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A REGENERATIVE ELECTROACOUSTIC METHOD OF TESTING ABRASIVE WHEELS BY RESONANCE FREQUENCY MEASUREMENT

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1. Introduction

The recent advance in the design and construction of automatic grinding machines and the increased requirements for the precision of the grinding process call for new and more accurate methods of testing and grading respective tools, particularly abrasive wheels. One of the most important characteristics of every abrasive wheel is its grade of hardness which is a measure of the tenacity with which the individual grinding grains are bonded together. The inner structure of an abrasive wheel is schematically shown in Fig.1. Many different methods have been hitherto used for measuring the hardness of abrasive wheels being all, as a rule, mechanical methods which consist, generally speaking, in acting upon the surface of the wheel's body with a hardened tool or with a normalized sand blast and then in measuring the depth of the depression produced in the abrasive material. All these methods, besides being destructive ones, are inaccurate, very laborious and depend to some extent on the operator's skill and judgment.

In this connection another methods for grading abrasive wheels were proposed being non-destructive acoustical methods based on resonance frequency measurements [1][2]. They are founded upon the justified theoretically by Kirchhoff [3] and verified experimentally by many other investigators well-known relations which exist between the geometrical and material parameters of definite mechanical structures, especially circular plates, and their vibration properties.

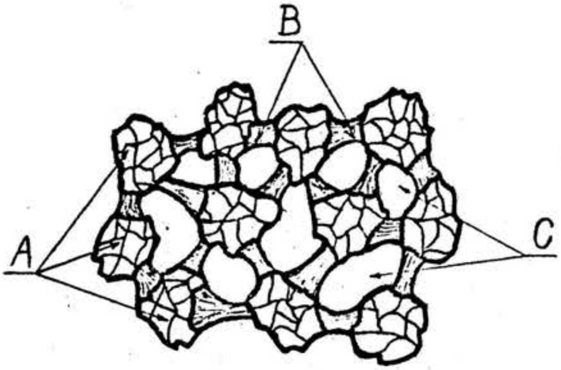

Fig.1. Simplified sketch showing the inner structure of an abrasive wheel
A - abrasive grains, B - bond, C - voids.

It was proved that the hardness of any abrasive wheel, depending on its material constants and expressed conventionally in several grades labelled according to the present standards with letters in the range from A /very soft/ to Z /very hard/, is closely related to the resonance frequency $F_{n,l}$ which corresponds to a definite mode of vibration defined by the occurrence of n diametric and l concentric nodal lines on the surface of the vibrating body and is given by the expression [3]

$$F_{n,l} = \frac{\lambda_{n,l}^{2}\, t}{4\pi r^{2}}\sqrt{\frac{E}{3\rho(1-\nu^{2})}} \quad \text{Hz} \qquad /1/$$

where: r - radius of the wheel /cm/,
t - thickness of the wheel /cm/,
E - Young's modulus of the wheels material /dyn/cm^2/,
ν - Poisson's ratio of the wheel's material,
ρ - density of the wheel's material /g cm^{-3}/,
$\lambda_{n,l}$ - constant for a given (n,l),
n - number of diametric nodal lines,
l - number of concentric nodal lines.

Theoretical distributions of diametric and concentric nodal lines on the surface of a freely vibrating circular plate are shown in Fig.2. The numerical relations of resonance frequencies at successive higher modes of vibration $F_{n,l}$ of a free circular plate to its fundamental frequency $F1 = F_{2,0}$ are given in Table 1.

Table 1. Frequency ratios of higher modes of vibration
$F_{n,l}$ to the fundamental frequency $F1 = F_{2,0}$
of a free circular plate with $\nu = 0.25$

$\underline{n}$ - number of diametric nodal lines,
$\underline{\underline{l}}$ - number of concentric nodal lines.

l \ n	0	1	2	3	4
0	-	-	1	2.3124	4.0485
1	1.6131	3.7032	6.4033	9.6445	13.3937
2	6.9559	10.8380	15.3050	20.3249	-

Many different methods for exciting self-vibrations of circular plates such as abrasive wheels are possible, using e.g. electrostatic, magnetic, acoustical or mechanical actuators. The simplest and most practical method consists in laing the wheel on four pointed supporting pins located

