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On the periodic wave propagation along the elastic fibre in the elastic matrix

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AXISYMMETRIC PROBLEM of plane wave propagation along the elastic rod of circular cross-section embedded in elastic space is considered. Approximation of plane uniformly deformed cross-section is employed for the rod. Shear stress continuity condition at the interface is replaced by the weaker integral condition of the axial momentum balance for the rod. Solutions for the elastic fields in the surrounding medium are constructed, Hankel functions of complex variable being used. The dynamic field obtained can be considered as the superposition of two elastic periodic waves emitted by the rod at strictly defined angles with respect to the rod axis. For certain sets of parameter values the characteristic equation has been numerically solved, the dispersive relations being obtained for the longitudinal wave in the rod. Relations describing the propagation angles and amplitude decay decrement changes versus the wave frequency have been also found.

1. Introduction

THE SIGNIFICANT role of the "micro-dynamic" effects in the fibre-reinforced composite fracture under quasi-static load is generally recognized at present.

It is rather evident that, for the understanding of the catastrophic "chain process" of the fibre ruptures caused by the ruptures of the neighbouring ones, one needs some information on the transmission of the dynamic pulse generated in the process of fibre rupture along the fibre axis as well as on the energy radiation from the fibre into the matrix.

As it has already been pointed out by the present author [1], some of the existing schemes of the process cannot be considered to be adequate for the description of the process under consideration. E.g. the model discussed by SAKHAROVA, OVCHINSKII *et al.* [2–7] being probably useful for the description of the large time scale process, seems to be improper for the short time scale problem of the dynamic interaction between the neighbouring fibres.

In the present paper the author tries to proceed a small step towards the understanding of this complex matter by presenting a description of the axisymmetric wave propagation along the fibre and the energy transmission from the fibre into infinite homogeneous surrounding elastic medium.

Solution of a similar plane problem for the elastic layer embedded in the elastic space was proposed earlier by the present author [1].

In the same manner as in the mentioned above paper [1], we shall seek for the leaky-wave type solution describing the energy radiation from the fibre rather than for a generalized Rayleigh (Stonley) wave type process of energy flux along the interface zone of the fibre and matrix materials.

One has to realize, of course, that any real dynamic process initiated by the fibre rupture is neither purely elastic nor periodic, and even most probably not linear. Nevertheless it seems to be worthwhile to study the present problem which, in author's opinion, can supply some additional (probably mostly qualitative) information, which can turn out to be useful for the composite material strength theory.

The author hopes also that the proposed solutions can be possibly interesting from the viewpoint of some practical acoustic problems, apart from the composite mechanics.

2. Formulation of the problem

We consider a system consisting of the linear elastic homogeneous isotropic circular cross-section rod (fibre) embedded in the same type material (matrix) with different elastic moduli and mass density. We shall focus our attention on the axisymmetric longitudinal wave propagation along the fibre axis.

We assume the continuity of the displacements as well as the continuity of the normal stress component at the fibre-matrix interface.

The continuity of the shear stress condition will be replaced with the weaker condition of the integral axial force and momentum balance for the entire fibre cross-section. This approach is compatible with the plane cross-section assumption, which we shall also adopt for our problem. The last assumption is commonly used for the long-wave approximate description of the longitudinal wave propagation in free rods, where the shear stress at the surface should be taken equal to zero. Thus we tacitly assume, that we restrict our considerations to the case of wavelengths exceeding the fibre radius and to low ratios of the matrix shear modulus to the fibre elastic moduli.

We shall assume at last, that the radial strain is constant over the entire fibre cross-section and, consequently, we disregard the radial dynamic terms. Thus we shall assume, that the radial equation of motion is — similarly as in the simplified theory of longitudinal waves in rods — fulfilled as an identity, which is sensible under the earlier mentioned restriction imposed on the wavelength.

We do not impose any restrictions on the dynamic fields in the matrix — outside the fibre — besides the boundary conditions at the interface and the sense of the energy flux, which should be directed outside the central axis of the system, expressing the energy radiation from the rod to infinity.

