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Some remarks on non-Newtonian flow in journal bearings

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THE FLOW of a non-Newtonian lubricant in a journal bearing is considered. The velocity field in the gap as well as the force acting on the shaft were calculated for the Prandtl-Eyring fluid for different orders of approximation of the Sommerfeld number with respect to the eccentricity ratio ε. It has been shown that in engineering calculations the real fluid behaviour may be approximated by the Newtonian behaviour with the differential viscosity used as the proper viscosity.

Rozważono przepływ smaru nienewtonowskiego w łożyskach poprzecznych. Pole prędkości w szczelinie łożyska jak również siłę działającą na wał obliczono dla cieczy Prandtla-Eyringa przy różnych rzędach przybliżenia liczby Sommerfelda w zależności od mimośrodu ε. Wykazano, że w obliczeniach inżynierskich zachowanie się płynu rzeczywistego przybliżyć można za pomocą cieczy newtonowskiej, zastępując zarazem lepkość rzeczywistą przez lepkość różnicową.

Рассматривается течение неньютоновской жидкости (смазки) в поперечных подшипниках. Сделан расчет поля скоростей в зазоре подшипника, а также силы, действующей на вал, для жидкости Прандтля-Эйринга при разных степенях приближения числа Саммерфельда, в зависимости от эксцентриситета ε. Показано, что в инженерных расчетах поведение реальной жидкости можно приблизить с помощью ньютоновской жидкостью, заменяя одновременно действительную вязкость дифференциальной.

1. Introduction

IN RECENT years non-Newtonian fluids have found increasing use as lubricants in diverse kinds of bearings in machines. Moreover, the lubricants in the joints of animals and man exhibit non-Newtonian behaviour. Therefore it is of some import to generalize the theory of lubrication, which is well developed for Newtonian lubricants, to non-Newtonian lubricating fluids. As prototype of a bearing, the simple cylindrical journal bearing, with the gap completely filled by the lubricant and the shaft rotating with constant speed and constant excentricity (Fig. 1), will be studied in this note.

The differences between the Newtonian and non-Newtonian behaviour of fluids can be roughly divided into three mutually overlapping groups, namely nonlinear flow behaviour (viscosity depending on the shear-rate), normal stress effects and relaxation or memory effects. The flow in a journal bearing and, consequently, the force on the rotating shaft is markedly affected by nonlinear flow behaviour, and also by memory effects. In a number of previous papers both types of effects were studied [1–6]. If memory effects are neglected, the force on the shaft has a direction perpendicular to the displacement between the centers of shaft and casing, just as for Newtonian fluids. In a first-order approximation which is linear in the excentricity ratio $\varepsilon = e/b$ this force is given by the same expression as for a Newtonian fluid, provided that the "differential viscosity"

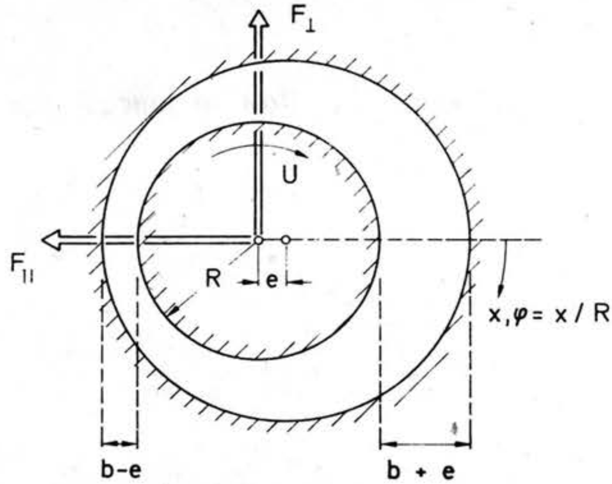

FIG. 1. Cylindrical journal bearing.

$\eta_d = d\tau/d\varkappa$ (τ — shear stress, $\varkappa$ — shear rate) is used in that expression instead of the viscosity:

$$F_\perp = 6\pi\varepsilon\eta_d UR^2/b^2. \tag{1.1}$$

The differential viscosity, as well as the viscosity $\eta = \tau/\varkappa$, depends on the shear rate for most non-Newtonian fluids, and in Eq. (1.1) the value of η_d pertaining to the mean shear rate $\varkappa = U/b$ in the gap has to be used [4].

