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Second Stokes problem in the presence of a magnetic field in porous media: Brinkman model

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THE VISCOUS incompressible flow, due to an infinite impermeable oscillating plate at the bottom of a porous medium of finite thickness is considered, in the presence of a uniform transverse magnetic field fixed relative to the fluid. In porous matrix Brinkman's equation has been used and, at the porous medium-fluid boundary, a modified set of boundary conditions are being applied. The effects of the permeability and the magnetic field on the distribution of fluid velocity are examined.

1. Introduction

THE EFFECTS of transverse magnetic field on the flow of an electrically conducting viscous fluid have been studied extensively in view of numerous applications to astrophysical, geophysical and engineering problems (CRAMER and PAI [1]).

The second Stokes problem—the familiar oscillating plate problem in classical hydrodynamics was discussed by SCHLICHTING [2]. RUDRAIAH [3] discussed this problem in magnetohydrodynamics and compared his results with those of hydrodynamic results of PANTON [4], who gave the transient solution for the second Stokes problem and discussed its importance in many practical applications. TOKIS [5] further studied this oscillating plate problem subjected to uniform suction or injection in the presence of a uniform magnetic field relative to the fluid or to the plate, and compared the results to that of ONG and NICHOLLS [6] in the absence of suction or injection. MURTHY [7] studied the Stokes First and Second problems in porous medium.

The object of the present paper is to study the flow in the porous medium due to an oscillating plate, in the presence of a magnetic field. In specifying our problem, we consider an incompressible electrically conducting viscous fluid contained in an infinite permeable bed of finite thickness h, and outside it in the semi-infinite region. The flow is due to an impermeable plate oscillating in its own plane at the bottom of the permeable bed, and a uniform transverse magnetic field fixed relative to the fluid is applied. We studied the coupled flow by dividing the whole flow field into two regions, (I) free fluid region, ($0 \leq y < \infty$), and (II) porous region ($-h \leq y \leq 0$). In a porous matrix Brinkman equation [8] is used, which allows for matching of the velocities and tractions at the boundary between the fluid and porous medium. Modelling the porous medium by the Brinkman equation enables us to avoid the difficulties by retaining the second-order viscous stress terms. However, then the effective viscosity of the Brinkman medium differs from that of the pure solvent. A modified set of boundary conditions is applied at the fluid-porous medium interface discussed by KIM and RUSSEL [9] in this light. We have examined the effect of the permeability and the magnetic field on the distributions of the fluid velocity.

2. Formulation and solutions

The viscous incompressible electrically conducting flow is considered, generated in a semi-infinite mass of fluid, due to an infinite impermeable flat plate oscillating in its own plane, with a constant amplitude U_0 and frequency w, at the bottom of an infinite porous medium of finite thickness h. The x-axis is taken along the fluid-porous medium interface, and the y-axis is normal to it. Pressure is constant throughout the flow field. A uniform transverse magnetic field fixed relative to the fluid is applied. In practice the fluid motion is set up from rest and, for some time after the initiation of the motion, the fluid velocity gradually becomes a harmonic function of t, with the same frequency as the velocity of the impermeable bottom. This steady periodic state is considered here.

The flow in the free fluid region ($0 \leq y < \infty$) is governed by the equation of motion:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 u, \tag{2.1}$$

and the flow in the porous region ($-h \leq y \leq 0$) is governed by the Brinkman equation:

$$\frac{\partial U}{\partial t} = -\frac{\overline{\nu}}{k} U + \overline{\nu} \frac{\partial^2 U}{\partial y^2} - \frac{\sigma B_0^2}{\rho} U, \tag{2.2}$$

where u and U are the velocities in the free fluid and porous region, respectively, ν is the kinematic viscosity; $\overline{\nu}$ is the effective kinematic viscosity in the porous medium; k is the permeability of the porous medium; ρ is the fluid density, σ is the electrical conductivity of the fluid, and B_0 is the magnetic field. Here the permeability in the Darcy's resistance term is redifined appropriately.