3. Dynamic field inside the fibre

Under the kinematic assumptions adopted in the previous section we can express the radial displacement field $u_r(r,z,t)$ and the axial displacement field $u_z(r,z,t)$ inside the fibre in the following form:

$$\begin{aligned} u_r(r,z,t) &= u_r^-(z,t)\frac{r}{R}, \\ u_z(r,z,t) &= u_z^-(z,t), \end{aligned} \tag{3.1}$$

where r and z are radial and axial coordinates in the polar cylinder coordinate system, R denotes the fibre radius, $u_r^-(z,t)$ and $u_z^-(z,t)$ are radial and axial displacements of the fibre material at the interface. Thus for strain tensor components we obtain

$$\begin{aligned} \varepsilon_{rr} &= u_{r,r} = u_r^-/R, \\ \varepsilon_{\varphi\varphi} &= u_r/r = u_r^-/R, \\ \varepsilon_{zz} &= u_{z,z} = u_{z,z}^-, \end{aligned} \tag{3.2}$$

(cf. [8, 9, 10]), where comma denotes partial derivative. Substituting these expressions into the stress-strain relations we obtain:

$$\sigma_{rr} = \lambda_f\left(\frac{1}{\nu_f}\frac{u_r^-}{R} + u_{z,z}^-\right),\qquad \sigma_{zz} = \lambda_f\left(\frac{2u_r^-}{R} + \frac{1-\nu_f}{\nu_f}u_{z,z}^-\right), \tag{3.3}$$

where λ_f and ν_f stand for the Lamé constant and the Poisson ratio of the fibre material.

Equation of motion along z-axis takes the form

$$\sigma_{zz,z} + f_z^e = \rho_f\ddot{u}_z, \tag{3.4}$$

where dot stands for material time-derivative, f_z^e denotes external force density (per unit volume) and ρ_f is the fibre material mass density. Integrating both sides of Eq. (3.3) over the arbitrary chosen segment of fibre with length equal to δ we obtain

$$\pi R^2\delta\overline{\sigma}_{zz,z} + \pi R^2\delta\overline{f}_z^e = \pi R^2\delta\rho_f\ddot{\overline{u}}_z, \tag{3.5}$$

where dash over the symbol denotes the mean value. The only external force is exerted by the shear stress at the interface, i.e. f_z^e, as a function of variable r behaves like δ-function ($f_z^e(r,z,t) = f_z^e(z,t)\delta(r-R)$), thus we can write:

$$\overline{f}_z^e = 2\tau/R,$$

where τ denotes the mean shear stress at the interface of the fibre segment. Finally dividing Eq. (3.5) by segment volume $V = \pi R^2\delta$, tending with δ to zero and expressing stress component σ_{zz} through displacements $u_r^-(z,t)$ (compare Eq. (3.3)) we obtain the following inhomogeneous equation of motion for any fibre cross-section:

$$\lambda_f\left(\frac{2u_{r,z}^-}{R} + \frac{1-\nu_f}{\nu_f}u_{z,zz}^-\right) - \rho_f\ddot{u}_z^- = -\frac{2}{R}\tau. \tag{3.6}$$

The second differential equation, which should be fulfilled by functions $u_r^-(z,t)$ and $u_z^-(z,t)$, yields from the normal stress continuity condition and takes the form

$$\lambda_f\left(\frac{1}{\nu_f}\frac{u_r^-}{R} + u_{z,z}^-\right) = t_n, \tag{3.7}$$

where t_n denotes external normal stress at the fibre surface.

Thus we have two differential equations for two unknown functions $u_r^-(z,t)$ and $u_z^-(z,t)$ of two variables: spatial variable z and time t. Right-hand side terms of Eqs. (3.6) and (3.7) (which are in fact the coupling terms between dynamic fields inside and outside the fibre) should be considered here as given functions of z and t variables $\tau = \tau(z,t)$, $t_n = t_n(z,t)$.

