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Isothermal and adiabatic flow laws of metallic elastic-plastic solids at finite strains and propagation of acceleration waves

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FROM THE EARLIER proposed constitutive equations for metallic elasto-plastic materials at finite strains and with induced anisotropy [18], the flow laws for isothermal and adiabatic processes are derived and examined. If certain minor coupling effects are disregarded, the difference between them consists only in the hardening functions. It is shown that deviation from plastic normality may arise from the plastic spin. On the basis of these equation, we consider the propagation of acceleration waves and give the expressions for the eigenvalues. They reduce in the case of small deformations to those of [10].

1. Introduction

IN [18] THE CONSTITUTIVE RELATIONS for metallic elastic-plastic solids at finite strains with induced anisotropy were derived under certain well-grounded assumptions. These equations are similar in form to those employed in problems of small strains but are applicable to the whole range of the deformation process. The objective of this paper is twofold.

First, it is to study the flow laws for two extreme idealizations of the real deformation process: the isothermal and the adiabatic ones. In the isothermal process, we assume that the heat transport is so intensive that the process does not affect the temperature of the particles and in the adiabatic process, the heat transport between the particles may be neglected. The flow laws for isotropic bodies were considered in [14, 19]. These processes are widely used in experiments, for example to determine the dynamic properties of materials. It is shown that when some small coupled effects are neglected, the only difference between the isothermal and adiabatic flow laws is due to the difference of the hardening functions. Another aspect is the deviation from plastic-normality arising from the plastic spin. It is widely known that non-normality is an important destabilizing feature for the localization process [15].

Secondly, motion of a second-order discontinuity surface in an infinite space is studied. Propagation of such acceleration waves in solids is intensively investigated [10] because the fundamental nature of them relates directly to the important issue of stability, static bifurcations and so forth. Moreover, recent developments in finite element method to analyze the localization of strain in thin zones within a solid have considerably intensified this interest. In the pioneering work of HADAMARD [2] elastic waves were studied. THOMAS [16, 17] investigated isothermal elastic waves. The main contributions to waves in elastic-plastic materials belong to MANDEL [4] and HILL [3]. Assuming certain particular forms of the constitutive equations, RANIECKI [13] and NOWACKI [8] investigated the problem of propagation of acceleration waves in metals and soils. The following papers are dealing with wave propagation at finite strains [1, 11, 12, 20]. For non-associated plasticity we can cite the work of MANDEL [6] and OTTOSEN and RUNESSON [10]. In this paper, neglecting the small terms due to the distortional elastic strains, the eigenvalues of the acoustic tensor are determined. For small deformations, these results reduce to those of [10]. Here, with the presence of plastic spin, 'flutter' instability may occur although for a very broad class of nonassociated plasticity it is shown in [10] that it can not take place.

Tensors will be written in boldface letters; summation over repeated indices is implied and the following symbolic operations are used: $\mathbf{AB} \to A_{ij}B_j$, $\mathbf{A}\cdot\mathbf{B} \to A_{ij}B_{ij}$, $\mathbf{A}\otimes\mathbf{B} \to A_{ij}B_{kl}$ with proper extension to tensor of different orders. The prefix 'tr' indicates the trace and a superposed dot - the material time derivative or rate. By $\overline{\mathbf{A}}$ we denote the deviatoric part of $\mathbf{A}$ and by $\mathbf{1}$ the identity tensor.

2. Basic equations

In the case of metallic elastic-plastic materials, the following equation in Eulerian description were derived [18]:

$$\check{\mathbf{T}} \equiv \dot{\mathbf{T}} - \boldsymbol{\omega}^e\mathbf{T} + \mathbf{T}\boldsymbol{\omega}^e = \beta(\mathbf{L}\mathbf{D}^e - \mathbf{B}\dot{\vartheta}), \tag{2.1}$$

where $\mathbf{T} = \beta\boldsymbol{\sigma}$, $\boldsymbol{\sigma}$ — Cauchy stress tensor, $\beta = \rho_0/\rho$, ρ_0 and ρ are the respective densities in the reference configuration and the actual configuration, according to the theory of MANDEL [5]. $\mathbf{L}$ and $\mathbf{B}$ are the generalized tensor of isothermal elastic moduli and elastic thermal stress, respectively,