Memory effects induce a force component parallel to the displacement of the shaft. If memory is "weak" [5, 6], this force component can be calculated; for small excentricity ratio, $\varepsilon \ll 1$, this force component is given by the following analytical expression:

$$F_{\parallel} = -\pi\varepsilon U^2 R/b^2 \cdot \left(N + \varkappa\frac{dN}{d\varkappa} - 2\beta\right). \tag{1.2}$$

Here, $N(\varkappa)$ is the first normal stress coefficient of the fluid and $\beta(\varkappa)$ is the "second flow function" which was introduced in [5, 6]. The case that the shaft vibrates and the case that the gap is not completely filled by the lubricant were also studied in [6], and formulas for the two force components were derived.

In [3] it was maintained, and supported by plausible arguments, that the approximation (1.1) for $F_\perp$, derived for $\varepsilon \ll 1$, is quite acceptable for values of ε up to 0.5. This statement is justified in the following sections by the results of numerical calculations for the Prandtl–Eyring fluid and by a direct calculation of the ε^3-contribution to $F_\perp$ for arbitrary fluids. Memory effects are neglected here.

2. Flow between parallel plates

The starting point for the following derivations is the flow law of the fluid, i. e. the relation between shear stress and shear rate in viscometric flow [4, 6, 7]:

$$\varkappa = \frac{\tau_*}{\eta_*} g\left(\frac{\tau}{\tau_*}\right). \tag{2.1}$$

Here, η_* and τ_* are reference quantities with the meaning of a characteristic viscosity and a characteristic stress; η_* is chosen as the lower Newtonian viscosity: $\eta_* = \lim_{\varkappa \to 0} \tau/\varkappa$. For each particular fluid g is assumed to be given.

As a prerequisite for studying the flow in the gap of the bearing we study the flow between two parallel walls, one of which is fixed whereas the other one moves with constant velocity U. A pressure gradient $\lambda = \partial p/\partial x$ acts in the direction of the motion of the wall (Fig. 2a). By using a frame of reference which moves with the velocity $U/2$ in the x-direction, we obtain the situation depicted in Fig. 2b. The shear stress is given by

$$\tau = \tau_0 + \lambda y. \tag{2.2}$$

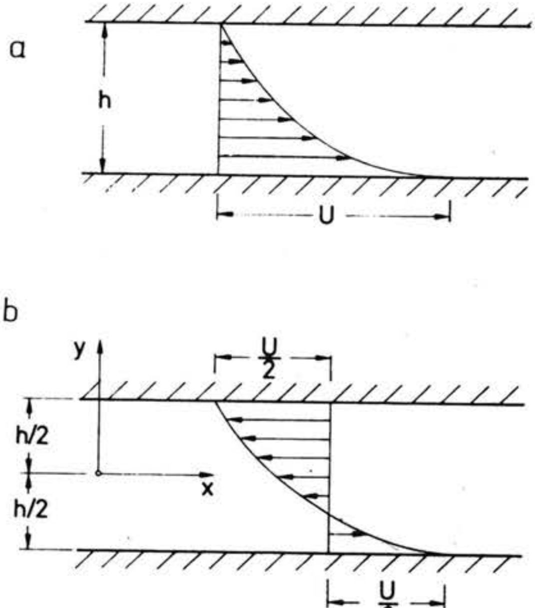

FIG. 2. Flow in a straight channel with moving wall.