4. Dynamic fields in the matrix (outside the fibre)

Outside the fibre the equations of axisymmetric motion in terms of displacements take the following form [8]:

$$
\begin{aligned}
c_l^2\left[\frac{1}{r}(ru_r)_{,r} + u_{z,z}\right]_{,r} + c_t^2[u_{r,z} + u_{z,r}]_{,z} &= \ddot{u}_r,\\
c_l^2\left[\frac{1}{r}(ru_r)_{,r} + u_{z,z}\right]_{,z} + c_t^2\frac{1}{r}[r(u_{r,z} + u_{z,r})]_{,r} &= \ddot{u}_z,
\end{aligned}
\tag{4.1}
$$

where

$$
c_l = \sqrt{\frac{\lambda_m + 2\mu_m}{\rho_m}}, \qquad c_t = \sqrt{\frac{\mu_m}{\rho_m}}
$$

are velocities of longitudinal and shear elastic plane waves, ρ_m is the matrix material density.

Using the following decomposition:

$$
\begin{aligned}
u_r &= \mathcal{U}_{,r} - \mathcal{V}_{,z},\\
u_z &= \mathcal{U}_{,z} + \frac{1}{r}(\mathcal{V}r)_{,r},
\end{aligned}
\tag{4.2}
$$

and looking for the solutions in the following form:

$$
\begin{aligned}
\mathcal{U}(r,z,t) &= U(r)\exp[-i(kz-\omega t)],\\
\mathcal{V}(r,z,t) &= V(r)\exp[-i(kz-\omega t)],
\end{aligned}
\tag{4.3}
$$

we are seeking in fact (bearing in mind the displacement continuity at the interface) for the decaying wave in the fibre propagating in z direction. If the sense of propagation has to be positive and energy is transmitted from the fibre to the matrix (i.e. wave amplitude in the fibre is decaying), then the following inequalities should hold:

$$
\begin{aligned}
\operatorname{Re} k &> 0,\\
\operatorname{Im} k &< 0,
\end{aligned}
\tag{4.4}
$$

ω is assumed to be real and positive.

Substituting Eqs. (4.2) and (4.3) into Eqs. (4.1) we reduce the dynamic Lamé equations to two Bessel equations

$$
U_{,rr} + \frac{1}{r}U_{,r} + \left(\frac{\omega_2^2}{c_l^2} - k^2\right)U = 0,
\tag{4.5}
$$

$$
V_{,rr} + \frac{1}{r}V_{,r} + \left[\left(\frac{\omega_2^2}{c_t^2} - k^2\right) - \frac{1}{r^2}\right]V = 0.
\tag{4.6}
$$

As long as we have to do with complex coefficients in the Bessel equations, it is most convenient to express the solutions in terms of Hankel functions (Bessel functions of the third kind) [11].

$$
U(r) = U_1H_0^{(1)}\left(r\sqrt{\frac{\omega_2^2}{c_l^2} - k^2}\right) + U_2H_0^{(2)}\left(r\sqrt{\frac{\omega_2^2}{c_l^2} - k^2}\right),
\tag{4.7}
$$

$$
V(r) = V_1H_0^{(1)}\left(r\sqrt{\frac{\omega_2^2}{c_t^2} - k^2}\right) + V_2H_0^{(2)}\left(r\sqrt{\frac{\omega_2^2}{c_t^2} - k^2}\right),
\tag{4.8}
$$

Symbol $\sqrt{(\cdot)}$ denotes here the "positive" root, i.e. the one preserving the sign of the imaginary part. U_1, U_2, V_1, V_2 are arbitrary constants.

Solutions (4.7), (4.8) contain four arbitrary constants; two of them — U_1 and V_1 in the case under consideration will be shown to vanish. Let us consider asymptotic behavior of solutions for $r \to \infty$.

For large arguments the following principal asymptotic representations are valid [11]:

$$H_\nu^{(1)}(Y) \cong \sqrt{2/(\pi Y)} \exp\left[i\left(Y - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\right], \tag{4.9}$$

$$H_\nu^{(2)}(Y) \cong \sqrt{2/(\pi Y)} \exp\left[-i\left(Y - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\right], \tag{4.10}$$

where $-\pi < \operatorname{Arg} Y < 2\pi$, $|Y| \to \infty$. In our case

$$Y = r\sqrt{\frac{\omega^2}{c^2} - k^2} \equiv \kappa r, \tag{4.11}$$

where

$$\kappa = \sqrt{\frac{\omega^2}{c^2} - k^2}, \qquad c = c_l \quad \text{or} \quad c = c_t, \qquad \operatorname{Re} k > 0, \qquad \operatorname{Im} k < 0.$$