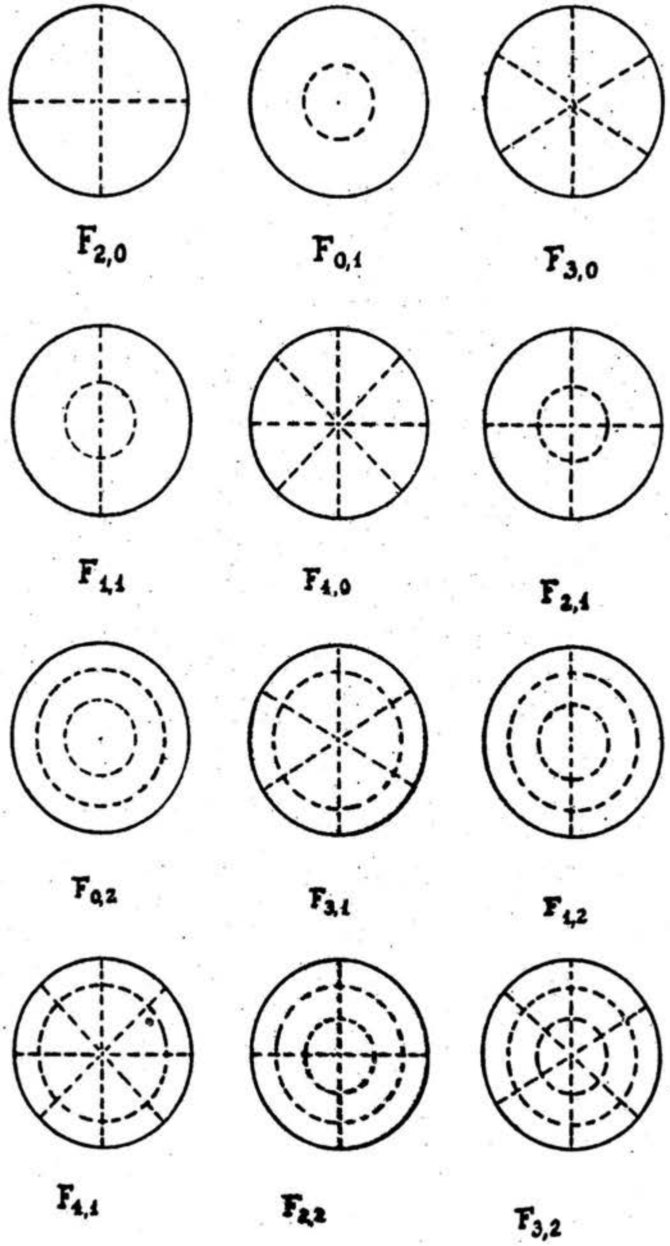

Fig.2. Distributions of **diametric** and concentric nodal lines of a **freely vibrating** circular plate

symmetrically on the nodal lines of the desired mode of vibration and in driving it by means of an electromechanical transducer in any antinodal point. The abrasive wheel is thus excited to forced vibrations whose amplitude reaches a maximum whenever the frequency of the driving force is equal to any resonance frequency $F_{n,1}$. This frequency may be measured by an arbitrary method using a probe microphone which receives the sound wave radiated by the vibrating wheel. Such a method, although used in practice [1][2][4], has many disadvantages being rather laborious and subjected to noise. To avoid these inconveniences we have proposed a new regenerative feedback method for measuring the resonance frequencies of mechanical structures, especially abrasive wheels [5].

2. Description of the method

The block diagram of the measuring equipment is given in Fig.3. The abrasive wheel ST under test, lying on four supporting pins located symmetrically on the hypothetical nodal lines of the desired mode of vibration, is mechanically coupled with two transducers PN and PO of the magnetoelectric type /see Fig.5/ which are electrically connected to the input and to the output of the broad-band electronic amplifier W with the gain $\hat{k}$. The four-pole 1-2-3-4 whose voltage transfer function according to Fig.3 is given by the expression

$$\hat{\beta} = U_{3,4} / U_{1,2} \qquad /2/$$

constitutes an electromechanical feedback loop. The electromechanical equivalent circuit of the feedback loop is shown in Fig.4 in which the following symbols are used:

E - e.m.f. of the voltage source, in the case being considered the open-circuit output voltage of the amplifier W,

R_g - source impedance, in the case being considered the output resistance of the amplifier W,

$Z_{e1} = R_{e1} + j\omega L_{e1}$ - electrical blocked impedance of the driving transducer PN,

$Z_{e2} = R_{e2} + j\omega L_{e2}$ - electrical blocked impedance of the receiving transducer PO,

α_1 - ideal gyrator coupling the electrical and the mechanical system of the driving transducer PN,

α_2 - ideal gyrator coupling the mechanical and the electrical system of the receiving transducer PO,

$Z_{m1} = R_{m1} + j\left(\omega M_{m1} - \frac{1}{\omega C_{m1}}\right)$ - mechanical open-circuit impedance of the driving transducer PN,

$Z_{m2} = R_{m2} + j\left(\omega M_{m2} - \frac{1}{\omega C_{m2}}\right)$ - mechanical open-circuit impedance of the receiving transducer PO,

ST - the abrasive wheel under test,

R_o - electrical load impedance of the four-pole 1-2-3-4, in the case being considered the input resistance of the amplifier W.

The whole system of Fig.3 becomes regenerative provided the well-known condition

$$\hat{k}\hat{\beta} \geqslant 1 \qquad /3/$$

is fulfilled. The frequency of the excited self-oscillations may be measured by means of the electronic counter MC.

The transfer function characteristic $\hat{\beta} = f(\omega)$ of the feedback loop depends both on the vibration properties of the abrasive wheel ST and on the resonance properties of the transducers PN and PO. This problem should be discussed separately in the low and in the high requency range.

