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Boundary layer — free stream interactions in horizontal and sloping open-channels

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THIS PAPER treats the open-channel flow problem as a boundary layer-free stream interaction problem, in analogy to viscous-inviscid interactions in gas dynamics. Convergent numerical results are obtained by an iterative solution procedure which alternately solves the boundary layer problem and the free stream equation. In a horizontal channel, the boundary layer is found to accelerate the external flow in the subcritical case. In the supercritical case the external flow is, on the other hand, retarded. A particularly strong interaction is observed for initial Froude numbers close to unity. In a downward sloping channel the interaction effects are found to oppose the effect of the sloping channel bed.

Przepływy w kanałach otwartych potraktowano jako problemy oddziaływania warstwy przyściennej ze strugą swobodną, analogicznie do oddziaływania płynów lepkich i nielepkich w gazodynamice. Otrzymano zbieżne rezultaty za pomocą procedury iteracyjnej rozwiązującej naprzemian zagadnienie warstwy przyściennej i równanie strumienia swobodnego. W kanale poziomym stwierdzono, że warstwa przyścienna przyspiesza przepływ zewnętrzny w przypadku podkrytycznym. W przypadku nadkrytycznym przepływ jest spowolniony. Szczególnie silne oddziaływanie występuje dla wstępnych wartości liczb Froude'a bliskich jedności. W kanałach pochylonych stwierdzono, że efekty oddziaływania przeciwstawiają się efektom nachylenia dna kanału.

Течения в открытых каналах трактуются как проблемы взаимодействия пограничного слоя со свободным потоком, аналогично взаимодействию вязких и невязких жидкостей в газодинамике. Получены сходные результаты при помощи итерационной процедуры, решающей попеременно задачу пограничного слоя и уравнение свободного потока. В горизонтальном канале констатировано, что пограничный слой ускоряет внешнее течение в докритическом случае. В сверхкритическом случае течение замедляется. Особенно сильное взаимодействие выступает для вступительных значений чисел Фруда близких единицы. В наклоненных каналах констатировано, что эффекты взаимодействия противопоставляются эффектам наклона дна канала.

1. Introduction

VISCOUS-INVISCID interactions are well-known phenomena in supersonic gasdynamics and have been extensively studied in the past, e.g. [1–3]. It is well known that the external flow cannot be predicted *a priori* as a known datum for calculation of the boundary layer problem. The boundary layer flow and the outer inviscid freestream must be treated simultaneously. CEBECI and BRADSHAW [4] proposed an iterative procedure for viscous-inviscid interactions in gasdynamics, and INOUE [3] has applied a similar scheme to incompressible boundary layer flows with separation and reattachment.

Recently ANDERSSON and YTREHUS [5] suggested that boundary layer-free stream interactions in open-channel flow, which is the hydraulic analogy to viscous-inviscid interactions in gasdynamics, could also be treated by this iterative procedure. Convergent numerical results were obtained for horizontal open-channel flow, and the results were compared favourably with the approximate calculations of subcritical flow by BINNIE [6].

In the present paper we first consider the effect of boundary layer-free stream interactions in a horizontal channel for initial Froude numbers in the range 0.2–10.0. Next, the iterative scheme is applied to subcritical and supercritical flows in a sloping open-channel.

2. Physical model

We consider the hydraulic flow entering a sloping open channel with velocity U_0 over the initial depth h_0, as indicated in Fig. 1. It is assumed that only negligible tangential forces are transmitted across the constant-pressure free-surface boundary between the flow and the ambient atmosphere. The channel is sufficiently wide for the motion to be considered two-dimensional. The fluid is incompressible and has constant kinematic viscosity ν. Only the steady case of laminar flow is considered.

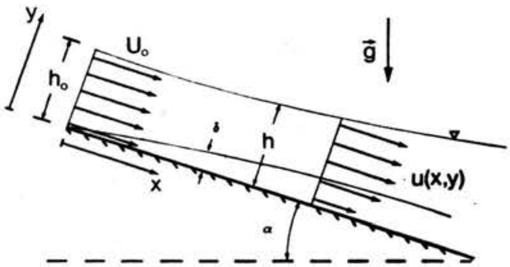

FIG. 1. Sketch of open-channel flow.

The classical boundary layer concept due to PRANDTL [7] is assumed; i.e. the flow is inviscid except in a thin layer adjacent to the channel bed through which it adapts to the viscous no-slip boundary condition. External to the viscous boundary layer, the flow is assumed quasi-one-dimensional.