The boundary conditions are

$$\begin{aligned} &\text{at } y = -h, \quad U = U_0 \cos wt, \\ &\text{at } y = 0, \quad u = U, \quad \nu \frac{\partial u}{\partial y} = \overline{\nu} \frac{\partial U}{\partial y}, \\ &\text{as } y \to \infty, \quad u \to 0. \end{aligned} \tag{2.3}$$

The equations of motion and boundary conditions, after introducing the following non-dimensional quantities

$$\begin{aligned} &\overline{u} = \frac{u}{U_0}, \quad \overline{y} = \frac{y}{h}, \quad \overline{t} = \frac{\nu t}{h^2}, \quad \overline{w} = \frac{h^2}{\nu} w, \\ &\overline{U} = \frac{U}{U_0}, \quad \overline{k} = \frac{k}{h^2}, \quad \phi = \frac{\nu}{\overline{\nu}} \text{ and } M^2 = \frac{\sigma B_0^2 h^2}{\rho \nu}, \end{aligned} \tag{2.4}$$

and dropping the bars, are

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2 u, \tag{2.5}$$

$$\frac{\partial U}{\partial t} = \frac{1}{\phi}\left(-\frac{U}{k} + \frac{\partial^2 U}{\partial y^2}\right) - M^2 U, \tag{2.6}$$

$$\begin{aligned} &\text{b.cs. at } y = -1, \quad U = \cos wt, \\ &\text{at } y = 0, \quad u = U, \quad \phi \frac{\partial u}{\partial y} = \frac{\partial U}{\partial y}, \\ &\text{as } y \to \infty, \quad u \to 0. \end{aligned} \tag{2.7}$$

Let

$$u(y,t) = \mathrm{Re}[f(y)e^{iwt}] \tag{2.8}$$

and

$$U(y,t) = \mathrm{Re}[F(y)e^{iwt}]. \tag{2.9}$$

Substituting (2.8) in Eqs. (2.5)–(2.7) and solving under the corresponding boundary conditions, we get the solutions as

$$u = e^{-\alpha_1 y}[d_1 \cos(wt - \beta_1 y) - d_2 \sin(wt - \beta_1 y)] \tag{2.10}$$

and

$$U = e^{-\alpha_2 y}[d_3 \cos(wt - \beta_2 y) - d_4 \sin(wt - \beta_2 y)] + e^{\alpha_2 y}[d_5 \cos(wt + \beta_2 y) - d_6 \sin(wt + \beta_2 y)], \tag{2.11}$$

where

$$\begin{aligned}
\alpha_1 &= \left(\frac{1}{2}\right)^{1/2} (M^2 + \sqrt{M^4 + w^2})^{1/2},\\
\beta_1 &= \left(\frac{1}{2}\right)^{1/2} (\sqrt{M^4 + w^2} - M^2)^{1/2},\\
\alpha_2 &= \left(\frac{1}{2}\right)^{1/2} \left[\left(\phi M^2 + \frac{1}{k}\right) + \sqrt{\left(\phi M^2 + \frac{1}{k}\right)^2 + \phi^2 w^2}\right]^{1/2},\\
\beta_2 &= \left(\frac{1}{2}\right)^{1/2} \left[\sqrt{\left(\phi M^2 + \frac{1}{k}\right)^2 + \phi^2 w^2} - \left(\phi M^2 + \frac{1}{k}\right)\right]^{1/2},\\
c_1 &= (\alpha_2 + \phi\alpha_1),\\
c_2 &= (\beta_2 + \phi\beta_1),\\
c_3 &= (\alpha_2 - \phi\alpha_1),\\
c_4 &= (\beta_2 - \phi\beta_1),\\
c_5 &= [e^{\alpha_2}(c_1 \cos\beta_2 - c_2 \sin\beta_2) + e^{-\alpha_2}(c_3 \cos\beta_2 + c_4 \sin\beta_2)],\\
c_6 &= [e^{\alpha_2}(c_2 \cos\beta_2 + c_1 \sin\beta_2) + e^{-\alpha_2}(c_4 \cos\beta_2 - c_3 \sin\beta_2)],\\
d_1 &= (d_3 + d_5),\\
d_2 &= (d_4 + d_6),\\
d_3 &= \frac{1}{d_7}(c_1 c_5 + c_2 c_6),\\
d_4 &= \frac{1}{d_7}(c_2 c_5 - c_1 c_6),\\
d_5 &= \frac{1}{d_7}(c_3 c_5 + c_4 c_6),\\
d_6 &= \frac{1}{d_7}(c_4 c_5 - c_3 c_6),\\
d_7 &= (c_5^2 + c_6^2).
\end{aligned} \tag{2.12}$$

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It is worthwhile to point out that the above solutions include the results of the previous investigations.