In the low frequency range when $f \ll F_{2,0}$, where $F_{2,0}$ stands for the first mode of vibration, the abrasive wheel ST behaves like a stiff mechanical lever with the force

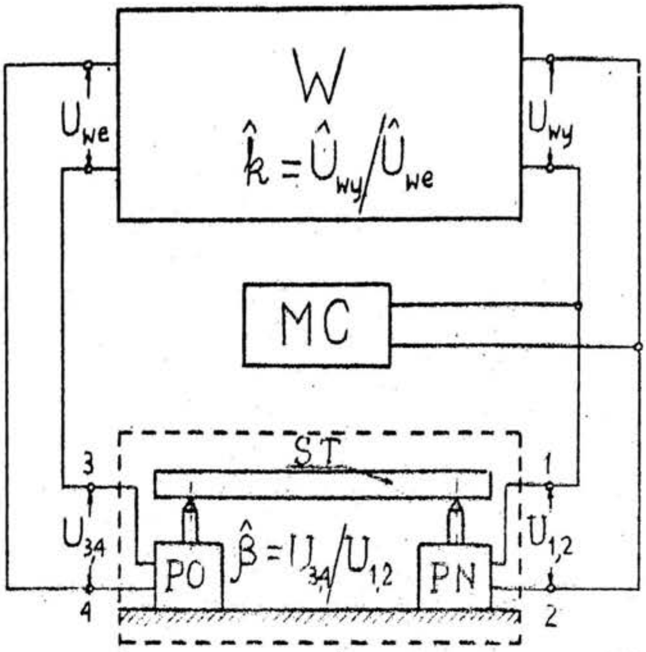

Fig.3. Schematic diagram of the equipment for measuring the resonance frequencies of abrasive wheels by the regenerative electromechanical feedback method

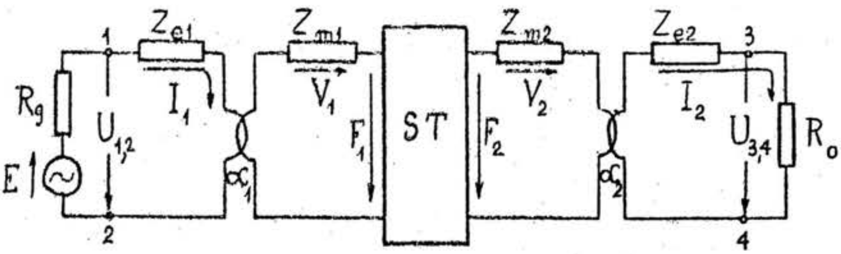

Fig.4. Electromechanical equivalent circuit of the four-pole 1-2-3-4 forming the feedback loop

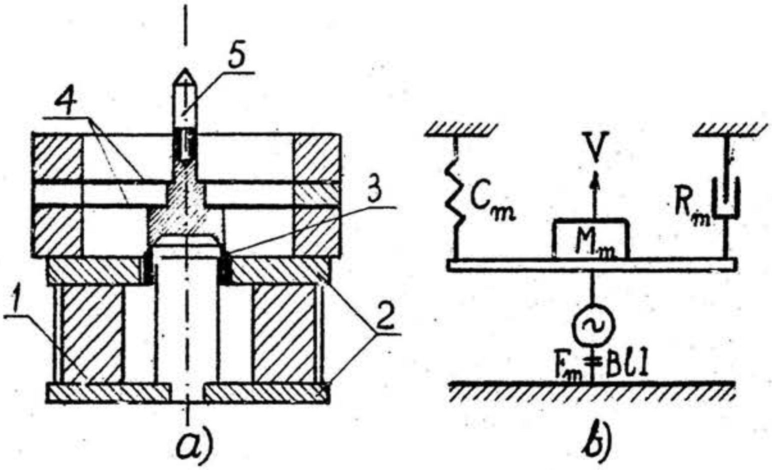

Fig.5. Simplified scheme of the magnetoelectric driving transducer /a/ and its equivalent mechanical circuit /b/

1 - permanent magnet, 2 - pole pieces, 3 - moving coil, 4.- moving-coil suspension, 5 - rivet shank coupling the transducer with the abrasive wheel.

ratio $\underline{n} = l_1/l_2$ which transfers the vibrations of the driving transducer <u>PN</u> to the receiving transducer <u>PO</u>. At the same time, the abrasive wheel <u>ST</u> being coupled with both transducers loads them with masses M_1 and M_2, respectively, and thus changes their own mechanical parameters. The simplified scheme of the abrasive wheel treated in the low frequency range as a coupling lever is given in Fig.6-a. Its electrical equivalent circuit in the impedance analogy system is shown in Fig.6-b. The coupling lever is here represented by the ideal transformer <u>TI</u> having the voltage /force/ ratio 1 : $\underline{n}$. In this case the transformer <u>TI</u> couples two mechanical series resonance circuits $Z_{m1} = R_{m1} + j\left(\omega M_{m1} - \frac{1}{\omega C_{m1}}\right)$ and $Z_{m2} = R_{m2} + j\left(\omega M_{m2} - \frac{1}{\omega C_{m2}}\right)$ corres-

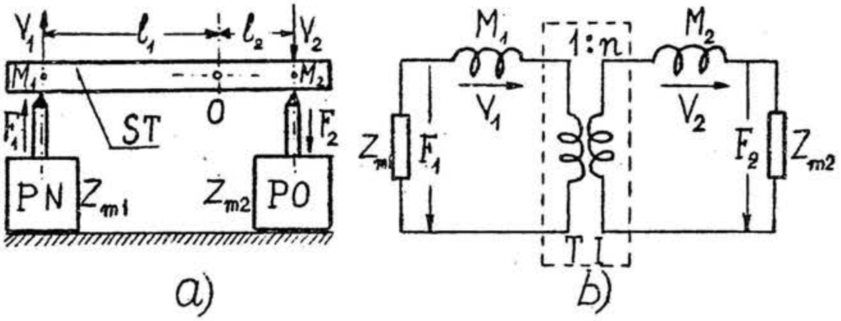

Fig.6. Mechanical /a/ and electrical /b/ equivalent circuit of the abrasive wheel considered in the low frequency range as a coupling lever

ponding to the driving and receiving transducer PN and PO, respectively.