Under our convention of "positive" roots we have $\operatorname{Im}\kappa > 0$, $\operatorname{Im} Y > 0$. Observe that, since ω and c are real,

$$\kappa^2 + k^2 = \frac{\omega^2}{c^2} \in \operatorname{Re} \tag{4.12}$$

and

$$\operatorname{Im}(\kappa^2 + k^2) = 2(\operatorname{Im}\kappa \operatorname{Re}\kappa + \operatorname{Im} k \operatorname{Re} k) = 0. \tag{4.13}$$

Taking into account that the term $\operatorname{Im} k \operatorname{Re} k$ is negative and $\operatorname{Im}\kappa$ is positive, one can see that real part of κ is positive.

Using asymptotic relations (4.9), (4.10) we can write two following expressions describing behavior of different terms of solutions $\mathcal{U}(r, z, t)$ and $\mathcal{V}(r, z, t)$ for large r:

$$\begin{aligned} H_\nu^{(2)}(\kappa r) \exp[-i(kz - \omega t)] \\ \cong \frac{C_2}{\sqrt{r}} \exp(r \operatorname{Im}\kappa + z \operatorname{Im} k) \exp[-i(r \operatorname{Re}\kappa + z \operatorname{Re} k - \omega t)], \end{aligned} \tag{4.14}$$

$$\begin{aligned} H_\nu^{(1)}(\kappa r) \exp[-i(kz - \omega t)] \\ \cong \frac{C_1}{\sqrt{r}} \exp(r \operatorname{Im}\kappa + z \operatorname{Im} k) \exp[-i(r \operatorname{Re}\kappa + z \operatorname{Re} k - \omega t)], \end{aligned} \tag{4.15}$$

where C_1 and C_2 are some complex constants. Expression (4.14) describes periodic concentric wave propagating at some angle $\psi = \operatorname{Atn}(\operatorname{Re}\kappa / \operatorname{Re} k)$ with respect to the fibre axis (Fig. 1). Real exponential factor $\exp(r \operatorname{Im}\kappa + z \operatorname{Im} k)$ describes the amplitude decaying in the direction orthogonal to the direction of propagation (the orthogonality of vectors $[\operatorname{Im}\kappa, \operatorname{Im} k]$ and $[\operatorname{Re}\kappa, \operatorname{Re} k]$ yields from Eq. (4.13)). Pre-exponential multiplier $1/\sqrt{r}$ provides the global energy flux balance. Wave described by Eq. (4.14) travels from the fibre surface to infinity, thus the energy is drained from the fibre into infinite matrix.

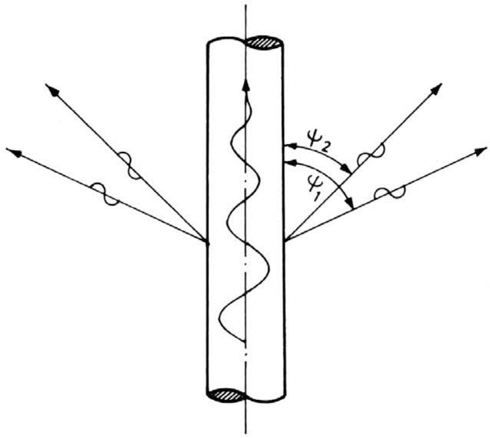

FIG. 1. Wave motion scheme.

On the contrary, expression (4.15) describes a wave coming from the infinity and propagating towards the interface. Thus, as long as we are looking not for any dynamic field but rather for the description of the wave emission from the fibre (e.g. in the process of fibre rupture), the terms containing $H_0^{(1)}(\kappa r)$ and $H_1^{(1)}(\kappa r)$ remain out of our interest and constants U_1 and V_1 should be taken equal to zero. Eventually, the dynamic fields in the matrix can be reduced to the following form:

$$\begin{aligned} \mathcal{U}(r,z,t) &= U_0 H_0^{(2)}(\kappa_l r)\exp[-i(kz-\omega t)], \\ \mathcal{V}(r,z,t) &= V_0 H_1^{(2)}(\kappa_t r)\exp[-i(kz-\omega t)], \end{aligned} \tag{4.16}$$

where

$$\kappa_l = \sqrt{\frac{\omega^2}{c_l^2} - k^2}, \qquad \kappa_t = \sqrt{\frac{\omega^2}{c_t^2} - k^2}.$$

