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An investigation of the Mach disc and the Riemann wave formation in impulse jets

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THE SHOCK waves in nonstationary jets of shock-heated N_2, Ar and CO_2 outflowing from flat and axisymmetrical sonic nozzles have been studied experimentally. The pressure range $N = P_{00}/P_\infty = 20 \div 200$ was investigated. Here P_{00} is the pressure behind the reflected shock wave, P_∞ is the pressure in the vacuum chamber. Schlieren photographs for the nonstationary jet structure were obtained for different spatial orientations of the flat nozzle using a schlieren installation IAB-451 and a spark light source. On the basis of the obtained schlieren photographs of jet formation, the nonstationary wave structure of flow was reproduced. The geometrical characteristics of the nonstationary Riemann wave for three nozzles and different values of n were generalized in dimensionless similarity parameters. A dependence between the location of the Mach disc and time was obtained up to the stationary location. Moreover, a comparison with the calculation was also made. An analysis was performed for the acceptance of the model of suddenly introduced source to the studied flow.

Przeprowadzono badania doświadczalne nad ogrzewanymi uderzeniowo niestacjonarnymi strumieniami N_2, Ar i CO_2 wypływającymi z płaskich i osiowosymetrycznych dyszy dźwiękowych. Rozważono ciśnienia w zakresie $N = P_{00}/P_\infty = 20 \div 200$, przy czym P_{00} oznacza ciśnienie za odbitą falą uderzeniową, a P_∞ jest ciśnieniem w komorze próżniowej. Uzyskano obrazy schlierena struktury niejednorodnego strumienia dla różnych orientacji przestrzennych dyszy płaskiej stosując instalację schlierena IAB-451 i iskrowe źródło światła. Na podstawie uzyskanych zdjęć schlierena powstającego strumienia odtworzono strukturę fali niestacjonarnej. Charakterystyki geometryczne niestacjonarnej fali Riemanna dla trzech dysz i różnych wartości n przedstawiono za pomocą bezwymiarowych parametrów podobieństwa. Otrzymano zależność między położeniem tarczy Macha i czasem, aż do położenia stacjonarnego. Przeprowadzono porównanie wyniku z obliczeniami. Rozważono kwestię stosowalności modelu w przypadku, gdy do rozważanego przepływu wprowadza się nagle źródło.

Проведены экспериментальные исследования ударно нагреваемых нестационарных потоков Na, Ar и CO_2, истекающих из плоских и осесимметричных звуковых сопел. Рассмотрены давления в интервале $N = P_{00}/P_\infty = 20 \div 200$, причем P_{00} — обозначает давление за отраженной ударной волной, а P_∞ — это давление в вакумной камере. Получены шлирен-изображения структуры неоднородного потока для разных пространственных ориентаций плоского сопла, применяя установку шлирена IAB-451 и искровый источник света. На основе полученных шлирен-пзображений возникающего потока, восстановлена структура нестационарной волны. Геометрические характеристики нестационарной волны Римана, для трех сопел и для разных значений n, представлены при помощи безразмерных параметров подобия. Получена зависимость между положением диска Маха и временем вплоть к стационарному положению. Проведено сравнение результата с расчетами. Рассмотрен вопрос применимости модели течения от внезапно включенного источника к исследованным течениям.

THE EXPERIMENTAL data on the formation of the wave structure of a three-dimensional impulse jet are presented in [1]. The development of inversion, which is known in hydrodynamics for jets that do not have axial symmetry, was discussed. The authors continued their studies and examined in detail the problem related to the motion and dimensions of the secondary wave of compression, i.e. the analogue of the Riemann wave in three-

-dimensional jets and the nonstationary Mach disc in axi-symmetrical jets. The results of these studies are presented in this paper.

A reliable calculation of the distribution of parameters in an impulse jet can be made if the wave structure and its rearrangement in time are known under the flow regimes considered. At present these data can be obtained only by experiments.

