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Boltzmann equation on a lattice global solution for non-Maxwellian gases

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WE STUDY the nonlinear Boltzmann equation in which the spatial variable is replaced by an infinite lattice. We prove the existence of a global strong solution in the case of non-Maxwellian molecules (unbounded collision kernel).

Rozważono nieliniowe równanie Boltzmanna, w którym zmienną przestrzenną zastąpiono nieskończoną siatką. Udowodniono istnienie silnego rozwiązania globalnego w przypadku cząsteczek niemaxwellowskich (nieograniczone jądro zderzeń).

Рассматривается нелинейное уравнение Больцмана, в котором пространственная переменная заменяется бесконечной сеткой. Доказывается существование сильного глобального решения в случае немаксвелловских частиц (неограниченное ядро столкновений).

1. Introduction

IN THIS PAPER we prove the global existence of a solution of the Boltzmann equation on a lattice. The Boltzmann equation on a toroidal lattice was considered recently by many authors [2, 3, 7]. We consider this equation on an infinite lattice and for non-Maxwellian molecules. A similar problem was treated in [3] and under the assumptions that the initial distribution has finite energy and entropy the existence of a weak solution was proved. In this paper we assume that the fourth moment of the initial data exists and we prove that the solution constructed is a unique strong solution of the Boltzmann equation on a lattice. The question of the existence of the lattice limit, i.e. the limit of solutions as the lattice spacing tends to zero, is in the case of an unbounded domain much more complicated than for a bounded domain (cf. [3]) and will be treated later on.

2. Formulation of the problem

The Boltzmann equation we consider is the following:

$$\frac{\partial f_{(i)}}{\partial t} + (Af)_{(i)} = J(f_{(i)}, f_{(i)}). \tag{2.1}$$

Here the index $(i) = (j, k, l)$ is a three-dimensional multi-index denoting the i-th lattice point in the three-dimensional infinite lattice. We assume the lattice spacing is 1 which assures the identity of multi-indices with coordinates of lattice points.

The operator A, which is a finite-difference approximation to the streaming term in the original Boltzmann equation, is defined as in [2, 3]. Namely, if

$$(A_x f)_{(j,k,l)} = \begin{cases} v_x[f_{(j,k,l)} - f_{(j-1,k,l)}]/2, & v_x > 0, \\ v_x[f_{(j+1,k,l)} - f_{(j,k,l)}]/2, & v_x < 0 \end{cases}$$

and A_y, A_z are given by similar expressions, then A is given by tensor products:

$$A = A_x \otimes I \otimes I + I \otimes A_y \otimes I + I \otimes I \otimes A_z.$$

The collision term is given by

$$(2.2)\quad J(f_{(i)}, f_{(i)}) = \int_{R^3}\int_{S^2} k(|v-v_1|, (v-v_1)\cdot u)\,[f_{(i)}(v_1')f_{(i)}(v') - f_{(i)}(v)f_{(i)}(v_1)]du\,dv_1,$$

$$u \in S^2 = \{u \in R^3 : |u| = 1\}.$$

Here v, v_1 are initial velocities and v', v_1' velocities after collision related to v, v_1 by the well-known relations.

We assume that the scattering kernel k is a measurable, non-negative function with

$$(2.3)\qquad k(|v-v_1|, (v-v_1)\cdot u) \leqslant K_k(1+|v|^{r_1}+|v_1|^{r_2}), \quad 0 \leqslant r_1, r_2 \leqslant 2.$$

Throughout the rest of the paper we write $k(v, v_1)$ instead of the full expression $k(|v-v_1|, (v-v_1)\cdot u)$.

We seek solutions of Eq. (2.1) in the Banach space

$$B = L_1(R^3, l_1)$$

with the norm

$$(2.4)\qquad \|f\| = \int_{R^3}\sum_{(i)} |f_{(i)}(v)|\,dv.$$

We also use the space B^s, which is the space B with the weight $w = (1+|v|^2)^{s/2}$ and norm

$$(2.5)\qquad \|f\|_s = \int_{R^3}\sum_{(i)} w|f_{(i)}(v)|\,dv.$$

We summarize now the properties of the collision operator J which we need in the subsequent sections.