$$\begin{aligned} L_{ijkl} &= \delta_{ij}\delta_{kl}(K_T - p) + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right), \\ B_{ij} &= \alpha_v K_T \delta_{ij}; \end{aligned} \tag{2.2}$$

here K_T — isothermal bulk modulus, $-p$ — mean pressure, α_v — volumetric thermal expansion, c_v — specific heat at constant volume, μ — Lamé constant. $\mathbf{D}^e$ and $\boldsymbol{\omega}^e$ denote, respectively, the elastic rate of deformation and spin tensor. ϑ is the temperature in actual configuration.

The flow laws for the plastic part with Huber–Mises yield criterion and combined isotropic-kinematic hardening are the following:

$$\begin{aligned} f &= \frac{3}{2}(\overline{\mathbf{T}} - \boldsymbol{\Pi})\cdot(\overline{\mathbf{T}} - \boldsymbol{\Pi}) - \sigma_Y^2(\vartheta, \Pi(\alpha)) = 0, \\ \mathbf{D}^p &= 3j\Lambda(\overline{\mathbf{T}} - \boldsymbol{\Pi}), \\ j &= \begin{cases} 1 & \text{if } f = 0 \quad \text{and} \quad \Lambda \geq 0, \\ 0 & \text{if } \quad f = 0 \quad \text{and} \quad \Lambda < 0 \quad \text{or} \quad f < 0, \end{cases} \\ \Lambda &= \frac{3(\overline{\mathbf{T}} - \boldsymbol{\Pi})\cdot\left(\check{\overline{\mathbf{T}}} + \dfrac{c_T}{c}\dot{\vartheta}\boldsymbol{\Pi}\right) - 2\sigma_Y\dfrac{\partial\sigma_Y}{\partial\vartheta}\dot{\vartheta}}{2\sigma_Y^2 H}; \end{aligned} \tag{2.3}$$

here $\mathbf{D}^p$ is the plastic rate of deformation; the combined plastic hardening modulus H is composed of the kinematic and isotropic hardening parts,

$$H = 3c + 2\left(\frac{\partial\sigma_Y}{\partial\Pi}\right)^2\frac{d\Pi}{d\alpha}, \tag{2.4}$$

α is a scalar variable modifying the 'size' of the yield stress and Π is the scalar internal thermodynamic force conjugate with α [14]. The evolution for α takes the form

(2.5)
$$\dot{\alpha} = \frac{1}{\sigma_Y}\frac{\partial \sigma_Y}{\partial \Pi}\mathbf{D}^p \cdot (\overline{\mathbf{T}} - \mathbf{\Pi}).$$

The function $\Pi(\alpha)$ is determined from the stored energy $\rho_0 u^*$ [14]. The latter represents the difference between the plastic work done and the heat release in the course of the isothermal cyclic process of straining. It is a measurable quantity in macroscopic experiments (cf. [9]).

The shift of the yield surface is represented by the back stress $\mathbf{\Pi}$, for which the evolution law has the form of linear kinematic modified by thermal effects

(2.6)
$$\breve{\mathbf{\Pi}} = c\mathbf{D}^p - \frac{c_T}{c}\dot{\vartheta}\mathbf{\Pi} \quad \text{with} \quad c = c_0 - c_T(\vartheta - \vartheta_0), \qquad c_0, c_T, \vartheta_0 = \text{const}.$$

The change in the temperature is described as

(2.7)
$$\rho_0 c_v \dot{\vartheta} = \left(1 - \frac{\Pi}{\sigma_Y}\frac{\partial \sigma_Y}{\partial \Pi}\right)(\overline{\mathbf{T}} - \mathbf{\Pi}) \cdot \mathbf{D}^p - \beta \operatorname{div} \mathbf{q} - \vartheta\beta\alpha_v K_T \operatorname{tr} \mathbf{D} - \vartheta\frac{c_T}{c}\mathbf{\Pi} \cdot \mathbf{D}^p.$$

Here $\mathbf{q}$ is the heat flux. This equation may be rewritten in the form

(2.8)
$$\rho_0 c_v \dot{\vartheta} = \overline{\mathbf{T}} \cdot \mathbf{D}^p - \rho_0 \dot{u}^* - \beta \operatorname{div} \mathbf{q} - \vartheta\beta\alpha_v K_T \operatorname{tr} \mathbf{D}.$$

In the equations of this section, the small terms involving $\overline{\mathbf{e}}$, the deviatoric part of the logarithmic left elastic stretch tensor [14] are neglected. The relation between $\overline{\mathbf{e}}$ and the Cauchy stress $\boldsymbol{\sigma}$ is $\overline{\boldsymbol{\sigma}} = 2\mu\overline{\mathbf{e}}$. The reader is referred to the paper [18] for more details.