The Mach disc and the Riemann wave are the determining elements of the wave structure of a nonstationary flow. But the assumptions made in [2–5] to substantiate the prerequisites of calculations of position and dimensions of the Mach disc and the Riemann wave are contradictory. Empirical relationships are available, which connect the position of the Mach disc on the axis of the stationary jet with flow parameters at the nozzle outlet [6, 7]. In two-dimensional jets the distance up to the Riemann wave can be determined with the help of formulae from [8, 9]. In [9] a generalized formula was proposed to determine the distance both up to the Mach disc in axisymmetrical jets and up to the Riemann wave in two-dimensional jets. A detailed study of the transformation of a nonstationary wave structure of the jet should help to reveal the nature of the dependence of the stationary structure on the determining parameter and to provide physical grounds for the selection of initial data for calculating the position and dimensions of the Mach disc and the Riemann wave.

The investigations were carried out in a 40×40 mm^2 shock tube fitted with a 4-*m* long low-pressure chamber with the help of a schlieren installation IAB-451. A plane or an axisymmetrical nozzle was installed at the end of the shock tube. The shock-heated gas flowed through this nozzle into the vacuum chamber. The tube and the vacuum chamber were filled with the gas under examination. The pressure of the gas was 10–50 mm of mercury column, the Mach number of the incident shock wave was $M = 2.5$–4. This ensured $N = 20$–200, where $N = P_{00}/P_\infty$ (P_{00} — pressure behind the reflected shock wave, P_∞ — pressure in the vacuum chamber). The parameters at the end remained unchanged ~ 500 μsec.

The successive stages of flow were recorded both by taking frame-by-frame photographs (flash time 1 μsec) on an aerial photographic film using spark light source (sensitivity of the film in GOST units was 0.85–1000, photograph size was 45×100 mm^2) and by continuous scanning through the use of flash light IFK-120 (flash time 1200 μsec) on a photographic recorder ZhFR-1 (the rate of rotation of prism was 1875 rpm). A 1.0 mm wide slit was installed on the window of the vacuum chamber along the axis of flow. These two methods of registration complemented each other ensuring large resolution of time and space. A detailed analysis of the jet structure was carried out from the photographs obtained with the help of the frame-by-frame photograph method. The development of flow in time was also analysed by the continuous scanning method.

Two series of experiments were carried out. In the first series the longitudinal axis of the nozzle was set parallel to the optical axis of the schlieren installation (Fig. 1a), and in the second series perpendicular to the installation optical axis (Fig. 1b).

Because of the timing system used in the experiments, both in the first series and in the second series the process could be photographed at different but pairwise identical instants reckoned from the beginning of the gas flow. Thus it was possible to construct the spatial model for the jet flow from a flat nozzle for different time instants.

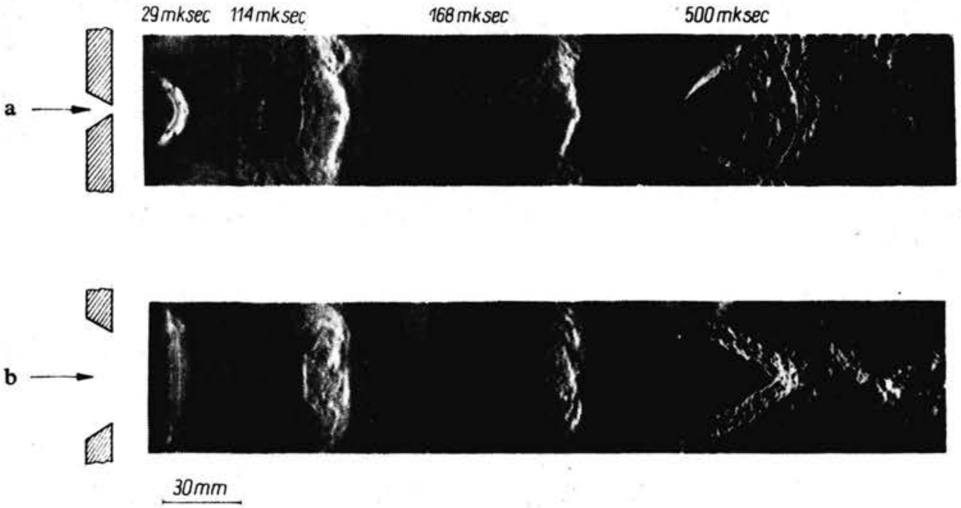

FIG. 1.