For $g \in B^2$ we have

$$(2.6)\qquad \int_{R^3} J(g, g)dv = 0,$$

$$(2.7)\qquad \int_{R^3} |v|^2 J(g, g)dv = 0$$

and for $g \in B^s$, $s > 2$ and a scattering kernel satisfying the inequality (2.3) we have

$$(2.8)\quad \|(1+|v|^2)^{s/2}J(g, g)\| \leqslant CK_k[\|g\|_{s+r_1-d}\|g\|_d + \|g\|_{s-d}\|g\|_{r_2+d}] \quad \text{for} \quad 0 \leqslant d \leqslant 2$$

(the last inequality is due to POVZNER [5]).

The plan of the paper is as follows. In Sect. 3 we consider the semigroup generated by the operator A. We find an explicit expression for this semigroup and prove that it is a contraction semigroup invariant on the cone of positive functions in B (Proposition 3.1).

In Sect. 4 the case of a bounded scattering kernel is investigated. The existence of a global, unique solution of Eq. (2.1) is proved. Section 5 is devoted to the general case (unbounded scattering kernel). Following ARKERYD [1], we construct a monotone sequence of approximations to the solution of an integral form of the Boltzmann equation. We prove that this sequence is bounded and by the Levy property convergent. We prove then that the limit function is a generalized solution of the Boltzmann equation. The last step is the proof of the strong differentiability of the limit function. This proves the existence of a global strong solution of Eq. (2.1) for non-Maxwellian molecules.

3. Semigroup properties

The operator A described in the previous section with

$$D(A) = \{f \in B : |v| f \in B\}$$

is a closable operator. Its closure $\bar{A}$ generates in B the semigroup

$$U(t) = e^{-t\bar{A}}.$$

Our aim is to prove this and investigate the properties of this semigroup. We collect the main results in the following proposition:

PROPOSITION 3.1

There is in B a contraction semigroup $U(t)$ invariant on the cone of non-negative functions $B^+ \subset B$. The generator of this semigroup is a closure of A.

Proof

The plan of the proof is as follows. First by "illegal operations" we find a good candidate for the semigroup. We prove that it is in fact a semigroup and that on a dense subset its generator is identical with A. It proves that the generator of the semigroup is a closed extension of A.

The "illegal operation" mentioned earlier is the Fourier transform with respect to the position variable. (For simplicity we present all calculations for the one-dimensional case.)

If $f = \{f_{(j)}\}_{j=-\infty}^{\infty}$ then $\hat{f}(c) = \sum_{j=-\infty}^{\infty} \exp(2\pi ijc) f_{(j)}$ and

$$Af = \{v_x/2(f_{(j)} - f_{(j-1)})\}_{(j)=-\infty}^{\infty} \rightarrow \hat{Af} = v_x/2[1 - \exp 2\pi ic]\hat{f}.$$

Using the inverse Fourier transform we find

$$\begin{aligned}(U(t)f)_{(k)} = (f(t))_{(k)} &= \int_0^1 \exp(-2\pi ikc)\exp[-v_x/2(1-\exp 2\pi ic)t] \\ &\times \sum_{j=-\infty}^{\infty} \exp(2\pi ijc) f_{(j)}(0)\, dc = \sum_{j=-\infty}^{\infty} f_{(j)}(0) \int_0^1 \exp\{-v_x/2(1-\exp 2\pi ic)t\} \\ &\times \exp[2\pi i(j-k)c]\, dc = \sum_{j=-\infty}^{\infty} f_{(j)}(0)\exp(-v_x t/2)\frac{1}{2\pi i}\int_{K(0,1)} \exp(v_x tz/2) z^{j-k-1} dz \\ &= \sum_{j=-\infty}^{k} f_{(j)}(0)\exp(-v_x t/2)\frac{(v_x t/2)^{k-j}}{(k-j)}.\end{aligned}$$

Hence

$$(3.1)\qquad \big(U(t)f\big)_{(k)} = \big(f(t)\big)_{(k)} = \exp(-v_x t/2)\sum_{j=-\infty}^{k} f_{(j)}(0)\,\frac{(v_x t/2)^{k-j}}{(k-j)!}.$$