3. Isothermal and adiabatic flow laws

We are in a position to derive the equations describing the isothermal and adiabatic processes. Combining Eqs. (2.1), (2.3) we obtain

(3.1)
$$\breve{\overline{\mathbf{T}}} = 2\mu\beta[\overline{\mathbf{D}} - 3j\Lambda(\overline{\mathbf{T}} - \mathbf{\Pi})].$$

Having calculated the term $\breve{\overline{\mathbf{T}}} \cdot (\overline{\mathbf{T}} - \mathbf{\Pi})$ from (3.1) the plastic rate of deformation can be rewritten in the form

(3.2)
$$\overline{\mathbf{D}}^p = \frac{3j(\overline{\mathbf{T}} - \mathbf{\Pi})}{2\sigma_Y^2 \mathcal{H}_i}[\mathbf{D} \cdot (\overline{\mathbf{T}} - \mathbf{\Pi}) - m_\vartheta \dot{\vartheta}] = \frac{3j(\overline{\mathbf{T}} - \mathbf{\Pi})}{2\sigma_Y^2 \mathcal{H}_i}\Psi,$$
$$j = \begin{cases} 1 & \text{if} \quad f = 0 \quad \text{and} \quad \Psi \geq 0, \\ 0 & \text{if} \quad f = 0 \quad \text{and} \quad \Psi < 0 \quad \text{or} \quad f < 0; \end{cases}$$

here
$$\mathcal{H}_i = 1 + \frac{H}{6\mu\beta}$$

isothermal hardening function,

(3.3)
$$m_\vartheta = \frac{1}{6\mu\beta}\left[2\sigma_Y\frac{\partial \sigma_Y}{\partial \vartheta} - 3\frac{c_T}{c}(\overline{\mathbf{T}} - \mathbf{\Pi}) \cdot \mathbf{\Pi}\right]$$

thermal coefficient of softening, measured in $\left[\frac{\mathrm{N}}{\mathrm{m}^2\,{}^\circ\mathrm{K}}\right]$.

From Eq. (2.7), it is easily obtained that

$$\dot{\vartheta} = j\frac{q_d + q_r}{\mathcal{H}_i}\Psi + \mathcal{Q} - h_e \operatorname{tr}\mathbf{D}, \tag{3.4}$$

where

$$q_d = \frac{1 - \dfrac{\Pi}{\sigma_Y}\dfrac{\partial\sigma_Y}{\partial\Pi}}{\rho_0 c_v}$$

thermal coefficient of heat produced by the dissipation of mechanical work, in $\left[\dfrac{{}^\circ\mathrm{Km}^2}{\mathrm{N}}\right]$,

$$q_r = -\frac{3c_T(\overline{\mathbf{T}} - \mathbf{\Pi})\cdot\mathbf{\Pi}}{2c\rho_0 c_v \sigma_Y^2} \tag{3.5}$$

thermal coefficient of heat of internal rearrangement, in $\left[\dfrac{{}^\circ\mathrm{Km}^2}{\mathrm{N}}\right]$,

$$h_e = \frac{\vartheta\beta\alpha_v K_T}{\rho_0 c_v}$$

measure of heat of elastic deformation, in [°K],

$$\mathcal{Q} = -\frac{\beta}{\rho_0 c_v}\operatorname{div}\mathbf{q} \tag{3.6}$$

measure of heat exchange with the surroundings, in $\left[\dfrac{{}^\circ\mathrm{K}}{\mathrm{s}}\right]$.