Figure 2a shows the initial region of the jet flow along the longitudinal axis of the zzle while Fig. 2b shows the flow along the lateral axis of the nozzle. In these diagrams tands for the jet boundary, *2* for the cylindrical intercepting shock, *3* for the Riemann

and

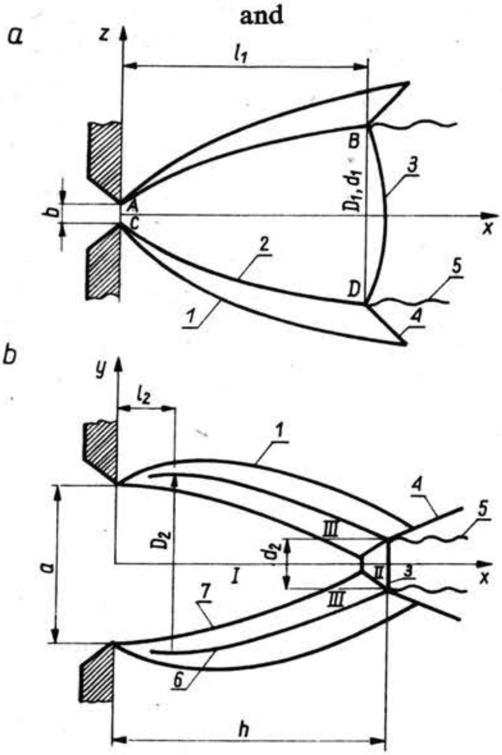

FIG. 2.

wave, *4* for the reflected shock waves, *5* for the slip surface, *7* for the projection of the interaction region of intercepting shocks *2*, visible in Fig. 2a, with the spatial intercepting shock *6* on the plane of the longitudinal axis.

Now consider the schematic representation of the initial region of the jet along the lateral axis of the nozzle (Fig. 2b). The diagram shows three distinct regions.

Region I is the projection of the surface of the cylindrical intercepting shock *2* on the plane. *AB* and *DC* is the projection of the shock *2* on the *zx*-plane. Region II is the projection of the Riemann wave *BD* on the *yx*-plane. Region III is the projection of intercepting shocks which limit the jet on the sides, i.e. in the direction of $\pm y$. Unlike the intercepting shocks *AB* and *DC* whose surfaces are cylindrical, the lateral intercepting shocks have a complicated three-dimensional surface. When the ratio a/b of the nozzle

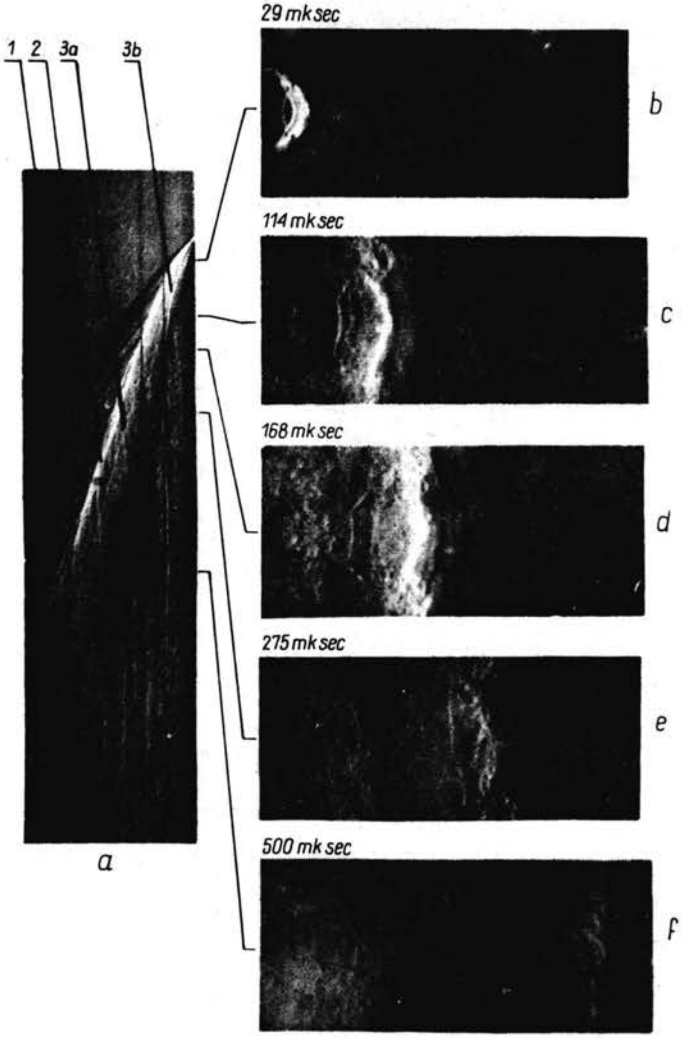

FIG. 3.

diameters is reduced, cylindrical regions are also formed on the lateral intercepting shock, and both the projections of the jet thus become qualitatively similar.

