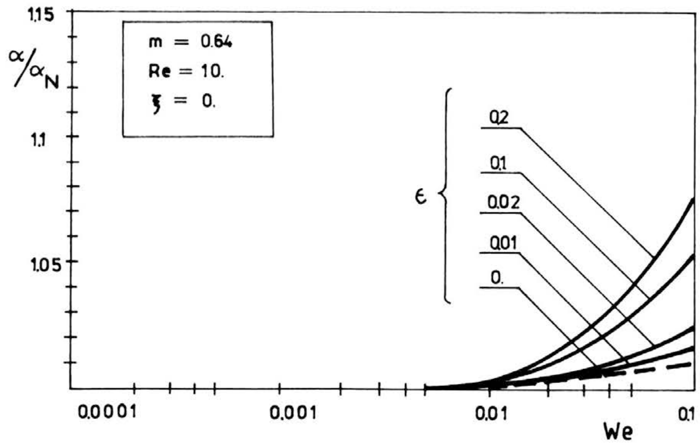

FIG. 3. Dependence of the discharge coefficient on the Weissenberg number and parameter ε.

Figure 2 shows the dependence of α/α_N upon the Weissenberg number for the parameter ξ. It is clear that any important changes of the discharge coefficient can be observed when We > 0.01; the changes are greater if the value of ξ increases. As for the fluid expressed by the Maxwell equation, for which $\varepsilon = 0$ and $\xi = 0$, these changes do not exceed 3% of the value of α for We $= 0 - 0.1$ and Re $= 10 - 100$. Influence of ε on α, shown in Fig. 3, is similar but in the considered range the influence is less than that

Newtonian fluid at the same fluid flux and the Reynolds number defined by

$$\mathrm{Re} = \frac{V_m D \rho}{\eta},$$

where

$$\eta = \eta_m + \eta_s.$$

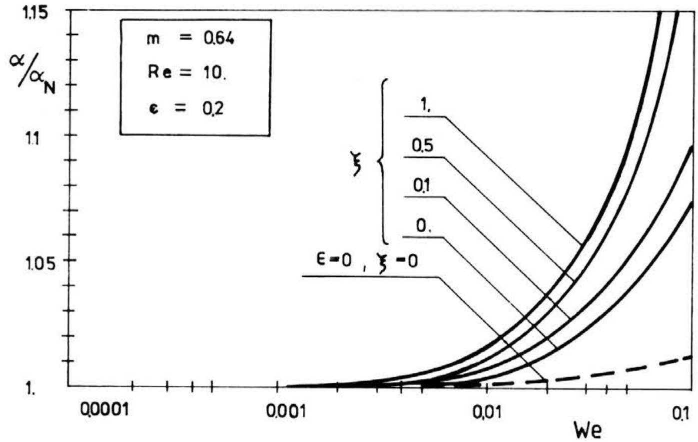

FIG. 2. Dependence of the discharge coefficient for the orifice on the Weissenberg number and parameter ξ.

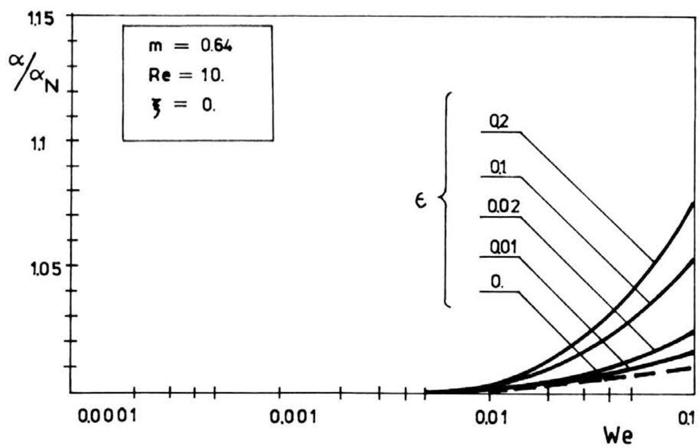

FIG. 3. Dependence of the discharge coefficient on the Weissenberg number and parameter ε.