Using the definitions (3.5), (3.6) we can write the equations (3.2), (3.4) in the forms:

$$\begin{gathered}\overline{\mathbf{D}}^p = \frac{3j(\overline{\mathbf{T}} - \mathbf{\Pi})}{2\sigma_Y^2\mathcal{H}_a}[\mathbf{D}\cdot(\overline{\mathbf{T}} - \mathbf{\Pi}) - m_\vartheta(\mathcal{Q} - h_e \operatorname{tr}\mathbf{D})] = \frac{3j(\overline{\mathbf{T}} - \mathbf{\Pi})}{2\sigma_Y^2\mathcal{H}_a}\Psi_a, \\ j = \begin{cases} 1 & \text{if} \quad f = 0 \quad \text{and} \quad \Psi_a \geq 0, \\ 0 & \text{if} \quad f = 0 \quad \text{and} \quad \Psi_a < 0 \quad \text{or} \quad f < 0; \end{cases} \\ \mathcal{H}_a = \mathcal{H}_i - m_\vartheta(q_d + q_r),\end{gathered} \tag{3.7}$$

adiabatic hardening function,

$$\dot{\vartheta} = j\frac{q_d + q_r}{\mathcal{H}_a}(\mathbf{\Lambda}^f\cdot\mathbf{D} - m_\vartheta\mathcal{Q}) + \mathcal{Q} - h_e \operatorname{tr}\mathbf{D}, \tag{3.8}$$

with

$$\mathbf{\Lambda}^f = (\overline{\mathbf{T}} - \mathbf{\Pi}) + m_\vartheta h_e \mathbf{1}. \tag{3.9}$$

When $\vartheta = \vartheta_0 = \text{const}$, the flow laws and the equation for temperature in the isothermal process are obtained from Eqs. (3.1), (3.2), (3.4):

$$\check{\mathbf{T}} = \beta\mathbf{L}\mathbf{D} - \frac{3j\mu\beta(\overline{\mathbf{T}} - \mathbf{\Pi})}{\sigma_Y^2\mathcal{H}_i}[\mathbf{D}\cdot(\overline{\mathbf{T}} - \mathbf{\Pi})], \tag{3.10}$$

$$0 = \left(1 - \frac{\Pi}{\sigma_Y}\frac{\partial\sigma_Y}{\partial\Pi} - \frac{3\vartheta_0 c_T(\overline{\mathbf{T}} - \mathbf{\Pi})\cdot\mathbf{\Pi}}{2\rho_0 c c_v \sigma_Y^2}\right)\frac{(\overline{\mathbf{T}} - \mathbf{\Pi})\cdot\mathbf{D}}{\mathcal{H}_i} - \beta\operatorname{div}\mathbf{q} - \vartheta_0\beta\alpha_v K_T \operatorname{tr}\mathbf{D}, \tag{3.11}$$

and when heat conductivity $\mathcal{Q} = 0$, the response of a solid to an instantaneous adiabatic process is described by the following equation

$$\check{\mathbf{T}} = \beta \mathbf{L}^a \mathbf{D} - \frac{3j\mu\beta}{\sigma_Y^2 \mathcal{H}_a}(\mathbf{\Lambda}^f \cdot \mathbf{D})\mathbf{\Lambda}^g \tag{3.12}$$

on account of Eq. (3.1), (3.7); here

$$\mathbf{\Lambda}^g = (\overline{\mathbf{T}} - \mathbf{\Pi}) + \frac{\alpha_v K_T (q_d + q_r)\sigma_Y^2}{3\mu}\mathbf{1}, \tag{3.13}$$

and

$$L^a_{ijkl} = \delta_{ij}\delta_{kl}[(1 + \alpha_v q_e)K_T - p] + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right) \tag{3.14}$$

is the generalized tensor of adiabatic elastic moduli. At moderate pressures, one can ignore the thermal expansion (the terms involving α_v), the heat of elastic deformation (the terms containing h_e) as discussed by MRÓZ and RANIECKI [7], so the second terms in the right-hand sides of Eqs. (3.9), (3.13) can be disregarded. Moreover, from (3.14) we have $\mathbf{L}^a = \mathbf{L}$ and the difference between Eqs. (3.10), (3.12) consists only in the fact that $\mathcal{H}_i \neq \mathcal{H}_a$