On increasing the ratio $n = P_a/P_\infty$ (P_a is the nozzle outlet pressure) the effect of rarefaction waves which destroy the jet along the y-axis is faster than along the z-axis. This accounts for the fact that as n increases, the width of the Riemann wave decreases while the distance between it and the nozzle section increases from the same phase of the flow.

After the general structure of the three-dimensional impulse jet has been established, we shall turn our attention to the study of the secondary shock wave. The Töpler pictures of the outflow of N_2 from a flat nozzle (Fig. 3a) and an axisymmetrical nozzle (Fig. 4a) are shown in the photographs. Typical surfaces are shown in schlieren pictures. In these pictures *1* stands for the trajectory of the motion of the surface of the primary shock wave, *2* contact surface of the flowing gas, *3a* and *3b* surfaces of the secondary shock wave in the flowing gas. Figures 3b–f show five successive stages of the outflow of N_2 from a flat nozzle. These figures were obtained making use of a spark light source.

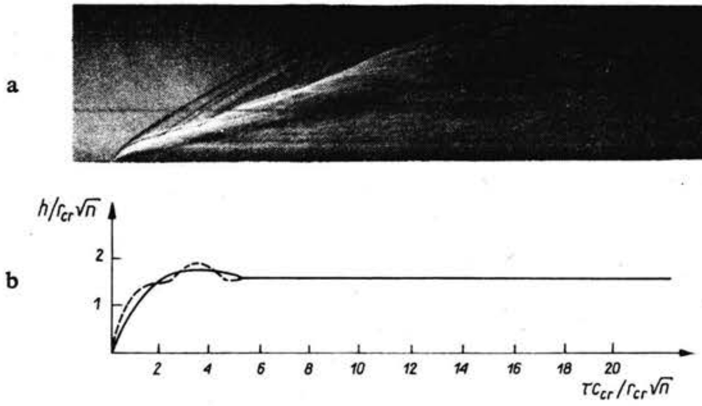

FIG. 4.

Now we shall consider the distinctive characteristics of the flow which can be noticed by correlating the photographs obtained by the frame-by-frame and continuous scanning photorecording method. Both in the plane and axisymmetrical flow the gas moves with intense formation of vortices. A very complicated interaction of vortices within the flowing gas also affects the trajectory of motion of its front by large and small pulses. This in turn results in the formation of sound and more intense density disturbances. On the photographs between the trajectories of the motion of the primary shock wave and the contact surface are seen the traces of theses disturbances which like any other disturbance behind the shock wave catch it up and absorb it. These disturbances are one of the causes that affect the self-similarity of flow.

In this paper we shall not examine the secondary wave *3a* that appears in the neighbourhood of the front of the flowing gas but shall study the wave *3b* which tends to occupy the quasi-stationary position of the Riemann wave and the Mach disc. Based on an analysis of a large number of photographs, the nature of the motion of the compression wave in the flowing gas, i.e. the non-stationary Mach disc (*3b*), was discovered. The curves of motion of the nonstationary Mach disc, when N_2 outflows from a sonic nozzle of a 4 mm diameter (τ — is the precession time, C_{cr} the velocity of sound in the critical section, h the distance along the axis measured from the nozzle outlet up to the nonstationary Mach disc, r_{cr} the radius of the critical section) are shown in Fig. 4*b* in the dimensionless similarity parameters $h/r_{cr}\sqrt{n}$ and $\tau C_{cr}/r_{cr}\sqrt{n}$. As may be seen from Fig. 4b, during jet formation the nonstationary Mach disc in the considered regimes exceeds its stationary position by 10–15%. Thereafter it returns back and occupies the quasi-stationary position after ~140 μsec from the beginning of the outflow. Moreover, as the Mach disc moves towards the quasi-stationary position, it (Mach disc) vibrates with a small amplitude (~7%) and frequency ~15 kHz, which are not reproduced from experiment to experiment. The vibrations obtained in one of the experiments are shown by dotted lines on the diagram. Now we shall examine how the quasi-stationary position of the Mach disc achieved in our experiments will correlate with the position determined by the formulae given in [6] and [7]. From the graph (Fig. 5) it is clear that the experimental points are sufficiently well-described by the formula from [6].

