Arch. Mech., 34, 5-6, pp. 685-689, Warszawa 1982

# The length of the separation bubble at turbulent shock-boundary layer interaction at curved walls

J. ZIEREP (KARLSRUHE)

IN THE FOLLOWING the problem of turbulent shock-boundary layer interaction at curved walls is treated. For the incipient separation the results of an analytical theory [1] are used. The length of the separation is determined with the help of gasdynamic similarity considerations.

Przedyskutowano zagadnienie oddziaływania fali uderzeniowej z warstwą przyścienną w pobliżu zakrzywionej ścianki. W procesie powstającego rozdziału wykorzystano wyniki teorii analitycznej [1]. Długość pęcherzyka rozdziału określono za pomocą rozważań dotyczących podobieństwa gazo-dynamicznego.

Обсуждена проблема взаимодействия ударной волны с пограничным слоем вблизи искривленной стенки. В процессе возникающего раздела использованы результаты аналитической теории [1]. Длина пузырька раздела определена при помощи рассуждений, касающихся газодинамического подобия.

## 1. Introduction

IN THE CASE of shock-boundary layer interaction at curved walls, separation with following reattachment is often observed (Fig. 1). Of special interest for various applications is the length of the separation bubble and its dependency on the dimensionless parameters of the problem. In the following we use the separation criterion derived from an analytical treatment of the problem for turbulent boundary layers [1, 2]. Some conclusions for separation bubbles are deduced. Subsequently a simple description of the length of the separation bubble is given by using similarity laws.

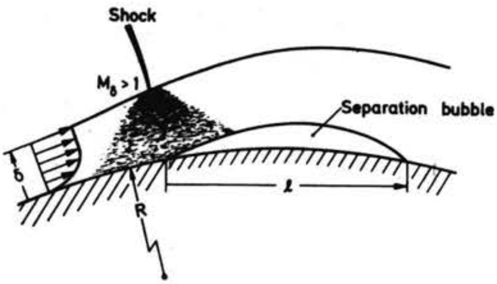

FIG. 1. Sketch of the shock-boundary layer interaction at a curved wall with separation.

## 2. Results of the analytical treatment of the shock-boundary layer interaction (separation criterion)

In the following the model used and the most important results are discussed. In Fig. 2 a sketch of the flow field and a survey of the assumptions is given. The turbulent boundary layer at a curved wall (Fig. 2, left) is disturbed by a weak normal shock (Fig. 2, middle). With a three-layer model I, II, III (Fig. 2, right) — as introduced by Lighthill — solutions in closed form are obtained for pressure and velocity in the boundary layer. Of special interest is the separation criterion (Fig. 3) that relates the most important dimensionless parameters of the problem:

$R/\delta$ — curvature parameter of the wall,
$\mathrm{Re}_\delta$ = Reynolds-number,
$\mathrm{M}_\delta$ = Mach-number.

For a given parameter of the wall curvature $R/\delta$, that combination of the Mach-number ($\mathrm{M}_\delta$) and Reynolds-number ($\mathrm{Re}_\delta$) can be found which leads to separation. Depend-

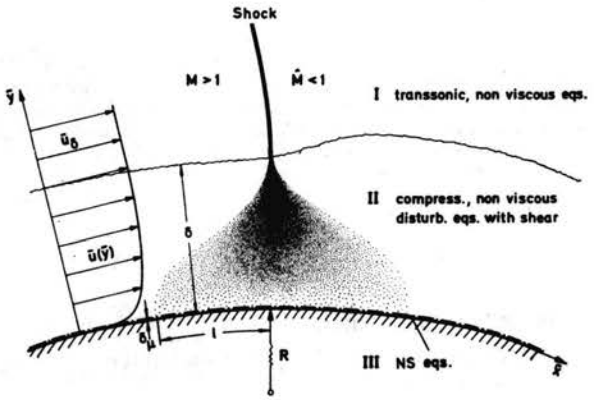

FIG. 2. Three layer model for calculation of shock-boundary layer interaction.