The Eqs. (2.6), (3.10), (3.12) are not convenient in applications since the elastic spin ω^e occurs at the left-hand side of them. The flow rules can be expressed in terms of usual Zaremba–Jaumann rates $\overset{\triangledown}{\mathbf{T}} = \dot{\mathbf{T}} - \boldsymbol{\omega}\mathbf{T} + \mathbf{T}\boldsymbol{\omega}$ for rate-independent materials in both isothermal and adiabatic processes (when certain small coupling effects are disregarded) as follows:

$$\overset{\triangledown}{\mathbf{T}} = \beta \mathbf{L}\mathbf{D} - \frac{3j\mu\beta \mathbf{D}\cdot(\overline{\mathbf{T}} - \mathbf{\Pi})}{\sigma_Y^2 \mathcal{H}}[(\overline{\mathbf{T}} - \mathbf{\Pi}) + \mathbf{P}], \tag{3.15}$$

$$j = \begin{cases} 1 & \text{if} \quad f = 0 \quad \text{and} \quad \mathbf{D}\cdot(\overline{\mathbf{T}} - \mathbf{\Pi}) \geq 0, \\ 0 & \text{if} \quad f = 0 \quad \text{and} \quad \mathbf{D}\cdot(\overline{\mathbf{T}} - \mathbf{\Pi}) < 0 \quad \text{or} \quad f < 0. \end{cases}$$

Here $\mathcal{H} = \mathcal{H}_i$ in isothermal and $\mathcal{H} = \mathcal{H}_a$ in adiabatic processes. $\mathcal{H}_a$ is usually smaller than $\mathcal{H}_i$. Tensor $\mathbf{P}$ appearing in (3.15) is obtained by expressing the term $(\boldsymbol{\omega}^p\mathbf{T} - \mathbf{T}\boldsymbol{\omega}^p)$ as a function of $\mathbf{D}^p$ where $\boldsymbol{\omega}^p$ is the plastic spin. Thus, non-normality is here a result of presence of the plastic spin. Under the assumption that $\boldsymbol{\omega}^p$ is of the same order as $\mathbf{D}^p$, one may neglect the term involving $\mathbf{P}$ if quantities of the order stress/ elastic moduli are small compared to unity, and in such circumstances the plastic spin does not play any important role in the relation between $\overset{\triangledown}{\mathbf{T}}$ and $\mathbf{D}$.

4. Propagation of acceleration waves

The motion of a so-called singular surface, across which variables may be discontinuous, is the subject of this section. Such a moving surface is called a wave. We consider now the propagation of the second-order discontinuity in a three-dimensional, unbounded, elastic-plastic medium. The propagation of waves may be analyzed in the space coordinates, or with respect to the material, that is in Lagrangian coordinates. However, in order to obtain the simplest results, let us assume the material configuration at time t as the reference configuration. It means that the motion of the wave in the time interval $[t, t + dt]$ is referred to the material particles at time instant t. The following relation between the velocities holds true:

$$W = \Omega + \mathbf{v} \cdot \boldsymbol{\nu}; \tag{4.1}$$

here W is the wave velocity in space, Ω is the local velocity (with respect to the material at time t), $\mathbf{v}$ is the particle velocity and $\boldsymbol{\nu}$ is the unit vector normal to the wave surface S. The jump conditions associated with such discontinuity were established by HADAMARD [2]. In Eulerian coordinate, if $\gamma(x,t)$ remains continuous at passing through S but its derivatives are discontinuous, there exists Γ such that

$$[\gamma_{,i}] = \left[\frac{\partial \gamma}{\partial x_i}\right] = \Gamma \nu_i, \quad [\gamma_{,t}] = \left[\frac{\partial \gamma}{\partial t}\right] = -\Gamma W, \quad [\dot{\gamma}] = -\Gamma \Omega, \tag{4.2}$$

where $[A]$ denotes the jump of A. Owing to the compatibility conditions, we obtain

$$[\gamma_{,i}] = -\frac{\nu_i}{\Omega}[\dot{\gamma}]. \tag{4.3}$$

The equations of continuity and of motion assume now the forms

$$\dot{\beta} = \beta \operatorname{div} \mathbf{v}, \qquad \operatorname{div} \boldsymbol{\sigma} = \rho \dot{\mathbf{v}}. \tag{4.4}$$