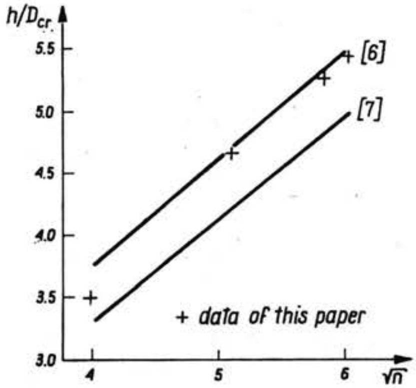

FIG. 5.

The formation of Riemann waves in three-dimensional jets was studied for the outflow of N_2, argon and carbon dioxide gases from flat sonic nozzles of dimensions 1.5×40 mm², 2.2×40 mm² and 0.8×32 mm² critical section. One of the main characteristics of the wave structure, i.e. the distance up to the Riemann wave in two-dimensional stationary jets, is proportional to n. We shall use this criterion to analyse the motion of the nonstationary Riemann wave during the formation of a three-dimensional jet. To this end we shall draw in dimensionless coordinates a dependence of the motion of the Riemann wave in time at the initial stage (up to 140 μsec) (Fig. 6). The dependences obtained for different gases were approximated with the use of power functions of the type $y = Ax^{\alpha}$, where A and α are tabulated in Fig. 6.

As the formation time of the stationary flow of a two-dimensional jet is much larger than that of the axisymmetrical one [10], the Riemann wave does not have time to occupy its stationary position during that period of time when the parameters at the end of the tube remain unchanged.

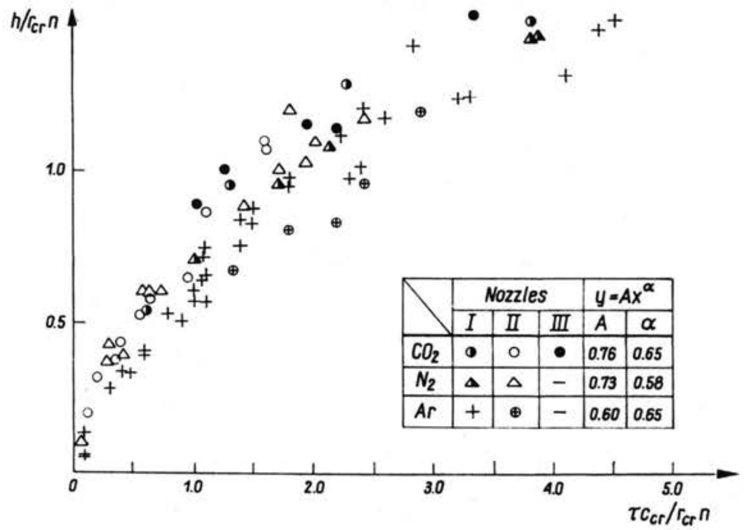

FIG. 6.

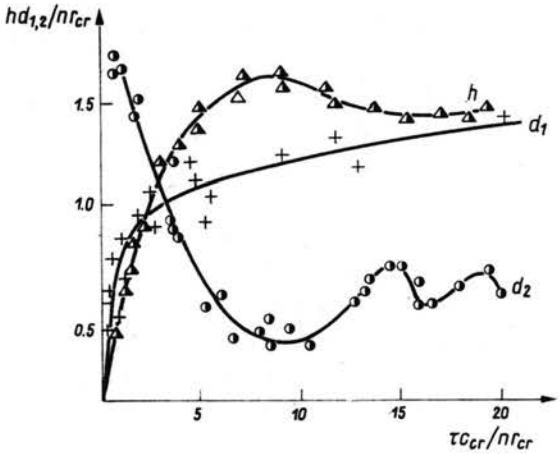

FIG. 7.