According to the general theory of inductively coupled circuits, the transfer function characteristic $|\hat{\beta}| = \varphi(f)$ in this frequency range has two maxima closely related to resonance frequencies $f_1 \approx f_2$ of both transducers PN and PO loaded with the partial masses M_1 and M_2 of the abrasive wheel and equal to

$$f' < f_1 = \frac{1}{2\pi\sqrt{(M_{m1} + M_1)\,C_{m1}}} \approx f_2 = \frac{1}{2\pi\sqrt{(M_{m2} + M_2)\,C_{m2}}} < f''. \quad /4/$$

The complete electrical equivalent circuit of the feedback loop is shown in Fig.7-a and its transfer function characteristic in Fig.7-b. At the frequencies f' and f'' the self-oscillations in the feedback circuit may easily be excited. As, however, these frequencies depend mainly on the resonance properties of the transducers themselves and are not correlated to the resonance frequency of the abrasive wheel under test, this effect should be treated as undesired one and must be eliminated by the suitable deemphasis of the amplifier's gain characteristic $|\hat{k}| = f(\omega)$ in this frequency range.

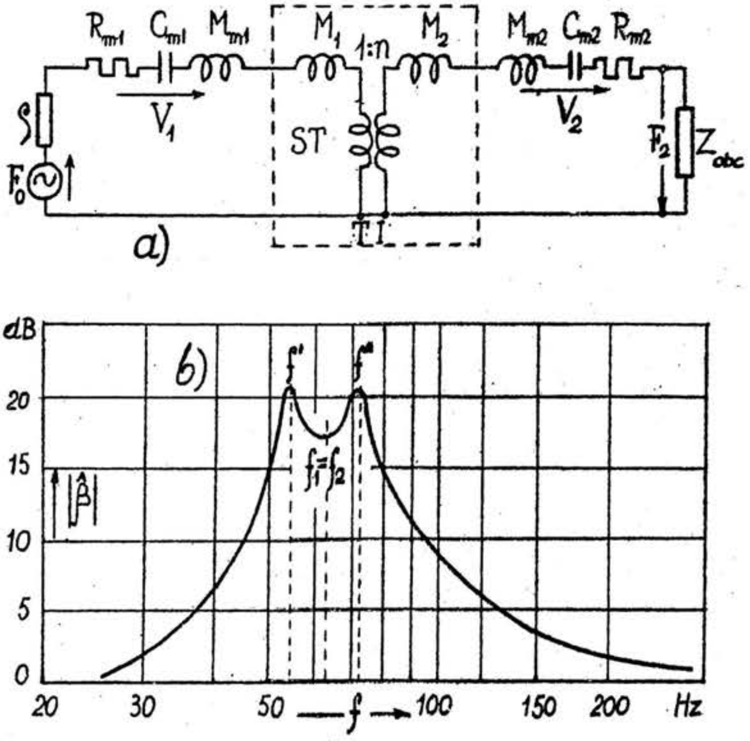

Fig.7. Equivalent circuit of the electromechanical feedback loop in the low frequency range /a/ and its transfer function characteristic /b/

In the high frequency range, on the other hand, where $f \gg F_{2,0}$, the abrasive wheel treated again as a coupling element behaves like a mechanical vibrating system with distributed constants and may be represented by an equivalent wave guide in which standing waves occur corresponding to the individual modes of vibration $F_{n,1}$. As an example, the schematic representation of standing wave's patterns corresponding to the fundamental resonance frequency F1 = $F_{2,0}$ of an abrasive wheel being supported in the points <u>a</u>,<u>b</u>,<u>c</u>,<u>d</u> located symmetrically on the nodal lines and coupled with the transducers <u>PN</u> and <u>PO</u> in the antinodal points <u>A</u> and <u>B</u> is shown in Fig.8-a. When considering this scheme it must be kept in mind that in fact the deformations of the abrasive wheel occur in the direction perpendicular to the plane of the drawing. In Fig.8-b the same standing wave´s patterns are shown in the cross-section perpendicular to the wheel´s plane along the unfolded circle <u>a</u>-<u>b</u>-<u>c</u>-<u>d</u> on which the supporting points are located.

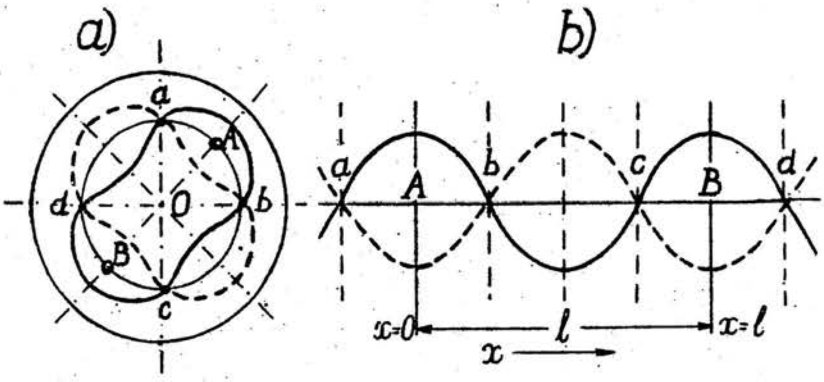

Fig.8. Schematic representation of standing wave´s patterns at the frequency F1 = $F_{2,0}$ on the surface of the abrasive wheel /a/ and in the cross-section perpendicular to its plane /b/