Taking into account Eqs. (4.2), (4.3), from (4.2) and (4.4) we obtain the equation which controls acceleration waves

$$(\mathbf{Q} - \rho \Omega^2 \mathbf{1})[\dot{\mathbf{v}}] = 0, \tag{4.5}$$

where the so-called acoustic tensor Q_{jl} is given by

$$\begin{aligned} Q_{jl} &= Q^e_{jl} + Q^p_{jl}, \\ Q^e_{jl} &= Q^{e_1}_{jl} + Q^{e_2}_{jl}, \\ Q^{e_1}_{jl} &= \left(K_T + \frac{\mu}{3}\right)\nu_j \nu_l + \mu \delta_{jl}, \\ Q^{e_2}_{jl} &= \mu[(\overline{e}_{pq}\nu_p\nu_q) - \overline{e}_{ij}\nu_i\nu_l - \overline{e}_{il}\nu_i\nu_j - \overline{e}_{jl}\nu_i\nu_i], \\ Q^p_{jl} &= -\frac{3j\mu}{\sigma_Y^2 \mathcal{H}} a_j b_l, \quad \text{where} \quad \begin{cases} a_j = [(\overline{T}_{ij} - \Pi_{ij}) + P_{ij}]\nu_i, \\ b_l = (\overline{T}_{kl} - \Pi_{kl})\nu_k. \end{cases} \end{aligned} \tag{4.6}$$

It appears that $\mathbf{Q}$ depends on the material parameters as well as on the direction $\boldsymbol{\nu}$. Here, the acoustic tensor $\mathbf{Q}$ is non-symmetric because $a_j \neq b_l$ in the presence of the plastic spin. The elastic acoustic tensor $\mathbf{Q}^e$ is symmetric and consists of the usual part $\mathbf{Q}^{e_1}$ and the part $\mathbf{Q}^{e_2}$ due to the geometric effects. Under the assumptions that $\mathbf{Q}^{e_1}$ is positive definite and the elastic distortions are small, as was shown in [20], $\mathbf{Q}^e$ is also positive definite. The eigenvalues of Eq. (4.5) give us the velocities of elastic waves (when $j = 0$) and those of plastic waves (when $j = 1$). Let us now determine the eigenvalues of the acoustic problem (4.5).

We shall use the identity

$$\begin{aligned} \det(\mathbf{1} + r\mathbf{a}\otimes\mathbf{b} + s\mathbf{c}\otimes\mathbf{d} + t\mathbf{e}\otimes\mathbf{f}) &= 1 + r\mathbf{a}\cdot\mathbf{b} + s\mathbf{c}\cdot\mathbf{d} + t\mathbf{e}\cdot\mathbf{f} \\ &+ rs[(\mathbf{a}\cdot\mathbf{b})(\mathbf{c}\cdot\mathbf{d}) - (\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})] + rt[(\mathbf{a}\cdot\mathbf{b})(\mathbf{e}\cdot\mathbf{f}) - (\mathbf{a}\cdot\mathbf{f})(\mathbf{b}\cdot\mathbf{e})] \\ &+ st[(\mathbf{c}\cdot\mathbf{d})(\mathbf{e}\cdot\mathbf{f}) - (\mathbf{c}\cdot\mathbf{f})(\mathbf{d}\cdot\mathbf{e})] \\ &+ rst[(\mathbf{a}\cdot\mathbf{b})(\mathbf{c}\cdot\mathbf{d})(\mathbf{e}\cdot\mathbf{f}) + (\mathbf{b}\cdot\mathbf{c})(\mathbf{d}\cdot\mathbf{e})(\mathbf{f}\cdot\mathbf{a}) + (\mathbf{a}\cdot\mathbf{d})(\mathbf{c}\cdot\mathbf{f})(\mathbf{b}\cdot\mathbf{e}) \\ &- (\mathbf{a}\cdot\mathbf{b})(\mathbf{c}\cdot\mathbf{f})(\mathbf{d}\cdot\mathbf{e}) - (\mathbf{c}\cdot\mathbf{d})(\mathbf{a}\cdot\mathbf{f})(\mathbf{b}\cdot\mathbf{e}) - (\mathbf{e}\cdot\mathbf{f})(\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})], \end{aligned} \tag{4.7}$$