3. Boundary layer problem

The boundary layer equations for continuity and momentum of two-dimensional steady, incompressible laminar flow can be written as [8]

$$\begin{gathered}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2},\\ u(x,0)=v(x,0)=0,\\ u(x,\delta)=U(x),\end{gathered}\tag{3.1}$$

where $U(x)$ is the velocity in the nonviscous part of the flow as obtained from the Bernoulli's theorem along the free surface, and $\delta(x)$ is the boundary layer thickness.

4. Free stream equation

Since at the free surface the pressure is constant and equal to the atmospheric pressure, the Bernoulli theorem reads

$$U_0^2/2g+h_0\cos\alpha = U^2(x)/2g+h(x)\cos\alpha-x\cdot\sin\alpha \tag{4.1}$$

which demonstrates the conservation of the total energy head. Equation (4.1) can be combined with the continuity equation

$$U_0 h_0 = U(x)[h(x)-\delta^*(x)]$$

to give

$$\begin{aligned} 1+2F_0^{-2} &= \tilde{U}^2(x)+hF_0^{-2}[\tilde{\delta}^*(x)-\tilde{x}\,\mathrm{tg}\,\alpha]+2F_0^{-2}\tilde{U}^{-1}(x),\\ \tilde{h}(x)-\tilde{\delta}^*(x) &= \tilde{U}(x)^{-1}, \end{aligned} \tag{4.2}$$

where $\delta^*(x)$ is the boundary layer displacement thickness

$$\delta^*(x) \equiv \int\limits_0^{\delta(x)} \left(1-\frac{u(x,y)}{U(x)}\right)dy$$

and

$$\begin{aligned} \tilde{h}(x) &= h(x)/h_0, \quad \tilde{\delta}^*(x) = \delta^*(x)/h_0,\\ \tilde{U}(x) &= U(x)/U_0, \quad \tilde{x} = x/h_0,\\ F_0 &= U_0/\sqrt{gh_0\cos\alpha} \end{aligned} \tag{4.3}$$

are dimensionless quantities, F_0 being the initial Froude number.

5. Solution of the interaction problem

The coupling between the boundary layer characteristic δ^* and the free stream velocity $U(x)$ which is expressed through Eq. $(4.2)_1$ above, clearly demonstrates that the resulting flow problem must be treated as an interaction problem between viscous and inviscid domains in the flow field. The iterative scheme proposed by CEBECI and BRADSHAW [4], chapt. 11.2, is:

1. calculate the inviscid flow, neglecting or crudely approximating the displacement effect from the boundary layer,

2. use the external velocity distribution to solve the boundary layer problem, Eqs. (3.1),

3. add δ^*, obtained from step 2, to the bottom shape to form a new displaced surface and recalculate the inviscid flow problem, Eqs. (4.2),

4. repeat steps 2 and 3 until the results converge.

A standard FORTRAN routine for algebraic equations identified the roots of the free-stream equation (step 1 and 3). The boundary layer problem (step 2) was solved numerically by a finite-difference method using the Keller Box scheme; KELLER [9]. A detailed account of the numerical method is given by CEBECI and BRADSHAW [4], chapts. 7–8.

The numerical calculations were carried out using 75×26 grid points, with nonuniform mesh spacing in the y-direction. The iterating process was stopped when the maximum relative changes in $\tilde{h}$, $\tilde{U}$ and $\tilde{\delta}^*$ became less than 10^{-3}, and the desired accuracy was reached after 2–10 global iterations, depending on the initial Froude number. The computer time required for one global iteration (step 2 and 3) on UNIVAC 1100/60 was about 10 s cpu.

6. Numerical results and discussion

6.1. Horizontal channels

In the horizontal case, the initial Reynolds number $\mathrm{Re}_0 = U_0 h_0/\nu$ is conveniently absorbed in the x-coordinate by the transformation

$$\tilde{\tilde{x}} = \tilde{x}/\mathrm{Re}_0 = \frac{x}{h_0}\frac{1}{\mathrm{Re}_0}$$

and thus no explicit Re_0-dependence occurs in the results displayed in Figs. 2–5.