By standard calculations we find

$$U(t+s) = U(t)U(s)$$

and

$$\|U(t)f-f\| \to 0 \quad \text{as} \quad t\to 0.$$

Hence $U(t)$ is a strongly continuous semigroup. By direct inspection of Eq. (3.1) we see that $U(t)f$ is differentiable and

$$\frac{d}{dt}U(t)f = AU(t)f$$

provided $U(t)f\in D(A)$. Since $D(A)$ is invariant under $U(t)$, the generator of $U(t)$ is the closure of A ([6] Th. X. 49). Inspection of formula (3.1) shows also that $U(t)$ is invariant on the cone of non-negative functions B^+ and is norm preserving on this cone. To prove that $U(t)$ is a contraction we decompose $f\in B$ as $f = f_1-f_2, f_i\in B^+$, then

$$\|U(t)f\| \leqslant \|U(t)f_1\|+\|U(t)f_2\| = \|f_1\|+\|f_2\| = \|f\|.$$

This ends the proof of the proposition.

4. Existence in the bounded case

In this section we assume that the scattering kernel $k(v, v_1)$ is bounded:

$$(4.1)\qquad 0\leqslant k(v, v_1|)\leqslant C_k.$$

Under this assumption we prove existence, uniqueness and some regularity conditions of the solution of the Cauchy problem for the equation (2.1). We do that in a series of propositions:

PROPOSITION 4.1

Let $f_0\in D(A)\cap B^+$ and $T_0\leqslant 1/9C_k\|f_0\|$. Then for $t\in[0, T_0]$ there exists a function $f(t)\in B^+$ strongly continuously differentiable in B, which is a unique solution of Eq. (2.1), with initial value $f(0) = f_0$.

The proof of this proposition is similar to that given in [4] for the continuous case with the additional assumption of essential boundedness of f_0 with respect to the position variable, which is automatically fulfilled in the discrete case $(\sup_{(i)}|g_{(i)}(v)|\leqslant \sum_{(i)}|g_{(i)}(v)|!)$.

PROPOSITION 4.2

Suppose $f_0(1+|v|^2)^{s/2}\in B^+$ with $s>2$, then there exists a unique solution $f(t)$ of (2.1) for $t\in[0,\infty[$ which satisfies

$$f(t)(1+|v|^2)^{s/2}\in B^+,$$
$$\|f(t)\| = \|f_0\|,$$
$$\|f(t)\|_2 = \|f_0\|_2,$$
$$\|f(t)\|_s\leqslant C_T\|f_0\|_s \quad \text{for} \quad t\leqslant T.$$

Here C_T depends on $\|f_0\|_2$, T, K_k and s only.

Proof

The solution constructed in Proposition 4.1 satisfies

$$||f(t)|| = ||f_0||.$$

This follows from Eq. (2.6) and direct integration of Eq. (2.1).

Then we can take $f(T_0)$ as the new initial value and obtain the solution for $t \in [T_0, 2T_0]$. By this procedure we can extend the solution on the whole semi-axis $[0, \infty[$.

We prove now that $f(t)(1+|v|^2)^{s/2} \in B^+$.

It is easy to repeat the proof of Proposition 4.1 in the space B^s. This guaranties the local existence of a solution. This solution satisfies

$$||f(t)||_2 = ||f_0||_2.$$

For $s > 2$ we apply Eq. (2.8) following ARKERYD [1].

For $2 < s \leqslant 4$ we take $d = s/2$ and Eq. (2.8) gives

$$||f||_s \leqslant \exp(CK_k||f_0||_2 T)||f_0||_s.$$

Using the result for $s = 4$ we prove this for $4 < s \leqslant 6$ taking $d = 2$ and then by induction for all $s > 2$ we have

$$||f||_s \leqslant C_T||f_0||_s.$$

The local existence of a solution together with this estimate gives a possibility of extention for every $t \in [0, \infty[$. *q.e.d.*

We summarise the results for the bounded case in the following:

THEOREM 1.

Let $f_0 \in D(A) \cap B^+$, then there exists a unique global solution $f(t) \in B^+$ of Eq. (2.1) *with the initial distribution f_0 and*

$$||f(t)|| = ||f_0||.$$

If f_0 has finite higher moments (i.e. $f_0(1+|v|^2)^{s/2} \in B^+$ for $s \geqslant 2$) then $f(t)$ has the same higher moments finite and

$$||f(t)||_2 = ||f_0||_2, \qquad \text{for all } t \geqslant 0,$$
$$||f(t)||_s \leqslant C_T||f_0||_s \quad \text{for } s > 2, \text{ for } 0 \leqslant t \leqslant T,$$

where C_T depends on K_k, T, $||f_0||_2$ and s only.