Expressions (4.16) describe (asymptotically) two waves propagating at different angles $\psi_1 = \mathrm{Atn}(\mathrm{Re}\,\kappa_l/\,\mathrm{Re}\,k)\pi/2$ and $\psi_2 = \mathrm{Atn}(\mathrm{Re}\,\kappa_t/\,\mathrm{Re}\,k)\pi/2$ with respect to the fibre axis. Amplitudes U_0 and V_0 are not independent, the relation between them as well as the dispersion relation $k = k(\omega)$ should be found from the solution of the boundary value problem.

5. Solution of the boundary value problem

Starting from the displacement continuity at the interface

$$\begin{aligned} u_r^- &= u_r^+, \\ u_z^- &= u_z^+, \end{aligned} \tag{5.1}$$

where index "plus" denotes a value in the matrix at the interface, and using relations (4.2) and (4.16) one obtains the following expression for the displacements in the fibre:

$$\begin{aligned} u_z &= u_z^- = A\exp[-i(kz-\omega t)], \\ u_r &= \frac{r}{R}u_r^- = \frac{r}{R}B\exp[-i(kz-\omega t)], \end{aligned} \tag{5.2}$$

where

$$
\begin{aligned}
A &= -ikU_0H_0^{(2)}(\kappa_l R) + \kappa_t V_0 H_0^{(2)}(\kappa_t R),\\
B &= -\kappa_l U_0 H_1^{(2)}(\kappa_l R) + ikV_0H_1^{(2)}(\kappa_t R).
\end{aligned} \tag{5.3}
$$

If a wave frequency ω is given, then the amplitude ratio U_0/V_0 and wavenumber value k (i.e. also values of κ_1, κ_t, ψ_1, ψ_2) can not be taken arbitrary. Dynamic displacement field (5.3) describing in fact the propagation of a one-dimensional wave in the fibre must satisfy, of course, the equation of motion (integral axial momentum balance condition) (3.6), and the normal stress continuity condition (3.7) must be satisfied as well. Let us try to fulfill these conditions.

The stress field in the matrix for the case of axial symmetry can be expressed as follows [8]:

$$
\begin{aligned}
\sigma_{rr} &= \lambda_m\left(u_{r,r} + \frac{u_r}{r} + u_{z,z}\right) + 2\mu_m u_{r,r},\\
\sigma_{rz} &= \mu_m(u_{r,z} + u_{z,r}),
\end{aligned} \tag{5.4}
$$

where λ_m and μ_m are Lamé constants of matrix material. At the interface we have

$$
\tau = \sigma_{rz}^+, \qquad t_n = \sigma_{rr}^+. \tag{5.5}
$$

Substituting relations (4.16) into (4.2) and then into (5.4), we are able, with the use of (5.5), to express right-hand sides of Eqs. (3.6) and (3.7) in the terms of unknown amplitudes V_0, U_0 and some functions of k and ω. Using relations (5.2) and (5.3) we are able to express in the same way also the left-hand sides of mentioned equations. After some rearrangement involving application of initial equations (4.1) we obtain finally the following set of two algebraic linear homogeneous equations for unknown amplitudes U_0 and V_0:

$$
\begin{aligned}
ikA_{11}U_0 + A_{12}V_0 &= 0,\\
A_{21}U_0 + ikA_{22}V_0 &= 0,
\end{aligned} \tag{5.6}
$$