The velocity distribution $u(y)$ satisfies the following equation obtained by combining Eqs. (2.1) and (2.2), with $\varkappa = du/dy$

$$\frac{du}{dy} = \frac{\tau_*}{\eta_*} g\left(\frac{\tau_0}{\tau_*} + \frac{\lambda y}{\tau_*}\right). \tag{2.3}$$

The following dimensionless quantities are now introduced:

$$v := \frac{u}{U}, \quad \xi := \frac{2y}{h}, \quad B := \frac{\lambda h}{2\tau_*}, \quad C := \frac{\tau_* h}{\eta_* U}, \quad S := \frac{\tau_0}{\tau_*}. \tag{2.4}$$

Thereby Eq. (2.3) is transformed into

$$\frac{dv}{d\xi} = \frac{C}{2} g(S + B\xi), \tag{2.5}$$

which is to be solved with the boundary conditions

$$v(-1) = \frac{1}{2}, \quad v(+1) = -\frac{1}{2}. \tag{2.6}$$

The flow volume in the original frame of reference (in which the upper wall is at rest) is

$$\dot{V} = \frac{Uh}{2} + \int_{-h/2}^{+h/2} u\,dy = \frac{Uh}{2}\left(1 + \int_{-1}^{+1} v(\xi)\,d\xi\right). \tag{2.7}$$

This can be nondimensionalized as follows:

$$q := \frac{\dot{V}}{Uh} = \frac{1}{2}\left(1 + \int_{-1}^{+1} v(\xi)d\xi\right). \tag{2.8}$$

Integration of Eq. (2.5) and taking into account the boundary condition $v(-1) = 1/2$ yields

$$v = \frac{1}{2} + \frac{C}{2B} \int_{S-B}^{S+B\xi} g(\sigma)d\sigma. \tag{2.9}$$

The boundary condition $v(+1) = -1/2$ leads to

$$\frac{C}{2B} \int_{S-B}^{S+B} g(\sigma)\,d\sigma = -1. \tag{2.10}$$

This equation determines the parameter S as a function of B and C. Inserting $S = S(B, C)$ into Eq. (2.9) yields $v = v(\xi; B, C)$ and, using this as integrand in Eq. (2.8), finally leads to

$$q = \frac{1}{2}(1+f(B, C)), \tag{2.11}$$

where

$$f(B, C) = \int_{-1}^{1} v(\xi; B, C)\,d\xi = \frac{C}{2B^2} \int_{S-B}^{S+B} (S-\sigma)\,g(\sigma)\,d\sigma. \tag{2.12}$$

This completes the determination of the flow between two parallel plates. It is clear that f is an odd function of B; furthermore $f(0, C) = 0$. Hence $f = a_1 B + a_3 B^3 + \ldots$, where the a_i are functions of C; (see Sect. 5).

3. Flow in the narrow gap of a journal bearing

In a journal bearing (Fig. 1) the mean gap width b is much smaller than the radius of the shaft: $b/R \ll 1$. Therefore, in order to determine the flow in the bearing, the gap can be unrolled into a straight gap of varying width h (Fig. 3):

$$h = b(1 + \varepsilon\cos\phi), \tag{3.1}$$

with

(3.2) $$\phi = x/R.$$

Furthermore, inertia forces are usually negligible for the motion of the lubricant. Under these circumstances, and because memory effects of the fluid are neglected, one can assume that locally, for every value of ϕ, the flow is the same as that between two parallel walls, which has been discussed in the preceding section. According to Eq. (2.11), the flow volume at every position ϕ within the gap is given by

(3.3) $$\dot{V} = \frac{Uh}{2}(1+f(B, C)) = \frac{Uh_0}{2}.$$

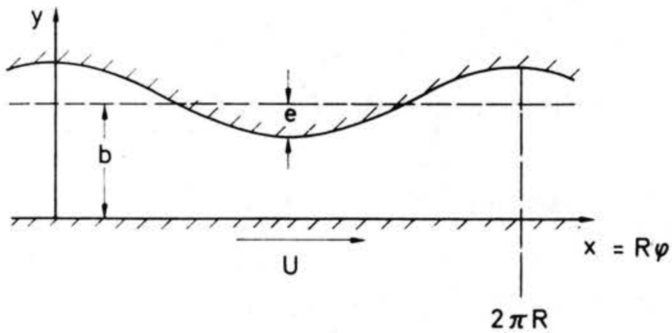

FIG. 3. Straight gap with variable width h.