5. The unbounded case

The proof of existence and uniqueness will be done following ARKERYD [1] by the method of monotone operators. We start by recalling some general ideas.

All our investigations will be made in a Banach space X which has the Levy property, i.e. X is a partially-ordered Banach space in which every non-negative, monotone bounded sequence is convergent. Namely, if $\{f^i\}_{i=1}^{\infty}$ is such that $0 \leqslant f^1 < \ldots \leqslant f^k \leqslant f^{k+1} \leqslant \ldots$ and $\sup ||f^i|| < +\infty$, then $\lim_{i\to\infty} f_i = f$ exists and $f \geqslant 0$.

An operator F in X is called positive and monotone if

$$0 \leqslant Ff \leqslant Fg \quad \text{for} \quad 0 \leqslant f \leqslant g.$$

Consider in X the following Cauchy problem:

$$(5.1) \qquad \left| \begin{array}{l} f_t + hf = G(f), \\ f(0) = f_0, \end{array} \right.$$

where h is a linear operator which generates in X the semigroup $Z(t) = e^{-ht}$ and all $Z(t)$ are positive, monotone contractions, and G is a nonlinear, positive and monotone operator.

We construct the solution of Eq. (5.1) by the method of successive approximations:

$$f^1(t) = 0,$$

$$f^{n+1}(t) = Z(t)f_0 + \int_0^t Z(t-s)G(f^n)(s)\,ds, \quad n = 1, 2, \ldots .$$

If $f_0 \geqslant 0$, then $\{f^i\}$ form a monotone, positive sequence. Suppose we can prove boundedness of $||f^i||$, then, according to the Levy property, there exists $f = \lim_{i\to\infty} f^i$ which is a solution of the following integral equation (generalized solution):

$$(5.2) \qquad f(t) = Z(t)f_0 + \int_0^t Z(t-s)G(f)(s)\,ds.$$

If we prove that $f(t)$ is continuously differentiable, as can be done under suitable conditions on G, then $f(t)$ is a strong solution of Eq. (5.1). Let us observe that if $g(t)$ is another non-negative solution of Eq. (5.2), then $f(t) \leqslant g(t)$ due to the construction of the series f^n. Let us consider a pair of Cauchy problems of the type (5.1) characterized by $(Z(t), G, f_0)$ and $(Z'(t), G'_0, f'_0)$. This pair is called a monotone pair if

$$Z'(t)g \leqslant Z(t)g, \quad G'(g) \leqslant G(g), \quad f'_0 \leqslant f_0 \quad \text{for} \quad g \geqslant 0.$$

The important property of a monotone pair is the following. If the problem $(Z(t), G, f_0)$ has a solution $f(t)$, then $(Z'(t), G', f'_0)$ has a generalized solution $f'(t)$ which satisfies

$$0 \leqslant f'(t) \leqslant f(t).$$

We are now able to prove our main result:

THEOREM 2.

Let $k(v, v_1)$ be given by Eq. (2.3) with $0 \leqslant r_1, r_2 \leqslant 2$ and $f_0(1+|v|^2)^{s/2} \in B^+$ with $s \geqslant 4$. Then there exists a unique strong solution $f(t)$ of Eq. (2.1) for $t \in [0, \infty[$ such that

$$f(t)\,(1+|v|^2)^{s/2} \in B^+,$$

$$||f(t)|| = ||f_0||, \quad ||f(t)||_2 = ||f_0||_2,$$

$$||f(t)||_s \leqslant C_T ||f_0||_s \quad \text{for} \quad t \leqslant T.$$

Here C_T depends on $||f_0||_2$, T, K_k and s only.

Proof.

a. Construction of a monotone sequence of approximations

We approximate the kernel $k(v, v_1)$ by the bounded functions

$$k_m(v, v_1) = \min\,[k(v, v_1), m]$$

and let J_m be the corresponding collision operator. According to the Proposition 4.1, the Cauchy problem

$$(5.3)\qquad \begin{vmatrix} f_t + Af = J_m(f,f), \\ f(0) = f_0 \end{vmatrix}$$

has a solution which we denote by $f_m(t)$.