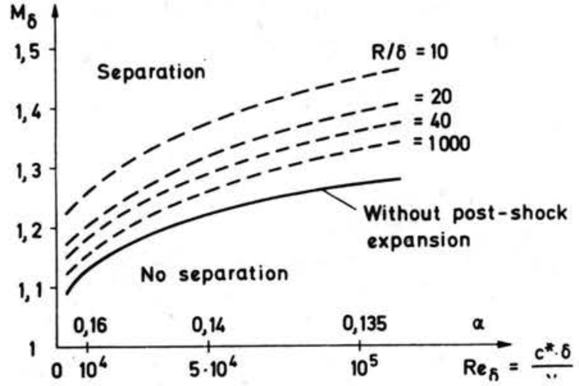

FIG. 3. Diagram for incipient separation. Mach-number as function of Reynolds-number that leads to separation. Influence of wall curvature. α is the form-parameter of the undisturbed boundary layer profile.

ing upon whether post-shock expansion is considered (- - -) or not (———), different curves are obtained. Beneath the solid curve there is no separation at all. Above the broken lines, corresponding to the different wall curvatures, the flow must separate. Of interest is the discussion of the variation of only one parameter. Increasing wall curvature ($M_\delta$ and $Re_\delta$ constant) diminishes the tendency towards separation. This is due to the increase of the post-shock expansion in this case. By increasing $M_\delta$ ($R/\delta$ and $Re_\delta$ constant), on the contrary, the tendency to separation rises. This is evident because the pressure rise in the shock increases. Lifting of $Re_\delta$ ($M_\delta$ and $R/\delta$ constant) diminishes the tendency towards separation. In this case the undisturbed velocity profile in the boundary layer becomes more rectangular and therefore allows a larger pressure rise.

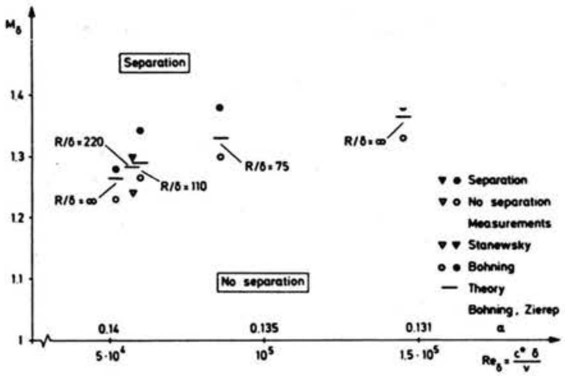

FIG. 4. Comparison between theory and experiment for incipient separation.

Figure 4 gives a comparison between theory and experiment for the incipient separation [3]. The filled symbols (▲, ●) belong to those measurements, where separation was definitely observed. The open symbols (△, ○), however, mark those measurements, where at the same Reynolds-number and wall curvature no separation was visible. The lines (———) give the result of the theory for incipient separation at the same parameters $Re_\delta$ and $R/\delta$. For all cases the result of the theory lies between the two limits given by the experiment. More cannot be expected! It should be realized, on the one hand, how difficult it is to ascertain separation in the experiment. On the other hand the many assumptions introduced for the theoretical consideration may not be forgotten.

## 3. Consequences for flows with separation bubbles

The shock-boundary layer interaction with separation bubble is sketched in Fig. 5 (top). The radius of curvature of the wall is $R_W$ and the radius of the separation bubble is $R_B$. From the separation diagram Fig. 3, given in Fig. 5 (bottom) with the corresponding notations, some conclusions can be drawn. To do this imagine the bubble as part of the wall. For sure it is $R_B < R_W$, and the curve belonging to $R_B/\delta$ lies above that for $R_W/\delta$. The corresponding starting values of the shock-boundary layer problem ($M_{\delta_1}$, $Re_{\delta_1}$) are between the two curves given by the parameters $R_W/\delta$ and $R_B/\delta$. This is very easy to understand. Since the fluid separates, the starting point $(\mathrm{M}_{\delta_1}, \mathrm{Re}_{\delta_1})$ must lie above the curve given by $R_W/\delta$ and while the fluid reattaches downstream, the starting point must lie below the curve corresponding to $R_B/\delta$. This consideration contributes to the understanding of the separating flow, but it says nothing about the magnitude of $R_B$. On the contrary, $R_B$ is assumed.