According to the general theory of wave guides it may be stated that in the vicinity of each resonance frequency $F_{n,1}$ the abrasive wheel may be considered as a mechanical series resonance circuit with the lumped constants $R_{n,1}$, $M_{n,1}$, $C_{n,1}$ representing its equivalent loss resistance, mass and complience, respectively, and with the factor of merit $Q_{n,1}$, the following relations being valid:

$$F_{n,1} = \frac{1}{2\pi\sqrt{M_{n,1} \cdot C_{n,1}}} \qquad /5/$$

$$Q_{n,1} = \frac{2\pi F_{n,1} M_{n,1}}{R_{n,1}} \qquad /6/$$

The electrical equivalent circuit of the feedback loop in the frequency range of interest, that is for $f \geqslant F_{2,0}$, is shown in Fig.9-a and the respective transfer function characteristic $|\hat{\beta}| = \varphi(f)$ in Fig.9-b. It may be observed that at any resonance frequency $F_{n,1}$ the mechanical impedance of the abrasive wheel decreases to a minimum value equal to $R_{n,1}$ and the modulus of the voltage transfer function $|\hat{\beta}|$ reaches a maximum thus creating the best conditions for the excitation of self-oscillations at these frequencies. The choice of the most convenient frequency of oscillations, which as a rule is $F1 = F_{2,0}$, depends on the suitable dislocation of the supporting points as well as of the driving point on the surface of the wheel.

The simplified electrical equivalent circuit of the abrasive wheel labelled in Figs.3 and 4 with letters <u>ST</u> and valid in the whole frequency range is given in Fig.10. When considering this scheme it must be kept in mind that the individual series resonance circuits shunting the transformer <u>TI</u> in the high frequency range are active successively according to the increase of frequency and that their rôle as coupling elements appears exclusively in the vicinity of their resonance frequencies.

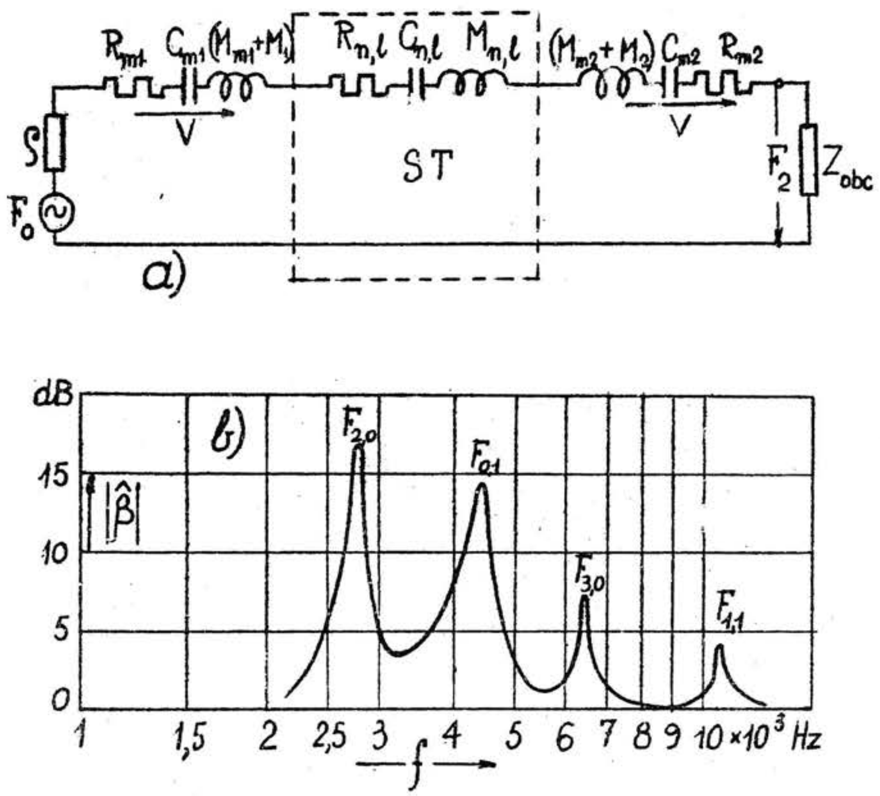

Fig.9. Equivalent circuit of the electromechanical feedback loop in the high frequency range /a/ and its transfer function characteristic /b/

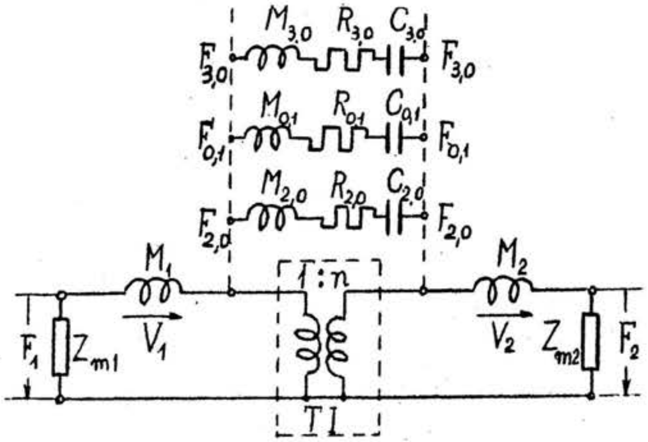

Fig.10. Simplified equivalent circuit of the abrasive wheel considered as the mechanical coupling element in the low frequency range /$f \ll F_{2,0}$/ and in the vicinity of resonance frequencies