Figure 2 shows the calculated variation of the dimensionless freestream velocity $\tilde{U}(x)$ for initial Froude numbers in the range 0.1–10. The corresponding variation of the dimensionless flow depth $\tilde{h}(x)$ is shown in Fig. 3. In the case of subcritical inflow conditions, the freestream is accelerated and the flow depth reduced in the streamwise direction. In the supercritical case, however, the inviscid part of the flow is retarded and the free surface is elevated in the downstream direction. The deviation from classical flat plate results, $\tilde{U}(x) = 1$ and $\tilde{h}(x) = 1$, increases as F_0 approaches 1. The enhancement of the interaction effects close to critical inflow conditions is analogous to the significant viscous-inviscid interaction effects in transonic gas dynamics.

Figure 4 shows the variation of the derivatives $d\tilde{U}/d\tilde{\tilde{x}}$ and $d\tilde{h}/d\tilde{\tilde{x}}$ with F_0. It is observed that the results for $F_0 = 10$ are close to the analytically derived asymptotic limits

$$d\tilde{U}/d\tilde{\tilde{x}} \to 0, \quad d\tilde{h}/d\tilde{\tilde{x}} \to 1/2\cdot 1.7208\cdot \tilde{\tilde{x}}^{-1/2}$$

as F_0 tends to infinity.

When Eq. $(3.1)_2$ was derived from the Navier-Stokes equations, the diffusion term $\nu\partial^2 u/\partial x^2$ was neglected under the assumption that this term is small compared with the remaining terms. In order to examine the validity of this assumption in the present case, the ratio of the neglected term $\nu\partial^2 u/\partial x^2$ to the driving term $U\cdot dU/dx$ is plotted in Fig. 5 as a function of y. We observe that the magnitude of this ratio is less than 0.03 throughout the boundary layer for $\mathrm{Re}_0 = 1000$. Since this ratio is proportional to Re_0^{-2}, the assumption that the term $\nu\partial^2 u/\partial x^2$ is negligible is justified for $\mathrm{Re}_0 \gtrsim 10^3$.

6.2. Sloping channels

Calculations were also carried out for subcritical and supercritical flows in a sloping channel, with the angle of inclination $\alpha = 1.0°$. The numerical results for flows with an initial Reynolds number $\mathrm{Re}_0 = 10^3$ are shown in Figs. 6 and 7. Results without interaction, i.e. $\delta^* = 0$ in Eqs. (4.2), are also shown for comparison.

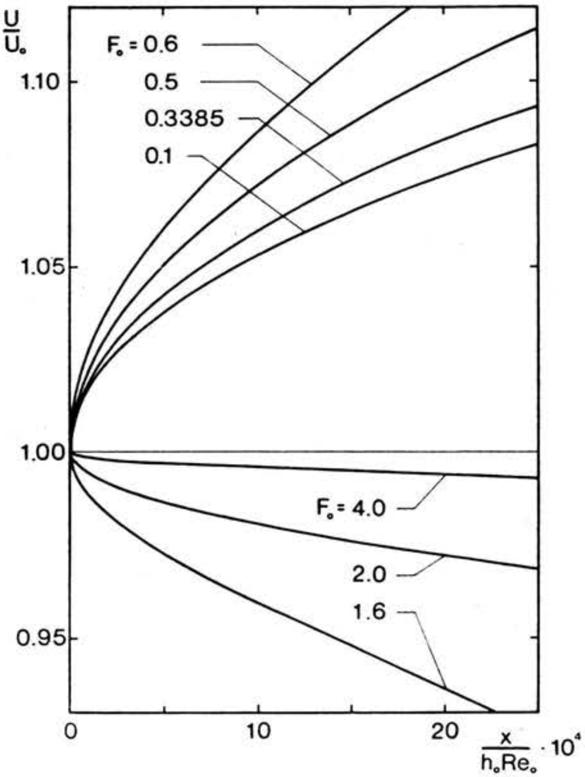

FIG. 2. Free stream velocity for different values of F_0. Horizontal channel.