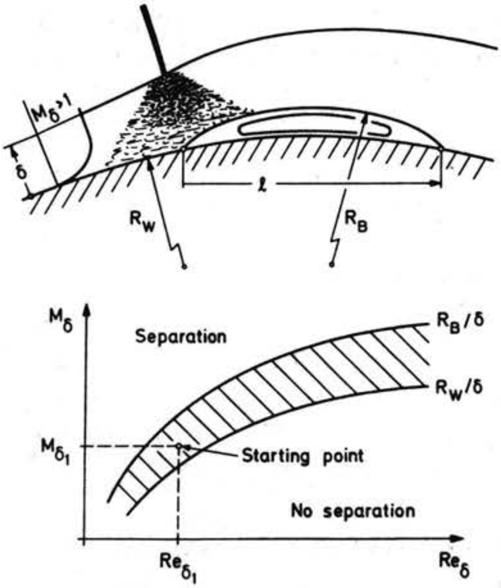

FIG. 5. Shock-boundary layer interaction with separation bubble. Discussion with help of the separation diagram.

In order to get some information about the extension of the region of separation, e.g. the *length l of the separation bubble*, similarity considerations are applied. Evidently there are four parameters involved in this problem, Fig. 5 (top):

$$\frac{\delta}{l}, \frac{R_W}{\delta}, \mathrm{Re}_\delta, \mathrm{M}_\delta.$$

Physical considerations lead to a dependency of the form

$$\frac{l}{\delta} = f(M_\delta, \mathrm{Re}_\delta, R_W/\delta). \tag{3.1}$$

The problem is to determine the function *f*. Here it is important to note that separation with reattachment is by no means a local problem. The bubble influences globally the whole flow field, especially the outer inviscid flow. For global similarity considerations, the *streamline analogy* is applied. The elements of gasdynamics lead with the *thickness parameter* $\delta/l$ and the Mach-number $\mathrm{M}_\delta$ to the relation

$$\frac{\delta}{l}\sqrt{M_\delta^2-1} = \mathrm{const}. \tag{3.2}$$

In this, $l$ is the length of the separation bubble and therefore it is *the* characteristic lengthscale in flow direction.

If we further introduce a formal lengthscale $l_x$ (e.g. 1 cm) we get from Eq. (3.2)

$$\frac{l}{l_x} \sim \frac{\delta}{l_x}\sqrt{M_\delta^2-1}. \tag{3.3}$$

Equation (3.3) expresses the length of the bubble by the boundary layer thickness $\delta/l_x$ and the Mach-number $\mathrm{M}_\delta$. The Reynolds-number determines the boundary layer thickness; the wall curvature is introduced by the form parameter of the velocity profile. If we assume the 1/7-power-law, we get

$$\frac{\delta}{l_x} \sim \frac{1}{\mathrm{Re}_\delta^{1/4}}. \tag{3.4}$$

Equation (3.3) leads to the simple expression

$$\frac{l}{l_x} \sim \frac{\sqrt{\mathrm{M}_\delta^2-1}}{\mathrm{Re}_\delta^{1/4}}. \tag{3.5}$$

One realizes immediately that the Mach-number and Reynolds-number have a different influence on the length of the bubble. Increasing $\mathrm{Re}_\delta$ diminishes $l$, increasing $\mathrm{M}_\delta$ enlarges $l$. This coincides completely with the conclusions drawn from the separation criterion in Chapter 2. KOOI [5] has collected all relevant measurements. They all confirm the strong Reynolds-number dependence of $l$. The length of the separation bubble seems to be very sensitive to variations of parameters. More carefully done measurements are required to check the above given formulas. It is doubtless that the influence of the channel walls will be of considerable importance in this case.

## References

1. R. BOHNING, J. ZIEREP, *Bedingung für das Einsetzen der Ablösung der turbulenten Grenzchicht an der gekrümmten Wand mit senkrechtem Verdichtungsstoss*, ZAMP, **29**, 190–198, 1978.
2. R. BOHNING, J. ZIEREP, *Separation of turbulent boundary layer at a curved wall with normal shock*, Arch. Mech., **30**, 353–358, 1978.
3. R. BOHNING, J. ZIEREP, *Normal shock turbulent boundary layer interaction at a curved wall*, AGARD Conf. Proc., No. 291, 1981.
4. E. STANEWSKI, *Transsonische Profile im Experiment*, DFVLR-Nachrichten, 30, 1980.
5. J. W. KOOI, *Influence of free-stream Mach-number on transonic shock-wave boundary-layer interaction*, NLR MP 78013 U, 1978.

INSTITUT FÜR STRÖMUNGSLEHRE UND STRÖMUNGSMASCHINEN
UNIVERSITÄT KARLSRUHE, FRG.

*Received October* 22, 1981.