where

$$
\begin{aligned}
A_{11} &= \left(\lambda_f \frac{1-\nu_f}{\nu_f}k^2 - \rho_f\omega^2\right)H_0^{(2)}(\kappa_l R) + 2\frac{\kappa_l}{r}(\lambda_f + 2\mu_m)H_1^{(2)}(\kappa_l R),\\
A_{12} &= -\kappa_t\left(\lambda_f \frac{1-\nu_f}{\nu_f}k^2 - \rho_f\omega^2\right)H_0^{(2)}(\kappa_t R)\\
&\qquad + \frac{2}{R}((\lambda_f + 2\mu_m)k^2 - \rho_m\omega^2)H_1^{(2)}(\kappa_t R),\\
A_{21} &= ((\lambda_f + 2\mu_m)k^2 - \rho_m\omega^2)H_0^{(2)}(\kappa_l R) + \frac{\kappa_l}{R}\left(\frac{\lambda_f}{\nu_f} + 2\mu_m\right)H_1^{(2)}(\kappa_l R),\\
A_{22} &= \kappa_t(\lambda_f + 2\mu_m)H_0^{(2)}(\kappa_t R) - \frac{1}{R}\left(\frac{\lambda_f}{\nu_f} + 2\mu_m\right)H_1^{(2)}(\kappa_t R).
\end{aligned} \tag{5.7}
$$

For brevity we shall omit here explicit expression for the condition of the existence of a nontrivial solution of the system (5.6),

$$
D(\omega, k) = k^2A_{11}A_{22} + A_{12}A_{21} = 0. \tag{5.8}
$$

It is clear, however, that solving numerically the nonlinear transcendental equation (5.8) one can find both the real and imaginary part of k as functions of frequency ω. If this is done, angles ψ_1 and ψ_2, wavelength and amplitude attenuation decrement in the fibre as well as amplitude ratio U_0/V_0 can be easily found.

We shall omit here this rather boring but simple procedures and focus our attention on the dispersion relations and on some other qualitative results following from the numerical solutions of the characteristic equation (5.8).

6. Preliminary results and discussion

Some characteristics of wave motion are plotted in Fig. 2 to 6 versus the wavenumber Re k. One can see, that such parameters like phase and group velocities or angles ψ_1 and ψ_2 are relatively intensitive to the wavelength in a wide range of Re k values.

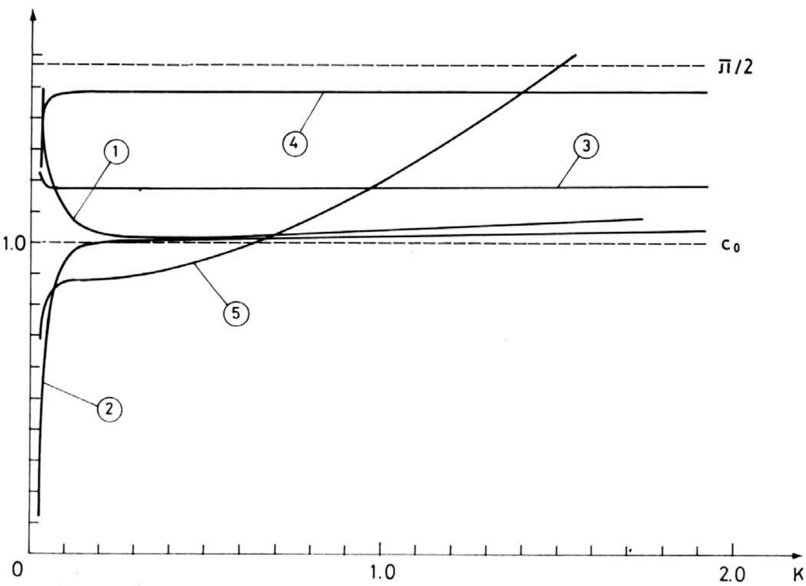

FIG. 2. Dispersion curves for "almost incompressible" matrix: $\mu_f/\mu_m = 50$, $\nu_m = 0.474$, $\nu_f = 0.3$, $\rho_f/\rho_m = 1$; *1* — dimensionless group velocity c_G/c_0; *2* — dimensionless phase velocity c_{ph}/c_0; *3* — ψ_1 angle; *4* — ψ_2 angle; *5* — wave amplitude decay decrement $\beta = |\operatorname{Im} kR|$ (scale 10 : 1). At the horizontal axis — dimensionless wave number $\kappa = |\operatorname{Re} kR|$, $c_0 = \sqrt{\frac{E_f}{\rho_f}}$ asymptotic long wave limit of longitudional wave velocity in free fibre, R — fibre radius.