Note that in Eq. (3.3) the quantities h, B, C depend on ϕ; h is given by Eq. (3.1) and, using Eq. (3.1), one can write B and C in the form

(3.4) $$B = B_0(1+\varepsilon\cos\phi), \quad C = C_0(1+\varepsilon\cos\phi),$$

where B_0 and C_0 have the following meaning:

(3.5) $$B_0 = \frac{\lambda b}{2\tau_*}, \quad C_0 = \frac{\tau_* b}{\eta_* U}.$$

Of course, the flow volume $\dot{V}$ must be independent of ϕ and hence constant. This constant value of $\dot{V}$ defines the parameter h_0 on the right hand side of Eq. (3.3). The parameter h_0 still has to be determined. For that purpose Eqs. (3.1) and (3.4) are inserted into Eq. (3.3); this yields

(3.6) $$(1+\varepsilon\cos\phi)\{1+f(B_0(1+\varepsilon\cos\phi), C_0(1+\varepsilon\cos\phi))\} = \frac{h_0}{b}.$$

Equation (3.6) can be solved, in principle, for B_0:

(3.7) $$B_0 = B_0(\varepsilon\cos\phi, C_0, h_0/b).$$

Since the pressure must be 2π-periodic in the variable ϕ, the relation $\int_0^{2\pi} \lambda d\phi = 0$ must be satisfied or

(3.8) $$\int_0^{2\pi} B_0 d\phi = 0.$$

From Eq. (3.8) one can now determine the parameter h_0/b as a function of the two given parameters ε and C_0. Inserting this result for h_0/b into Eq. (3.7) yields B_0 as a function of ϕ and the parameters ε and C_0.

The force acting on the rotating shaft per unit length is given by [3, 5, 6]:

$$(3.9)\qquad F_\perp = R^2 \int_0^{2\pi} \lambda \cos\phi \, d\phi = \frac{2R^2\tau_*}{b} \int_0^{2\pi} B_0 \cos\phi \, d\phi .$$

Since, according to Eq. (3.7), B_0 depends on ϕ only through $\cos\phi$, it is clear that the force component $F_\parallel$ is zero; this is due to the neglect of memory effects. The result of the integration (3.3) can be written in the form

$$(3.10)\qquad F_\perp = \frac{R^2 U \eta_*}{b^2} \cdot \mathrm{So}(\varepsilon, C_0).$$

Here the dimensionless quantity So is the "Sommerfeld number" [4].

4. Results for the Prandtl–Eyring fluid

It is obvious that for a general flow law, i. e. for a general function $g(\tau/\tau_*)$, the calculations described in Sects. 2 and 3 can be performed only numerically, with the exception of a few particularly simple cases. One of these cases is the Newtonian fluid with viscosity η_*, for which $g(\tau/\tau_*) = \tau/\tau_*$. For that fluid one obtains the well-known results (see [8])

$$(4.1)\qquad f(B, C) = -\frac{BC}{3}$$

and

$$(4.2)\qquad \mathrm{So} = \frac{6\pi\varepsilon}{\sqrt{1-\varepsilon^2}\left(1+\frac{\varepsilon^2}{2}\right)} .$$

The Sommerfeld number depends on ε only. This is to be expected for a Newtonian fluid because the flow law $\varkappa = \tau/\eta_*$ is independent of an arbitrary reference stress value τ_*; therefore the result for So must also be independent of τ_* and hence of C_0. The series expansion of So with respect to powers of ε starts with

$$(4.3)\qquad \mathrm{So} = 6\pi\varepsilon + O(\varepsilon^5).$$

Because the term following the linear term is already a fifth-order term it is to be expected that the linear approximation is quite satisfactory up to moderately high values of ε. A comparison of the linear approximation with the exact result for So in Fig. 4 confirms this surmise: for $\varepsilon \leqslant 0.5$ the deviation of the linear approximation from the exact result is practically negligible.