Figure 7 shows the height d_1, width d_2 and the position h of the Riemann wave in similarity parameters obtained for the N_2 jet. The parameters were recorded throughout the "clear" experiment time (up to 500 μsec). The curves of the position and the width of the Riemann wave show that the wave structure formation is an oscillatory process and the height d_1 measurement accuracy and the data scatter probably exceed the amplitude of these oscillations. Therefore it was not possible to find a relationship between the behaviours of d_1 and d_2 during the jet structure formation process.

Now we shall examine the problem of using the model of flow formation from the source up to the flow under study. The wave diagram of such a model in *x-t* corrdinates is shown in Fig. 8. In this diagram 1 and 3 stand for the primary and secondary shock waves, 2 contact surface. The flow from cylindrical and spherical sources of interest to

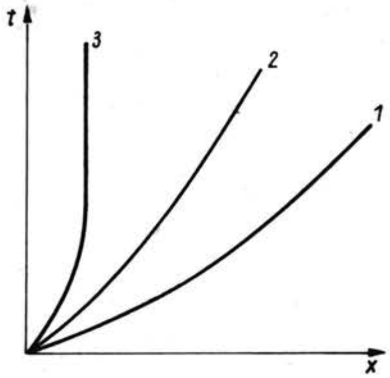

FIG. 8.

us is found to be qualitatively similar. For this reason we shall not draw a distinction between them unless specified. The primary shock wave *1* which appears with the outflow of gas reduces its speed with time and at infinity degenerates into a sonic wave. The contact surface *2* also moves with deceleration and as the time tends to infinity, the velocity of the contact surface decreases to zero and the pressure behind it asymptotically approaches the ambient pressure. The decrease in the motion of the contact surface is due to the fact that the mass of the gas enclosed between the wave *1* and the contact surface *2* increases with time, the area of the contact surface also increases with time but the inflow of impulse into the expanding system of the gas remains constant. The deceleration of the surface *2* even at the initial stage of expansion results in compression of the inflowing gas. At first compression takes place isentropically, but with the increase in time it occurs in the shock wave *3*, the intensity of which increases as the flow process it developed. The shock wave tends to occupy the stationary position and there are regimes, as pointed out by SIMONS [11], when it runs its stationary position and only after that is returns to it. At the initial stage of expansion the wave *3* moves along the nonstationary rarefaction wave and in the course of time cuts it (see, for example, the experiments in nozzles carried out by SMITH [12]). Further it travels along the stationary rarefaction wave. Under certain conditions the shock wave 3 starts moving almost immediately along the stationary rarefaction wave (hypersonic source, ambient pressure is not very small). Such is the qualitative picture of flow in general terms. This is in compliance with the papers [11, 13].

Within the framework of the ideal fluid theory SIMONS [11] found asymptotic laws of the behaviour of typical surfaces of the flow *1*, *2*, *3* (Fig. 8). The approximated laws for these surfaces at the initial stage of expansion close to the supersonic source were found by CHEKMAREV [13]. The proposed formulae give results with an accuracy of 10%. Chekmarev assumed that the wave *3* moves along the stationary rarefaction wave (this has also been mentioned earlier) and the thickness of regions r_1-r_2 and $r_2-r_3 \ll r_1$. Using certain analogies with the self-similarity problems related to the motion of the convergent shock wave and the flow caused by the expanding piston, he developed a mass and energy balance equation. From this equation appropriate relationships for typical surfaces were obtained. Apart from this, Chekmarev solved numerically the nonstationary problem related to the flow of stationary cylindrical and spherical sources with due regard to viscosity and heat conductivity at all stages of flow formation. The numerical calculations corroborated that qualitative picture (described above) of the process and revealed that the effect of Reynolds number on the flow is very small. It was also noted that the wave *3* close to its stationary position varies slightly (~3–4%). At the initial stage of the process compression occurred isentropically in the wave *3*. The approximate relationships which describe the position of typical surfaces in the field of flow, are in good agreement with the calculated data. On the basis of the numerical data Chekmarev examined some problems of modelling such flows. Now we shall see how the problems solved by Simons and Chekmarev and the jet formation problem will correlate with each other. It is known that the solution for the steady-state flow from the stationary source is applicable to the central part of the stationary jet. It would be a negligence to apply this method to the nonstationary jet because:

a. The stationary isentropic rarefaction wave, which is present throughout the devel-