The only parameter which is very sensitive to both the wavelength and the elastic moduli ratio μ_f/μ_m is the imaginary part of k, i.e. exponent β describing the amplitude decay along the fibre, or in other words — the energy exchange ratio between the fibre and the surrounding matrix material. It should be noted that the scale of β at Fig. 6 is ten times smaller than that at the remaining figures.

General behavior of the wave motion characteristics for very long waves can be easily observed, e.g. at Fig. 6. For higher values of the μ_f/μ_m ratio certain caution in the interpretation must be suggested in view of some numerical instabilities observed in the course of solution obtained for very long waves.

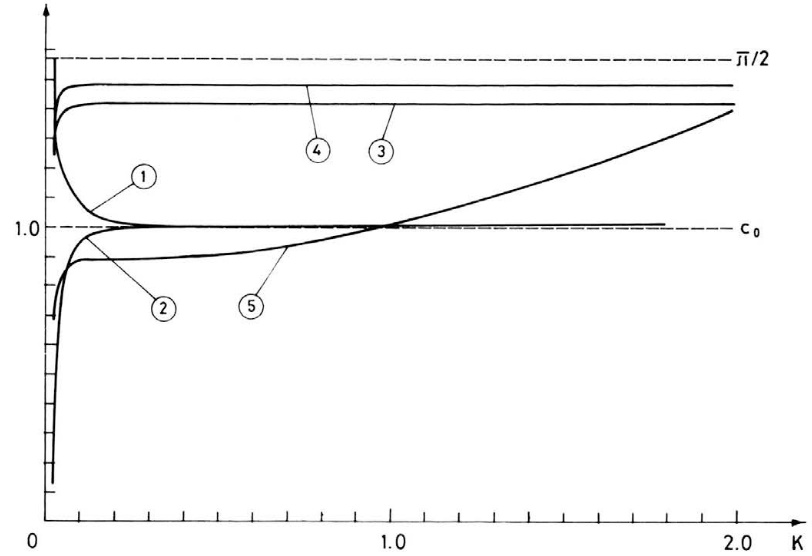

FIG. 3. Dispersion curves for "compressible" matrix:
$\mu_f/\mu_m = 50$, $\nu_m = 0.250$, $\nu_f = 0.3$, $\rho_f/\rho_m = 1$.
Remaining notation the same as in Fig. 2.

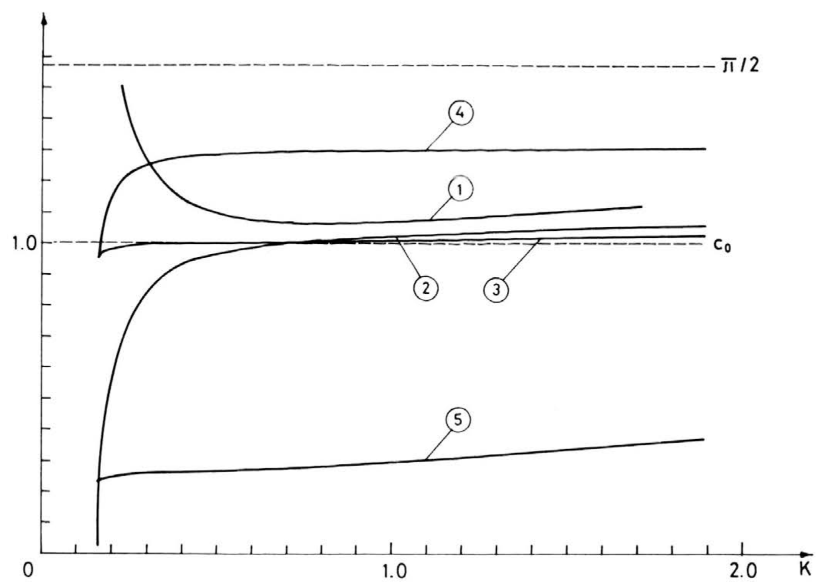

FIG. 4. Dispersion curves for "soft" fibres
$\mu_f/\mu_m = 5.0$, $\nu_m = 0.333$, $\nu_f = 0.3$, $\rho_f/\rho_m = 1$.
Remaining notation the same as in Fig. 2 except the scale of $\beta(1:1)$.