that may be verified by direct expansion of the above determinant. This identity reduces when $s = t = 0$ to that used in [13]. Using Vieta's theorem of roots for the characteristic equation (4.5) and the identity (4.7), we therefore obtain

$$
\begin{aligned}
\rho\Omega_I^2 &= \mu(1 + \overline{e}_{rs}\nu_r\nu_s),\\
\rho\Omega^2_{\mathrm{II,III}} &= \frac{1}{2}\left\{\left(\mu + M - \frac{3j\mu}{\sigma_Y^2\mathcal{H}}a_jb_j\right) \pm \sqrt{\Delta}\right\}.
\end{aligned} \tag{4.8}
$$

Here the eigenvalues are determined, the terms of the order $O(|\overline{e}|^2)$ due to the distortional elastic strains being disregarded, and

$$
\begin{aligned}
M &= K_T + \frac{4\mu}{3},\\
\Delta &= \left(\mu + M - \frac{3j\mu}{\sigma_Y^2\mathcal{H}}a_jb_j\right)^2 - 4\mu M\Lambda_A\left[1 + \frac{9(K_T + 2\mu)(3K_T + \mu)}{M\mu^2(3K_T + 4\mu)^2}(\overline{e}_{rs}\nu_r\nu_s)\right],
\end{aligned}
$$

with

$$
\begin{aligned}
\Lambda_A = 1 - \frac{3j\mu}{\sigma_Y^2\mathcal{H}}\Bigg\{&\frac{1}{\mu}\left[(a_kb_k) - \frac{3K_T + \mu}{(3K_T + 4\mu)}(a_j\nu_j)(b_k\nu_k)\right]\\
&- \frac{1}{\mu}\left[(\overline{e}_{rs}\nu_r\nu_s)(a_kb_k) - \frac{2(\overline{e}_{rs}\nu_r\nu_s)(3K_T + \mu)^2}{(3K_T + 4\mu)^2}(a_j\nu_j)(b_k\nu_k)\right.\\
&\left. -(\overline{e}_{kj}a_jb_k) + \frac{(3K_T - 2\mu)}{(3K_T + 4\mu)}(\overline{e}_{rs}\nu_rb_s)(a_j\nu_j) + \frac{(3K_T - 2\mu)}{(3K_T + 4\mu)}(\overline{e}_{rs}\nu_ra_s)(b_k\nu_k)\right]\Bigg\}.
\end{aligned} \tag{4.9}
$$

For small deformations, these results reduce to those of [10] ($\overline{e}_{ij} \to 0$ and $\beta \to 1$) for non-associated plasticity and those of Hill in the case of associated plasticity [3]. When $j = 0$ we obtain the familiar expressions for elastic waves, which are independent of the direction $\mathbf{v}$. If the wave speed is zero, the condition for static localization of RICE [15] is obtained from Eq. (4.8).

5. Conclusions

The flow laws for the isothermal and adiabatic processes were derived and on the basis of this, we have investigated the propagation of acceleration waves. The similar problems in small deformations were considered by RANIECKI [13] and by NGUYEN [20] for finite deformation of isotropic materials. Acceleration waves relate directly to the problem of stability. When plastic normality applies and $\mathbf{Q}$ is symmetric, then all velocities $\rho\Omega^2$ are real, as shown in [4], for each direction $\mathbf{v}$, the plastic wave velocity is not greater than the corresponding elastic wave velocity. So, in this case there is stability or not according to whether the smallest plastic wave velocity is positive or negative. When it is negative, we have 'divergence' instability.

For nonassociated plasticity, tensor $\mathbf{Q}$ is nonsymmetric, i.e. it is possible to have complex velocity $\rho\Omega^2$: 'flutter' growth may occur. It is shown in [10] that for a very broad class of nonassociated plasticity models, 'flutter' instability cannot occur. Here, deviation from plastic normality is caused by the plastic spin and, depending on the form of the plastic spin applied, the possible complex values of velocity may exist. The basic theory of uniqueness in relation to localizations and stationary waves is not yet adequately developed for materials deviating from normality and the present paper suggest that it deserves further study.

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