3. Instrumentation

According to the above given considerations proved by experiments made with various assortments of abrasive wheels, the laboratory model of the equipment for measuring their resonance frequencies was designed and constructed. The general view of the measuring equipment is given in Fig.11 where two fundamental parts may be distinguished, viz. the measuring table and the electronic device.

The measuring table is in fact a heavy iron plate weithing about 140 kG, on which four supporting pins and two electromechanical transducers may be suitably fixed to fit the chosen distribution of the arbitrary mode of vibration on the surface of any abrasive wheel with the diameter ranging from 150 to 600 mm. The measuring table with the abrasive wheel under test lying on supporting pins is shown in Fig.12.

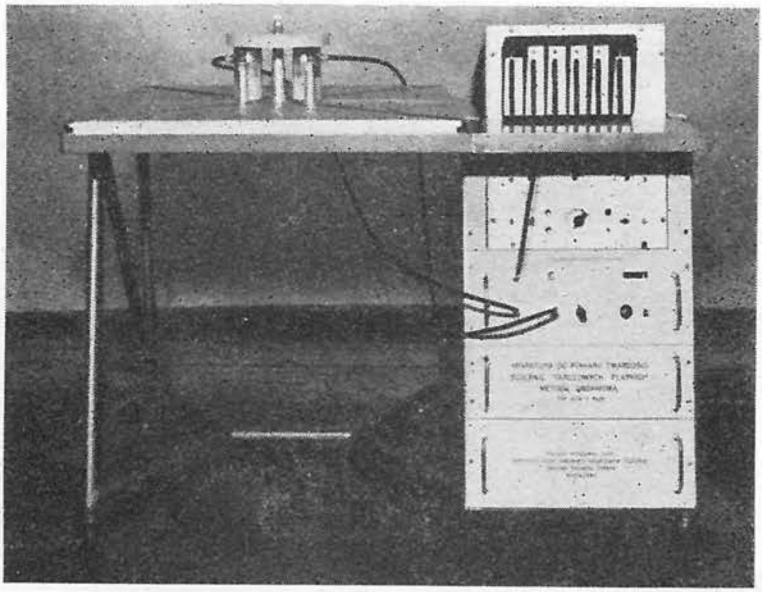

Fig.11. General view of the measuring equipment

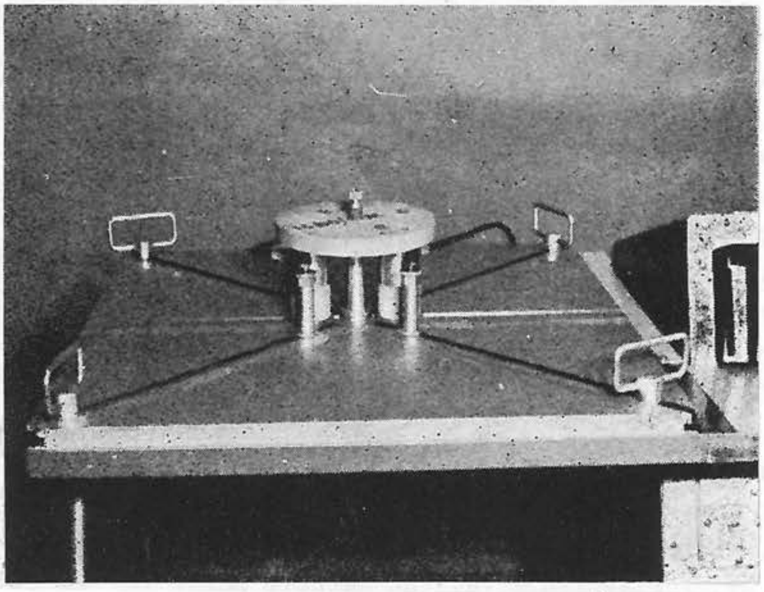

Fig.12. The measuring table with the abrasive wheel

The electronic device consists of the voltage amplifier, compressor, power amplifier and digital frequency counter and is so designed as to fulfil the condition /3/ of generating electrical oscillations in the case of various types of abrasive wheels to be investigated. The compressor is necessary to keep the amplitude of the excited self-oscillations within the range of linearity. The time necessary to measure the resonance frequency of an individual object does not exceed 1 second, the accuracy is better than $10^{-5} \pm 1$ Hz, the distribution of results in iterative measurements is less than 0.2% and the long-time stability better than 0.01% per hour.

4. Results and conclusions

Using the above described method and equipment, laboratory tests were carried out aiming at empirical prove of relations which exist between the hardness of various types of abrasive wheels measured by conventional methods and their resonance frequency $F1 = F_{2,0}$. Two examples of nodal lines visible on the surface of an abrasive wheel, viz. the diametric nodal lines at the first mode of vibration $F1 = F_{2,0}$ and the concentric nodal line at the second mode of vibration $F2 = F_{0,1}$ are shown in Figs.13 and 14, respectively.