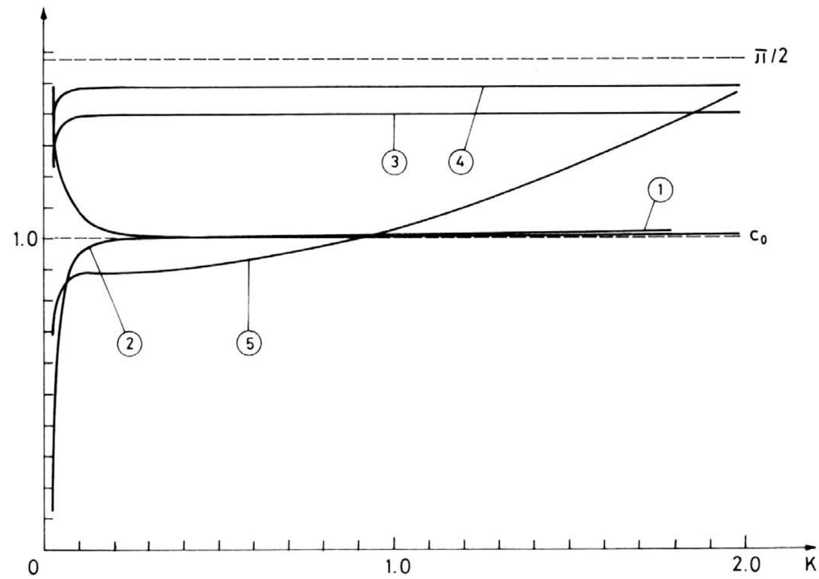

FIG. 5. Dispersion curves for "normal" fibres
$\mu_f/\mu_m = 50, \quad \nu_m = 0.333, \quad \nu_f = 0.3, \quad \rho_f/\rho_m = 1.$
Remaining notation the same as in Fig. 2.

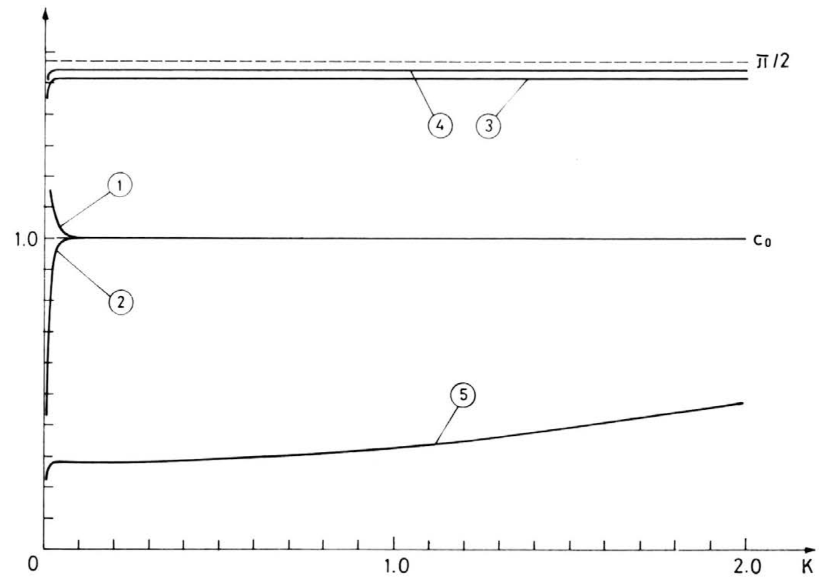

FIG. 6. Dispersion curves for "rigid" fibres
$\mu_f/\mu_m = 500, \quad \nu_m = 0.333, \quad \nu_f = 0.3, \quad \rho_f/\rho_m = 1.$
Remaining notation the same as in Fig. 2.

The behavior of wave motion for short waves, when the $\mathrm{Re}\, k \cdot R$ product exceeds 2, seems to be interesting, but needs further studies without the simplifying assumptions concerning the wave motion inside the fibre. From some qualitative considerations one can expect decreasing values of β in very short wave regions, but this question being outside the application range of the present simplified model must remain open for further investigations.

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