Another fluid for which simple analytical results are at least partly possible is the Prandtl–Eyring fluid. The flow law is here

$$(4.4)\qquad g\left(\frac{\tau}{\tau_*}\right) = \sinh\left(\frac{\tau}{\tau_*}\right).$$

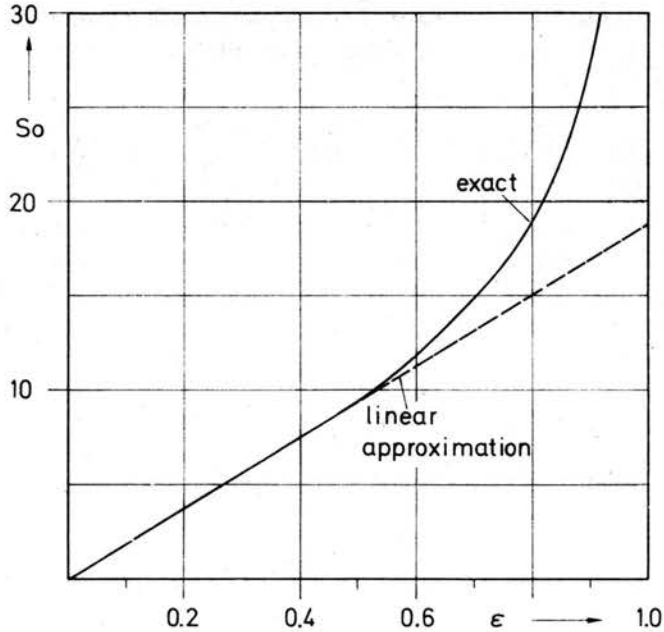

FIG. 4. Sommerfeld number for Newtonian lubricant. Exact result: Eq. (4.2); linear approximation: Eq. (4.3).

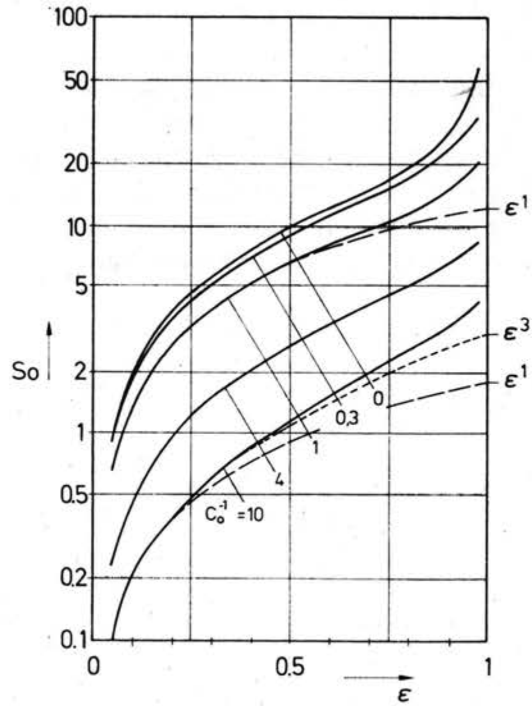

FIG. 5. Sommerfeld number for Prandtl–Eyring fluid as lubricant. Linear ε^1-approximation: Eqs. (4.6), (4.7); third order ε^3-approximation: Eqs. (5.16), (5.18).

This is a shear-thinning fluid; the viscosity decreases with increasing stress τ or increasing shear rate $\varkappa$. For the Prandtl–Eyring fluid the calculations of Sect. 2 can be performed analytically with the result [5, 7, 9]

$$f(B, C) = -\left[1+\left(\frac{C}{B}\sinh B\right)^2\right]^{1/2}\left(\operatorname{Coth} B-\frac{1}{B}\right). \tag{4.5}$$

However, the calculations of Sect. 3 have to be performed numerically. Some of the results (obtained by G. BÖHME, HAMBURG [7], and also by W. OCHS, DARMSTADT) are shown in Fig. 5 as the solid curves. Also shown are the approximations which are linear in ε. As has been shown elsewhere [4, 6, 7], these linear approximations are given by

$$\mathrm{So} = 6\pi\varepsilon\eta_d/\eta_*, \tag{4.6}$$

where η_d is the differential viscosity pertaining to the mean shear rate, $\varkappa_0 = U/b$, in the gap of the bearing. The ratio η_d/η_* is a function of C_0. For the Prandtl–Eyring fluid

$$\frac{\eta_d}{\eta_*} = \frac{C_0}{\sqrt{1+C_0^2}}. \tag{4.7}$$

(Note that without loss of generality $C_0 > 0$ is assumed).