Figure 2 shows the dependence of α/α_N upon the Weissenberg number for the parameter ξ. It is clear that any important changes of the discharge coefficient can be observed when We > 0.01; the changes are greater if the value of ξ increases. As for the fluid expressed by the Maxwell equation, for which $\varepsilon = 0$ and $\xi = 0$, these changes do not exceed 3% of the value of α for We $= 0 - 0.1$ and Re $= 10 - 100$. Influence of ε on α, shown in Fig. 3, is similar but in the considered range the influence is less than that for a flow where We = 0.08 of a fluid with $\varepsilon = 0.01$. The calculations were made for two different Reynolds numbers. Except for $\xi = 0 - 0.1$, the changes of α are larger for flows for which Re is less, i.e. when influence of fluid viscosity on the fields of hydrodynamical quantities is more visible.

Results of digital simulation of flows through the orifices having different moduli, as well as influence of ξ on α for various m are shown in Fig. 6. In relation to the discharge coefficient for a Newtonian fluid, the differences of α are much larger when the modulus of the orifice is smaller, because smaller modulus is connected with larger deformation of the velocity field and with more visible effects resulting from viscoelastic properties of the fluid.

The discussed numerical investigations were made in order to determine influence of fluid elasticity on differential pressure during the flow through the orifice. In experiments this problem is rather difficult because the value of Δp depends also on other rheological properties of the considered fluids. Therefore, it is difficult to verify the experimental results. From all the papers accessible to the author only the experimental investigation by SHIMA [15] concerns the results for a flow through the orifice. In [15] a flow of polyacryloamide solutions, considered as viscoelastic fluids, through orifices with different moduli, installed in a pipe of 10.5 m in diameter, has been analyzed. From the investigations (made also by this author — see [13, 14]) it results that the analyzed solutions are also power - law fluids diluted with shear. In [15], however, there are no data concerning rheological parameters of the examined fluids. Basing on the results obtained by SHIMA one can state that, as for the turbulent flows, the discharge coefficient for the polymer flow is distinctly larger than that for water. In the transient zone Re = 10^3–3.10^3, in the case of an orifice with modulus 0.32, α is larger for the polymer (the difference is not large), and when the modulus is 0.6 the discharge number for the polymer is less than that for water. It should be noticed, however, that in the considered case the measuring points are distinctly scattered. From the measurements made by BATE [1] (polymer solution flows through the orifice with modulus 0.285 for Re = 500 – 3000) and investigations by TOMITA [17] ($m = 0.8, \mathrm{Re} \cong 2.10^4$) it results that the discharge coefficient for the orifice during the polymer flow is larger than that for water. On the other hand, investigations carried out by BILGEN [2], GILES [7] and HASEGAWA [8] proved that the flow of a polymer solution was characterized by a smaller value of α than the flow of a Newtonian fluid. However, the investigations of BILGEN, GILES and HASEGAWA were carried out for flows of the polymer solutions through holes with extremely small diameters (below one milimeter). According to SHIMA [15], such systems have different properties than those with a common orifice.

5. Conclusions and directions for future investigations

From the investigations based on digital simulation of laminar flow of viscoelastic fluids it results that, for the flows characterized by We < 0.01, the value of α is, in practice, the same as for Newtonian fluids. If We is higher than 0.01, a dependence of the discharge coefficient on the viscoelastic properties of the medium can be observed. The difference between the values of α for viscoelastic and Newtonian fluids increases together with the values of We, ε and ξ. From the results of numerical calculations it appears that the discharge coefficient for a viscoelastic fluid is always higher than that for a Newtonian fluid. It has been found that there are distinct differences between the data from experiments, given in references. However, the authors used viscoelastic fluids with different rheological properties and various flow systems for their experiments. Such a situation causes many problems connected with unification and interpretation of the measurement data and their application for verification of the numerical results. The mentioned difficulties strongly influence the directions of the future investigations on metrological properties of the conventional flowmeters used for measurements of viscoelastic fluid flows. These directions may be formulated in the following way:

1. A group of viscoelastic fluids which are most often used in practice should be separated. Next, suitable rheological models should be assigned to them. Their material parameters must be determined on the basis of the available data.