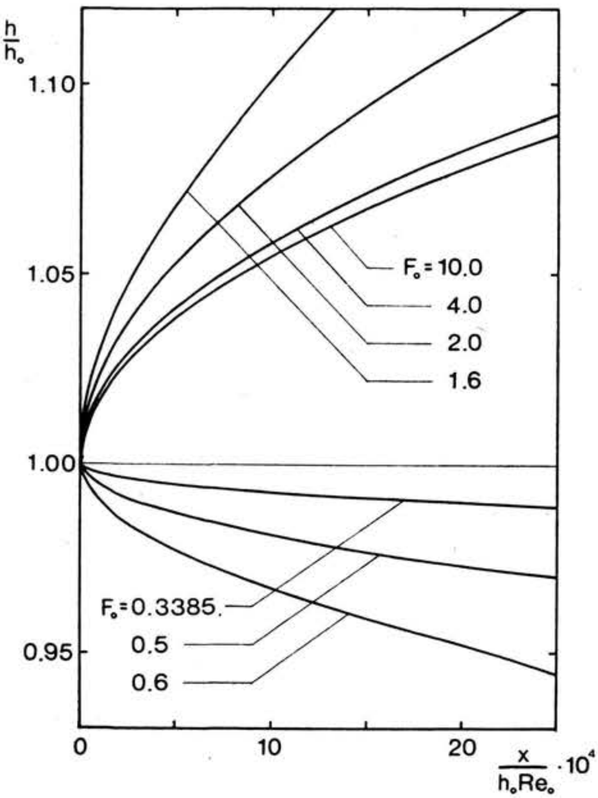

FIG. 3. Flow depth for different values of F_0. Horizontal channel.

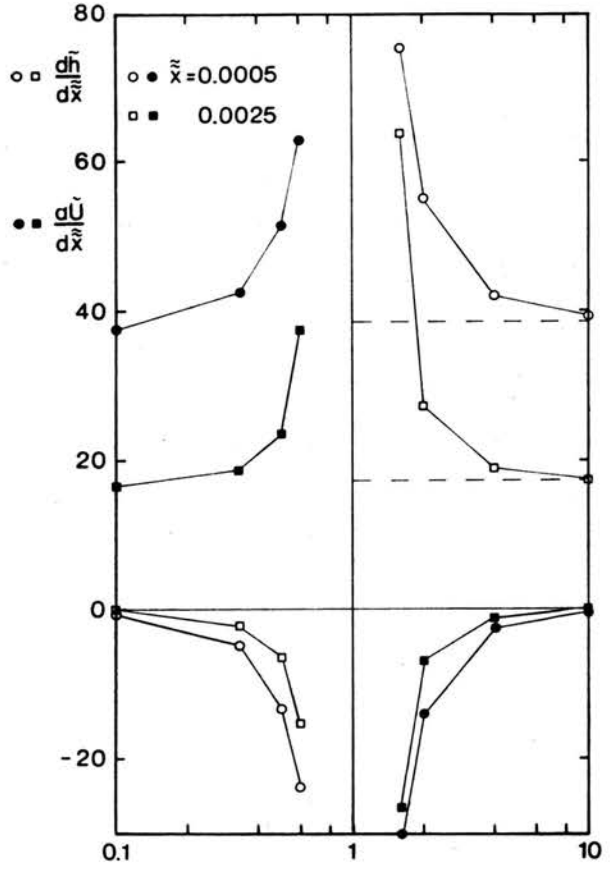

FIG. 4. Variation of dh/dx and dU/dx with F_0. Hotrizontal channel.

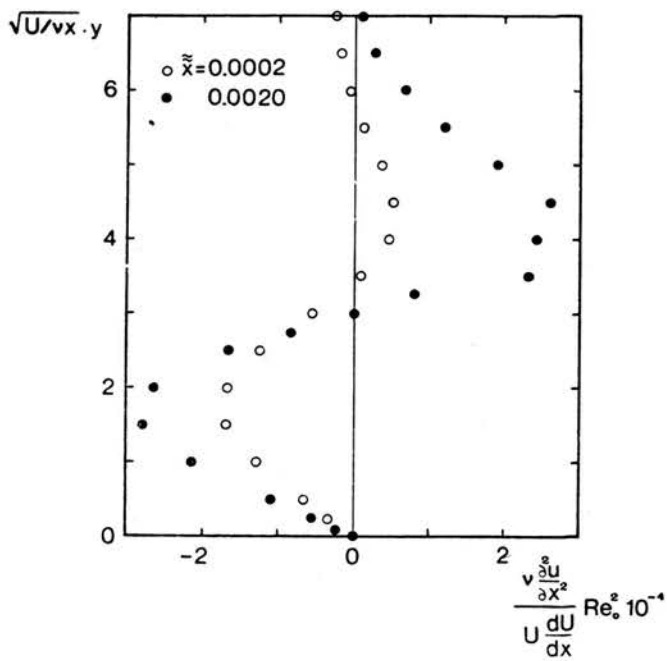

FIG. 5. Ratio between neglected diffusion term $\nu\partial^2 u/\partial x^2$ and driving term $U \cdot dU/dx$ in a horizontal channel.