5. Third order analytical results

The diagram in Fig. 5 allows to assess the range of ε for which the simple and useful approximation (4.6) is practically sufficient if the lubricant is a Prandtl–Eyring fluid. In order to establish the range of validity of the approximate result (4.6) in the case of a general fluid, a third order approximation in ε is derived in this section. The derivation is based on the fact that the pressure gradient λ, and hence also B, vanishes if $\varepsilon = 0$. As a matter of fact, B is proportional to ε for sufficiently small values of ε. To derive a third order result in ε it is therefore sufficient to use approximations to the formulas of Sects. 2 and 3 which are of third order in B.

The details of the routine calculations are omitted here, only the most important steps on the way to the final result are sketched. For a given value of S the expansion of the boundary condition (2.10) yields

$$g(S) = -\frac{1}{C}-\frac{g''(S)}{6}B^2+O(B^4). \tag{5.1}$$

The expansion of the velocity v is (cf. Eq. (2.9))

$$v(\xi) = -\frac{\xi}{2}+\frac{CB}{4}\left\{g'(S)(\xi^2-1)+B\frac{g''(S)}{3}(\xi^3-\xi)+B^2\frac{g'''(S)}{12}(\xi^4-1)\right\}+O(B^4). \tag{5.2}$$

Integrating this expression yields, according to Eq. (2.12),

$$f(B, C) = -\frac{BC}{3}g'(S)\left\{1+\frac{g'''(S)}{g'(S)}\frac{B^2}{10}\right\}. \tag{5.3}$$

One has to keep in mind that Eq. (5.3) is not yet the final third order result for f because S depends on B, C, too (cf. Eq. (2.10)); therefore $g'(S)$ and $g'''(S)$ must also be expanded with respect to powers of B. In order to perform this expansion, we assume that the flow function g satisfies the following differential equation:

(5.4) $$g' = \phi(g).$$

This equation permits a simple interpretation:

(5.5) $$g' = \frac{dg}{d(\tau/\tau_*)} = \frac{\tau_* dg}{d\tau} = \eta_* \frac{d\varkappa}{d\tau} = \frac{\eta_*}{\eta_d}.$$

Hence

(5.6) $$\eta_d = \frac{\eta_*}{\phi(g)} = \frac{\eta_*}{\phi\left(\dfrac{\eta_* \varkappa}{\tau_*}\right)}.$$

Therefore the function $\phi(g)$ is known for every fluid once the dependency of the differential viscosity on the non dimensional shear rate, $\eta_* \varkappa/\tau_*$, is known. We note the following identities (the dot denotes differentiation of ϕ with respect to g):

(5.7) $$g' = \phi(g), \quad g'' = \dot{\phi}\phi, \quad g''' = \ddot{\phi}\phi^2 + \dot{\phi}^2\phi.$$

Taking account of Eq. (5.1), one derives

(5.8) $$g'(S) = \phi\big(g(S)\big) = \phi\left(-C^{-1} + \frac{g''(S)}{6} B^2\right) + O(B^4)$$

or

(5.9) $$g'(S) = \phi(-C^{-1}) - \dot{\phi}(-C^{-1}) \cdot \frac{g''(S)}{6} B^2 + O(B^4).$$

Since the second term on the right side of Eq. (5.9) is already of second order in B, we can approximate the coefficient $g''(S)$ by

(5.10) $$g''(S) = \phi(-C^{-1})\, \dot{\phi}(-C^{-1}).$$

Likewise the term $g'''(S)/g'(S)$ appearing in Eq. (5.3) can be approximated by

(5.11) $$g'''(S)/g'(S) = \ddot{\phi}(-C^{-1})\, \phi(-C^{-1}) + \dot{\phi}^2(-C^{-1}).$$

This leads to the final third order result

(5.12) $$f(B, C) = -\frac{B}{3\psi}(1 + B^2\gamma) + O(B^5),$$

where ψ and γ are the following functions of C:

(5.13) $$\psi(C) = \frac{1}{C\phi}, \quad \gamma = \frac{1}{30}(3\ddot{\phi}\phi - 2\dot{\phi}^2).$$

The argument of ϕ, ϕ, ϕ is $-C^{-1}$, or, because ϕ is an even function, C^{-1}.