Let us introduce

$$h(v) = K(1+|v|^2)\sum_{(i)}\int (1+|v_1|^2)(f_0)_{(i)}(v_1)\,dv_1,$$

$$J_m'(f,f) = J_m(f,f) + K(1+|v|^2)f(v)\sum_{(i)}\int (1+|v_1|^2)f_{(i)}(v_1)\,dv_1,$$

then

$$(5.4)\qquad \begin{vmatrix} f_t + Af + hf = J_m'(f,f), \\ f(0) = f_0 \end{vmatrix}$$

is the Cauchy problem of the type (5.1) as $A+h$ generates the positive, monotone semigroup $U(t)e^{-ht}$ and J_m' is positive and monotone for K sufficiently large.

Since the solution of Eq. (5.3) is a solution of Eq. (5.4), a sequence of successive approximations of Eq. (5.4) is bounded by f_m.

Let us introduce now

$$J_m''(f,f) = \int k_m(v,v_1)f(v')f(v_1')\,du\,dv_1 + K(1+|v|^2)f(v)\sum_{(i)}\int (1+|v_1|^2)f_{(i)}(v_1)\,dv_1$$
$$-\int k(v,v_1)f(v)f(v_1)\,du\,dv_1,$$

J_m'' is positive and monotone for sufficiently large K and

$$J_m'(f,f) \geqslant J_m''(f,f), \quad f \geqslant 0,$$
$$J_m''(f,f) \geqslant J_k''(f,f), \quad m \geqslant k \text{ and } f \geqslant 0.$$

Consider the Cauchy problem:

$$(5.5)\qquad \begin{vmatrix} f_t + Af + hf = J_m''(f,f), \\ f(0) = f_0. \end{vmatrix}$$

Then Eqs. (5.4) and (5.5) constitute a monotone pair. Hence there exists a generalized solution f_m'' of Eq. (5.5) and

$$f_m'' \leqslant f_m,$$
$$\|f_m''\|_s \leqslant \|f_m\|_s \leqslant C_T\|f_0\|_s.$$

Since $\left(U(t)e^{-ht}, J_m'', f_0\right)$ and $\left(U(t)e^{-ht}, J_k'', f_0\right)$ constitute a monotone pair too, then

$$f_k'' \leqslant f_m'' \quad \text{if} \quad k \leqslant m.$$

Hence we have a monotone sequence $\{f_j''\}_{j=1}^{\infty}$ and since

$$\|f_m''(t)\|_2 \leqslant \|f_m(t)\|_2 = \|f_0\|_2,$$

the sequence is bounded, this implies convergence

$$f(t) = \lim_{m\to\infty} f_m''(t)$$

We have also

$$\|f(t)\|_s \leqslant C_T \|f_0\|_s$$

as for $f_m(t)$.

b. f_m'' is a strong solution of Eq. (5.5)

Let us consider the equation (we write for simplicity f, f_t^i instead of f_m'', $(f_m'')^i$):

$$f^{i+1} = U(t)e^{-ht}f_0 + \int_0^t U(t-s)e^{-h(t-s)}J_0''(f^i, f^i)(s)\,ds.$$

We show now that if f^i is continuously differentiable and $\left\|\frac{df^i}{dt}\right\|_2$ is bounded, then the same holds for f^{i+1}.

Let us observe that

$$\left\|\frac{d}{dt}J_m''(f^i, f^i)\right\| \leqslant 4K\left\|\frac{df^i}{dt}\right\|_2 \|f^i\|_2.$$