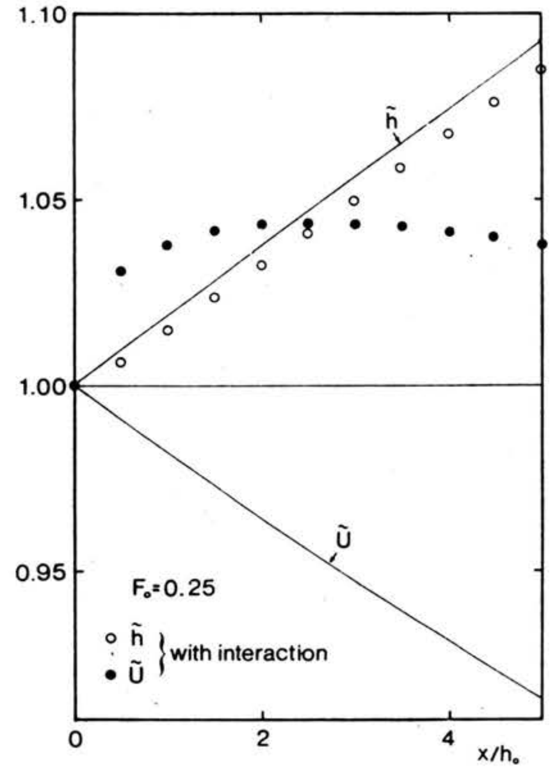

FIG. 6. Subcritical flow in a sloping channel. The lines denote results without interaction. $\alpha = 1.0°$ and $Re_0 = 10^3$.

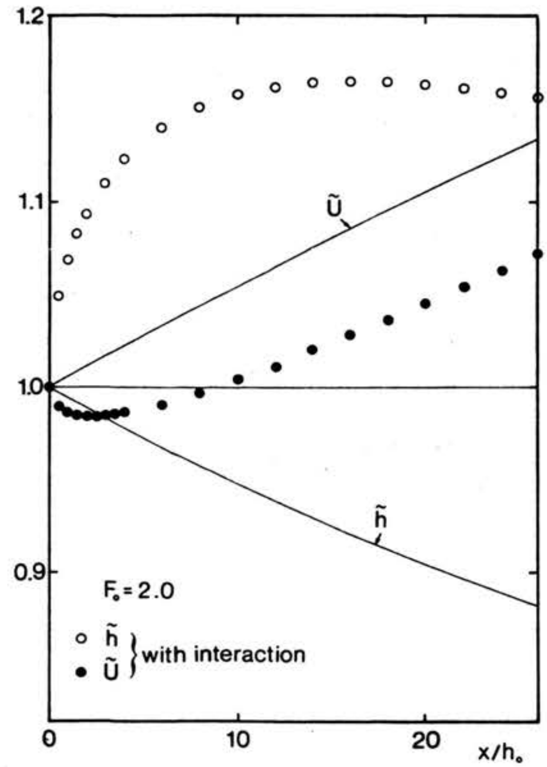

FIG. 7. Supercritical flow in a sloping chanpel. The lines denote results without interaction. $\alpha = 1.0°$ and $Re_0 = 10^3$.

The solid lines in Fig. 6 show that in the case of subcritical inflow ($F_0 = 0.25$) the free stream is retarded and the flow depth increased in the streamwise direction when boundary layer-free stream interactions are neglected. As the interactions are taken into consideration, however, the increase in the flow depth is slightly reduced, and the free stream is accelerated in the region $x/h_0 \lesssim 2.5$.

Figure 7 shows that the flow depth decreases and the surface velocity increases downstream in the supercritical case if the displacement effect is neglected. The circles show that h is increased and U is reduced owing to the interaction effect.

7. Conclusions

The present formulation of free-surface channel flow leads to qualitative different flow behaviour in the subcritical and supercritical flow regime. The initial Froude number $U_0/(gh_0 \cos\alpha)^{1/2}$, the initial Reynolds number $U_0 h_0/\nu$ and the angle of inclination α are the fundamental parameters of the problem. Particulary strong interaction effects are observed for initial Froude numbers close to unity. In the case of a downward sloping channel the interaction effects are found to oppose the effect of the sloping channel bed.

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