For the Prandtl–Eyring fluid Eq. (5.14) yields

(5.14) $$\psi = \frac{1}{\sqrt{1+C^2}}, \quad \gamma = \frac{1}{30}\frac{3C^2-2}{1+C^2}.$$

Inserting these expressions into Eq. (5.12) gives the result

$$f(B, C) = -\frac{B\sqrt{1+C^2}}{3}\left\{1+\frac{B^2}{30}\frac{3C^2-2}{1+C^2}\right\}. \tag{5.15}$$

The same result is easily derived by direct expansion of Eq. (4.5).

With the generally valid approximation (5.12) one can now perform the calculations outlined in Sect. 3 in order to find an approximation to the Sommerfeld number which is of third order in the excentricity ratio ε.

The necessary calculations are straightforward but tedious; therefore they are omitted here. The result can be written in the form

$$\mathrm{So} = 6\pi\varepsilon\eta_*/\eta_d \cdot (1-\Gamma\varepsilon^2), \tag{5.16}$$

with

$$\Gamma(C_0) = \frac{1}{4}\left\{\frac{3C_0^2\psi''\psi+4C_0\psi'\psi-4C_0^2\psi'^2}{2\psi^2} -27\gamma\psi^2-1\right\}. \tag{5.17}$$

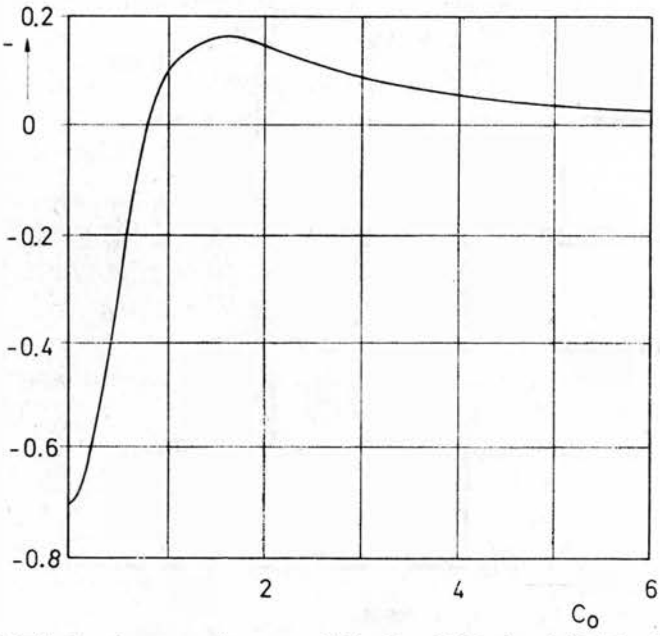

FIG. 6. Third order correction term Γ for Prandtl–Eyring fluid, Eq. (5.18).

The argument of ψ, γ, ψ', ψ'' in Eq. (5.17) is C_0. Specialisation to the Prandtl-Eyring fluid gives

$$\Gamma = \frac{7}{20}\frac{3C_0^2-2}{(1+C_0^2)^2}. \tag{5.18}$$

Figure 6 shows Γ, according to Eq. (5.18), as a function of C_0. One notes that $\lim_{C_0\to\infty}\Gamma = 0$; in this limit the fluid is Newtonian with viscosity η_*. In that case the third order term in Eq. (5.16) vanishes in agreement with Eq. (4.3). However, the third order term vanishes also for $C_0 = \sqrt{2/3}$ because for that value of C_0 the coefficient Γ is also zero.