Hence $J_m''(f^i, f^i)$ is continuously differentiable and f^{i+1} is continuously differentiable as well. We have in addition

$$f^{i+1} + (A+h)f^{i+1} = J_m''(f^i, f^i)$$

and

$$\|f_t^{i+1}\|_2 \leqslant (1+2K)C_T\|f_0\|_4\|f_0\|_2.$$

We conclude now that since f_t^i are uniformly bounded, all f^i are equicontinuous with respect to t. Since $\{f^i\}_{i=1}^{\infty}$ is a pointwise convergent sequence of equicontinuous functions, it is convergent uniformly in $C([t_1, t_2], B^2)$ on every bounded interval $[t_1, t_2]$. We have

$$\|f_t^k - f_t^i\| \leqslant \|A(f^k - f^i)\| + \|h(f^k - f^i)\| + \|J_m''(f^k, f^k) - J_m''(f^i, f^i)\|$$
$$\leqslant \|f^k - f^i\|_2(1 + K\|f_0\|_2 + 2K\|f_0\|_2).$$

Hence $\{f_t^i\}_{i=1}^{\infty}$ converges uniformly on every bounded interval and the function $f(t) = \lim_{i\to\infty} f^i(t)$ is differentiable. This ends the proof since if f_m'' is differentiable it is a strong solution of Eq. (5.5).

c. $\{f_m\}_{m=1}^{\infty}$ converges to $f(t)$

Let us consider integral equations which describe f_m and f_m'':

$$f_m(t) = U(t)f_0 + \int_0^t U(t-s)[J_m'(f_m, f_m) - hf_m](s)\,ds,$$

$$f_m''(t) = U(t)f_0 + \int_0^t U(t-s)[J_m''(f_m'', f_m'') - hf_m''](s)\,ds.$$

Then for $s_m = f_m - f''_m$ we have

$$\|s_m(t)\|_2 \leqslant \int_0^t \|J'_m(f_m, f_m) - hf_m\|_2\, ds + \int_0^t \|J''_m(f''_m, f''_m) - hf''_m\|_2\, ds$$

$$= \int_0^t \sum_{(i)} \int J_m\big((f_m)_{(i)}, (f_m)_{(i)}\big)(1+|v|^2)\, dv\, ds + \int_0^t \sum_{(i)} \int J_m\big((f''_m)_{(i)}, (f''_m)_{(i)}\big)(1+|v|^2)\, dv\, ds$$

$$+ \int_0^t \sum_{(i)} \int [k(v, v_1) - k_m(v, v_1)]\, \big(f''_m(v, s)\big)_{(i)} \big(f''_m(v_1, s)\big)_{(i)}\, dv_1\, dv\, ds$$

$$+K \int_0^t \sum_{(i)} \int (1+|v|^2)^2 (1+|v_1|^2)\, \big(f''_m(v, s)\big)_{(i)} \big[\big(f_0(v_1, s)\big)_{(i)} - \big(f_m(v_1, s)\big)_{(i)}\big]\, dv_1\, dv\, ds$$

$$\leqslant 0 + 0 + o(1) + tKC_T\|f_0\|_4 \sup_{0\leqslant s\leqslant t} \|s_m(s)\|_2$$

and

$$\lim_{m\to\infty} \|s_m(t)\|_2 = 0 \quad \text{for} \quad t \text{ sufficiently small}.$$

In the estimates above we have utilized the fact that $(1+|v|^2)$ is the collision invariant and

$$k(v, v_1) - k_m(v, v_1) = 0 \quad \text{for} \quad \max(|v|, |v_1|) \leqslant (mK_k^{-1} - 1)^{1/2}.$$

If $\lim \|s_m(t)\|_2 = 0$ for small t, we can prove step by step that this is true for every t and hence

$$f(t) = \lim_{m\to\infty} f_m(t).$$

d. $f(t)$ **is a strong solution of Eq.** (2.1)

It is easy to see that $f(t)$ is a solution of the following integral equation:

$$f(t) = U(t)f_0 + \int_0^t U(t-s)J(f, f)\, ds$$

hence a generalized solution of Eq. (2.1).

We are now able to prove that $f(t)$ is continuously differentiable and is a strong solution of Eq. (2.1).

To this end let us observe that $\{f_m\}_{m=1}^{\infty}$ form an equicontinuous family of functions with respect to t, as

$$\left\|\frac{df_m}{dt}\right\|_2 \leqslant (2K+1)C_T\|f_0\|_4\|f_0\|_2.$$

Hence $f_m(t)$ converge to $f(t)$ uniformly on every bounded interval. Considerations analogous to that in **b.** show that $\dfrac{df_m}{dt} \to \dfrac{df}{dt}$ uniformly. This ends the proof.

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