The investigated inventory contained more than 500 wheels of the type NSAa /according to Polish Standards PN-62/M - 59119/ with different characteristics, viz. three kinds of abrasive grains /corundum EA, corundum EB, silicon carbide SZ/, three grain sizes / 25/60, 32/46, 40/36 /, five structures /from 5 to 9/ and several hardness grades ranging from F to Q. An example of the obtained results is given in Fig.15. The curves are valid for NSAa abrasive wheels of the size 150 mm×20 mm×20 mm, structure 6, grain size number 40/36, with grains made of corundum EA, corundum EB and silicon carbide SZ, respectively, all on ceramic bond. The ordinate scale is given both in hardness grades expressed in letters

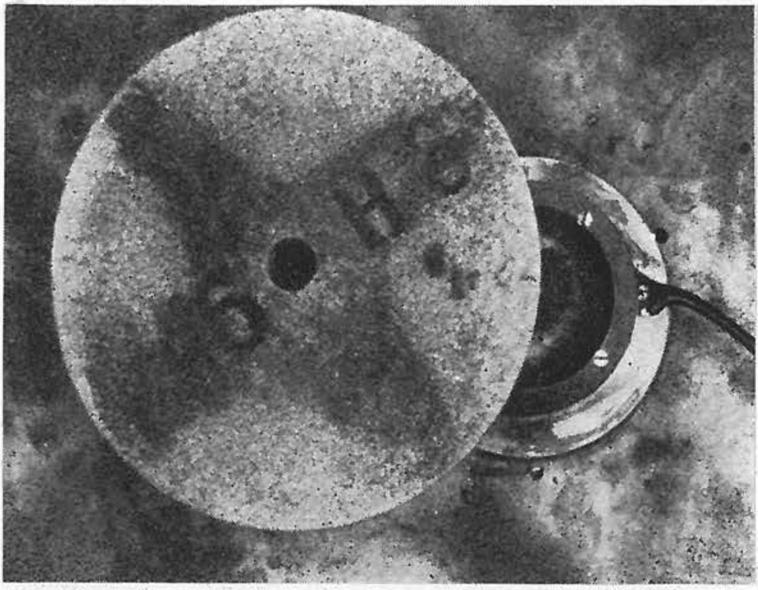

Fig.13. Diametric nodal lines at F1 = $F_{2,0}$

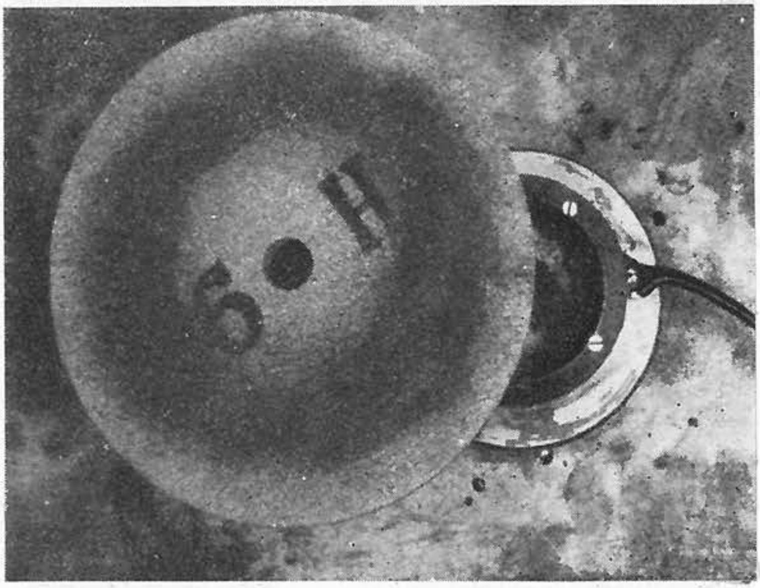

Fig.14. Concentric nodal line at F2 = $F_{0,1}$

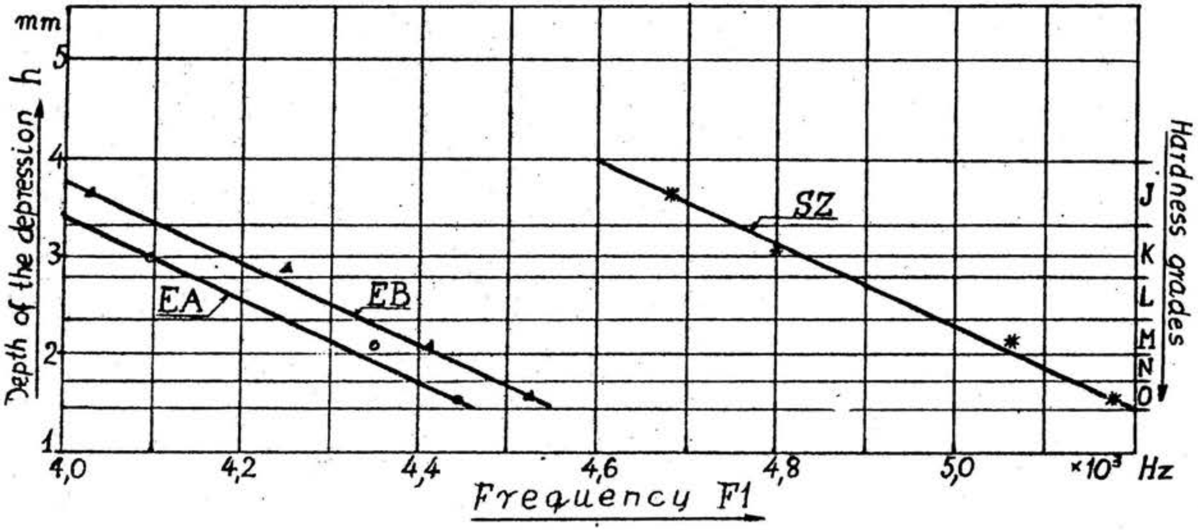

Fig.15. Curves representing empirical relations between the resonance frequency F1 = $F_{2,0}$ and the hardness of NSAa abrasive wheels of the size 150×20×20 mm, structure 6, grain size number 40/36, with grains made of corundum EA, corundum EB and silicon carbide SZ on ceramic bond.

of the alphabet code and in depths h of the depression in the wheel's body produced by the sand blast when grading it by means of Mc Kensen's apparatus.

It follows that the electroacoustic regenerative feedback method proposed for grading abrasive wheels according to their hardness is a non-destructive comparative method. Wheel grading by resonance frequency measurement is based on the formerly found empirical relation being representative for a given type of wheels. It is worth while saying that the results obtained for wheels of a given diameter D_1 and thickness t_1 may be directly translated to wheels of another diameter D_2 and another thickness t_2 when using simple arythmetical relations.

The method proved to be very useful for many research and practical applications. It makes it possible not only to measure quickly and accurately the hardness of all exemplars of abrasive wheels being produced, but also to find out within a given inventory of wheels with *similar* characteristics those exemplars which are *identical* from the point of view of their hardness. This fact is very important in view of the increasing demands for the uniformity and repreducibility of tools in the grinding process by automatic machines when the abrasive wheels are periodically consumed and have to be replaced without any readjustment of the machine.

At the moment the model of the measuring equipment is subjected to detailed exploitation tests under normal production conditions in the manufactory and is used in technological research work in the abrasive industry.