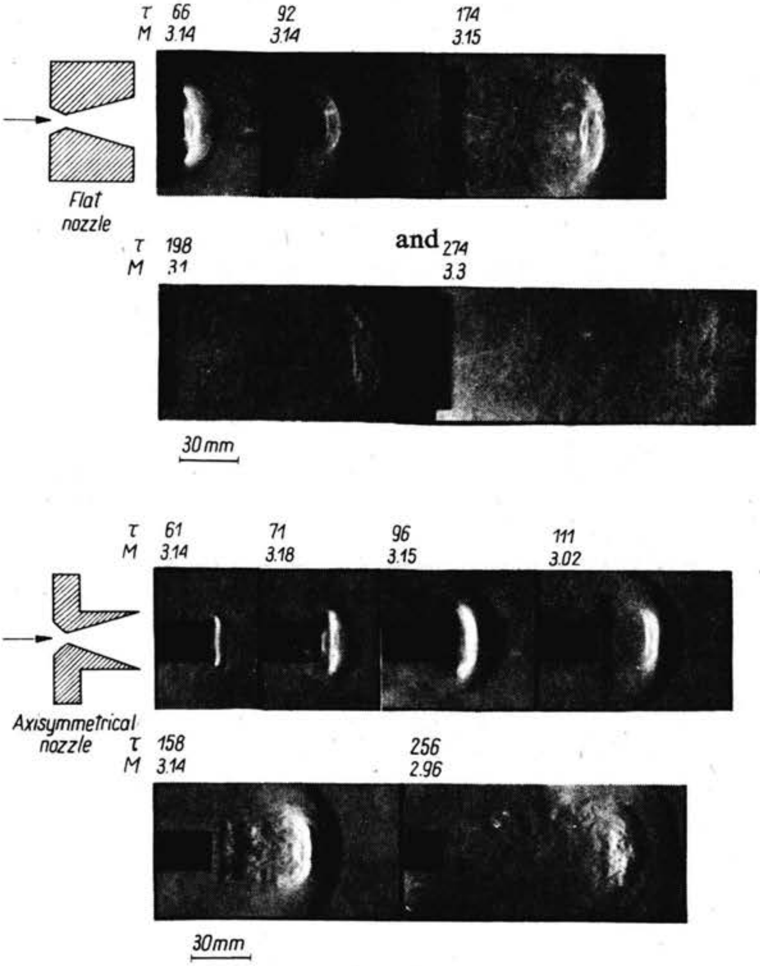

FIG. 9.

opment process in the flow from the source, cannot be matched with any area of the flow in the nonstationary jet;

b. In the model of nonstationary flow from the source, the secondary shock wave *3* after some time occupies the stationary position. But in the experiments carried out by us for $N = 20 \div 200$ the jet formation is accompanied by the formation of two secondary shock waves. One of them, i.e. the one located nearer to the contact surface (*3a*), is noticed in the experiments carried out with different parameters (Figs. 3, 4, 9). Apparently its existence is associated with the presence of a very vast zone of back-vortex motion close to the front of the flowing gas. In distinction to the wave *3b* the secondary wave *3a* present in the zone of backward motion of gas should be designated as "dynamic secondary wave". The existance of the other wave (*3b*) is determined by the value n and for $n < 1$ this wave should not exist. Figures 3 and 4 show how the secondary shock wave (*3b*) for $n > 1$ tends to occupy the quasi-stationary position of the Riemann wave and the Mach disc for the two-dimensional and axisymmetric flows, respectively.

Figure 9 shows schlieren photographs of jet structure formation when the jet outflows from a flat and axisymmetrical nozzles with the geometric number. $M_{\mathrm{outlet}} = 3.5$ ($\gamma = 1.4$) and $n = 0.4$. The figures on the photographs indicate the precession time in micro-seconds reckoned from the instant when the incident shock wave approaches the end of the tube, and the Mach number of this wave. The Schlieren photographs show the development of the dynamic secondary wave in the absence of the secondary wave *3b*. This development is related to the overexpansion of flow.

c. Numerically calculated positions of typical surfaces in the gas flowing from the source only qualitatively agree with the experimentally measured positions of corresponding surfaces in nonstationary jets. The disparity is 30% and more.

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