6. Conclusion

Figure 7 presents a comparison of different approximations for the Sommerfeld number. The figure shows the exact value of the Sommerfeld number for a Prandtl–Eyring lubricant with $C_0 = 0.1$ as solid line. The ε^1-approximation is given by the dashed line and the ε^3-approximation by the dotted line. The dash-dotted line in Fig. 7 was obtained by assuming that the fluid behaves strictly Newtonian for all values of ε, and that the viscosity of the fluid is the differential viscosity η_d pertaining to the mean shear rate U/b. Under these circumstances the Sommerfeld-number is given by the expression (4.2), multiplied by the ratio η_d/η_* (as given by Eq. (4.7)). For $\varepsilon \to 0$ the exact result is approached asymptotically because the linear approximation to the dash-dotted line is, of course, given by Eq. (4.6) and is therefore identical with the linear approximation to the exact result.

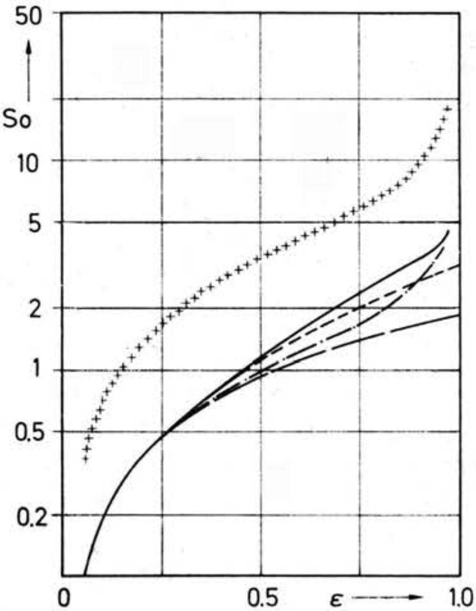

FIG. 7. Comparison of different approximations with exact result for Sommerfeld number; $C_0 = 0.1$. Linear approximation: — — —; third order approximation: - - -; Newtonian result with η_d as viscosity: —.—.—; Newtonian result with η as viscosity: + + + +.

As can be seen the dash-dotted line is a fairly good approximation to the solid line, representing the exact result, in the whole range of excentricity ratios ε for which the curves have been drawn (up to $\varepsilon \approx 0.9$). The approximation is still better for higher values of the parameters C_0 (for $C_0 \to \infty$ the fluid becomes progressively more Newtonian; for $C_0 = 0.1$ the fluid behaves markedly non-Newtonian). Therefore it seems expedient to recommend for engineering purposes the use of the Newtonian expression (4.2) for the Sommerfeld number multiplied by the ratio η_d/η_*, where η_d is determined by the mean shear rate U/b. This amounts to a completely Newtonian theory for the force $F_\perp$ with the viscosity given by η_d. The approximation obtained thereby is asymptotically correct in the limit $\varepsilon \to 0$.

If one uses, unjustifiably, the viscosity η instead of the differential viscosity η_d in such a calculation, the result is far off the mark if the fluid behaves strongly non-Newtonian. The value of So thus calculated for $C_0 = 0.1$ is for all values of ε three times the value calculated by using η_d as viscosity. This causes an unacceptably large deviation from the correct value, as may be inferred from Fig. 7, where the Newtonian result based on η is shown as the crossed curve. These results make clear that in certain situations it is useful to approximate the real fluid behaviour by Newtonian behaviour, as is done often in engineering calculations. However, the viscosity to be used is not always the viscosity proper but the differential viscosity. The flow in a journal bearing illustrates this statement.

A final conclusion may be drawn from the foregoing discussion: The result that a Newtonian approximation is a fairly good one, and is even asymptotically correct for $\varepsilon \to 0$, is a consequence of the fact that the pressure distribution in the gap of the bearing is also approximately the same as for a Newtonian fluid. This means that for a given value of the force $F_\perp$ the maximum and the minimum value of the pressure in a non-Newtonian lubricant will be approximately the same as in a Newtonian lubricant. If $\varepsilon \to 0$, the pressure distribution becomes exactly the same as in a Newtonian lubricant. Engineers claim that the use of shear-thinning lubricants decreases the pressure peaks for a given load on the bearing. The remarks just made show that the effect cannot be very drastic and that it must disappear for small values of ε, i.e. for weakly-loaded bearings; numerical calculations of the pressure distribution confirm this (see, e.g. [10]).

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