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ELEKTROAKUSTYCZNA METODA GENERACYJNA BADANIA ŚCIERNIC TARCZOWYCH PRZEZ POMIAR CZĘSTOTLIWOŚCI REZONANSOWYCH

Streszczenie

W oparciu o uzasadnione teoretycznie przez Kirchhoffa i potwierdzone wielu badaniami doświadczalnymi ogólnie znane związki między parametrami materiałowymi brył o określonych kształtach geometrycznych, w szczególności zaś płyt kołowych, a ich właściwościami drganiowymi, opracowano oryginalną generacyjną metodę elektroakustyczną wyznaczania twardości ściernic tarczowych płaskich. Metoda polega na wykryciu związków, jakie istnieją między twardością ściernicy określonego typu, wyrażaną w konwencjonalny sposób w stopniach twardości według kodu literowego od A do Z, a częstotliwością rezonansową $F_{n,l}$ odpowiadającą określonemu rozkładowi drgań własnych na jej powierzchni, który charakteryzuje się w ogólnym przypadku występowaniem n średnic węzłowych i l koncentrycznych okręgów węzłowych.

Pomiar częstotliwości rezonansowej badanej ściernicy tarczowej polega na jej sprzęgnięciu mechanicznym z dwoma przetwornikami elektromechanicznymi typu magnetoelektrycznego i utworzeniu w ten sposób pętli sprzężenia zwrotnego w układzie elektronicznego wzmacniacza szerokopasmowego. W powstałym układzie generacyjnym następuje wzbudzenie się drgań elektrycznych o częstotliwości $F_{n,l}$ odpowiadającej określ-

lonemu rozkładowi drgań ściernicy, o którego wyborze decyduje rozmieszczenie punktów jej podparcia oraz jej sprzęgnięcia z obu przetwornikami. Pomiaru częstotliwości rezonansowej $F_{n,1}$ dokonuje się częstotliwościomierzem liczącym.

Opracowana metoda elektroakustyczna wyznaczania twardości ściernic tarczowych wykazuje szereg zalet w porównaniu ze stosowanymi dotychczas metodami konwencjonalnymi /np. metody wciskowe, metoda piaskowa za pomocą aparatu Mc Kensena itp./, a ponadto przewyższa je pod względem dokładności, prostoty stosowania oraz czasu zużytego na pomiar. Należy podkreślić, że w odróżnieniu od wszystkich dotychczas stosowanych metod jest to metoda nieniszcząca, pozwalająca na badanie na bieżąco wszystkich egzemplarzy ściernic produkowanego lub użytkowanego asortymentu bez zmiany ich właściwości eksploatacyjnych.

W pracy przeprowadzono szczegółową analizę metody ze szczególnym uwzględnieniem warunków wzbudzania się drgań w układzie generacyjnym o elektromechanicznym sprzężeniu zwrotnym. Podano krótki opis opracowanej modelowej aparatury pomiarowej oraz przytoczono przykładowe wyniki badań eksperymentalnych, obejmujących pomiary częstotliwości drgań własnych określonych asortymentów ściernic tarczowych pod kątem wyznaczania ich twardości.