2. Experiments concerning viscoelastic fluid flows through constrictions must be realized. The obtained results of measurements will allow to perform numerical calculations and verify the applicability of the considered method for the problem analyzed.

3. On the basis of the theoretical considerations and experiments it would be possible to formulate some criterial relationships determining the range of applicability — mainly of constriction flow meters — for measurements of viscoelastic fluid flows.

References

1. H. G. BATE, *Orifice plate calibration in dilute polymer solution*, Nature, **216**, 1100, 1967.
2. E. BILGEN, *On the orifice flow of dilute polymer solutions*, **12**, 3, pp. 375–391, 1973.
3. M. B. BUSH, R. I. TANNER and N. PHAN-THIEN, *A boundary element investigation of extrudate swell*, J.Non-Newtonian Fluid Mech., **18**, pp. 143–162, 1985.
4. B. DEBRANT, I. M. MARCHAL and M. J. CROCHET, *Numerical simulation of highly viscoelsatic flows through an abrupt constriction*, J. Non-Newtonian Fluid Mech., pp. 119–146, 1988.
5. B. DOBROWOLSKI and J. POSPOLITA, *Theoretical estimation of the differential pressure type flowmeters in presence of pulsation of the mass flux*, Arch. Mech., **39**, pp. 571–588, 1987.
6. B. DOBROWOLSKI and J. POSPOLITA, *Numerical analysis of power law fluid flow through the orifice* [in Polish], Arch. Hydrotech., **37**, 1–2, pp. 157–173, 1990.
7. W. B. GILES, *Orifice flows of polyethylene oxide solutions*, Nature, **224**, 584, 1969.
8. T. HASEGAWA and T. IWAIDA, *Experiments on elongational flow of dilute polymer solutions, Part 1. Jet reaction and excess pressure drop for the flow through small apertures*, J. Non-Newtonian Fluid Mech., **15**, pp. 257–277, 1984.
9. З. КАБЗА, *Математическое моделирование расходомеров с сузающчими устройствами*, Машиностроение, Ленинград 1981.
10. R. KEUNINGS and M. J. CROCHET, *Numerical simulation of the flow a viscoelastic fluid through an abrupt constriction*, J. Non-Newtonian Fluid Mech., **14**, pp. 279–299, 1984.
11. P. P. КРЕМЛЕВСКИЙ, *Раходомеры и счотуики количества*, Мшиностроение, Ленинград 1975.
12. M. G. N. PERERA and K. WALTERS, *Long range effects in flows involving abrupt changes in geometry. Part 1. Flow associated with L-shaped and T-shaped geometries*, J. Non-Newtonian Fluid Mech., **2**, pp. 49–81, 1977.
13. J. POSPOLITA, *Experimental investigations of power law fluid flows through the orifice* [in Polish], Arch. Hydrotech., 1-2, pp. 175 - 184, 1990.
14. J. POSPOLITA and K. WOJS, *First estimate of metrological properties of an orifice in power law fluid flow measurements* [in Polish] Reports of VIII Polish Fluid Mech. Conference, Bialystok-Bialowieza 1988.
15. N. SHIMA, *Loss and discharge characteristics of a flow of polymer solutions through pipe orifices*, Bull. ISME, **27**, 225, pp. 443–449, 1984.
16. W. L. WILKINSON, *Non-Newtonian liquids*, WNT, Warszawa 1963.
17. S. ZAHORSKI, *Mechanics of visco-elastic liquid flows* [in Polish], Bibl. Mech. Stos., PWN Warszawa–Poznań 1978.

TECHNICAL UNIVERSITY OF OPOLE, OPOLE.

Received April 5, 1991.