(i) When $M \to 0$ and $k \to \infty$,

$$u = U = e^{-(w/2)^{1/2}(y+1)} \cos\left(wt - \left(\frac{w}{2}\right)^{1/2}(y+1)\right),$$

which is the solution of Stokes for the familiar oscillating plate problem, when the impermeable plate is oscillating at $y = -1$, in the free fluid.

(ii) When $m \to 0$, $k \to 0$, $u = U = 0$. Since in the absence of permeability porous medium acts as impermeable layer, no oscillation will be transmitted to the fluid.

(iii) When $k \to \infty$,

$$u = U = e^{-\alpha_2(y+1)}[\cos(wt - \beta_2(y+1))],$$

where

$$\alpha_2 = (\frac{1}{2})^{1/2}(M^2 + \sqrt{M^4 + w^2})^{1/2} \quad \text{and} \quad \beta_2 = (\frac{1}{2})^{1/2}(\sqrt{M^4 + w^2} - M^2)^{1/2}$$

which is the solution of ONG and NICHOLLS [6] for the magnetic field fixed to the fluid; the plate is oscillating at $y = -1$.

3. Discussion

The present note involves the unsteady interior flow in a porous medium and over its surface. The fluid motion is due to an impermeable plate oscillating harmonically at the bottom of porous medium, and a uniform transverse magnetic field fixed relative to the fluid is applied. The oscillating plate performs the corresponding oscillations in the fluid, in which the velocity decays as the distance from the plate increases. This velocity distribution is shown in Fig. 1, for different magnetic and permeability parameters and for $wt = 0$ and $wt = \pi/2$. The symmetrical curves for $wt = \pi$ and $wt = 3\pi/2$ are not included.

It is observed that, by the introduction of magnetic field, the disturbances are decaying more rapidly in comparison to the classical Stokes profiles. By increasing the strength of the applied magnetic field, the profiles decay more rapidly for the same values of the permeability parameter. However, the depth of penetration of the viscous wave increases as a result of the introduction of permeable material, since then flowability becomes low due to the effective Brinkman viscosity in the medium and large mass of fluid is dragged along with the oscillating impermeable plate. On the other hand, in absence of the permeable medium, the fluid moves easily, ignoring the plate except in a thin boundary layer.

The results may find applications in the ground water hydrology, petroleum engineering, and in the study of variation of temperature in the surface layers of the ground, etc.

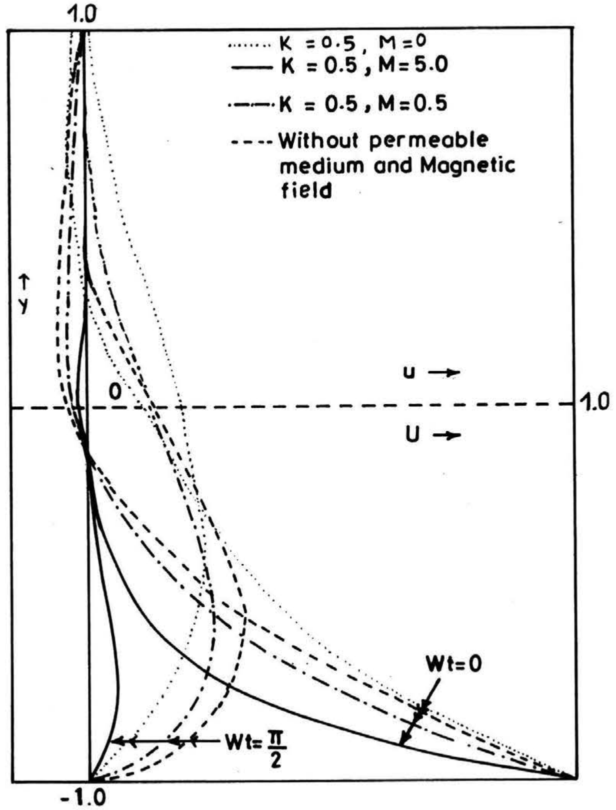

FIG. 1. Velocity profiles for $\phi = 0.4$ and $W = 8.0$.

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