STOCHASTIC INSTABILITY OF CARBON NANOTUBES VIA NONLOCAL CONTINUUM MECHANICS

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1. Introduction

Dynamic stability of distributed systems has been an object of considerable attention over the past half of century. Numerous papers are available of isotropic and laminated beams, shafts, plates and shells under periodic and random forces. Most of papers have applied finite dimensional or modal approximations in analysis of vibration and stability. The Liapunov direct method is a quite different approach and can be successfully used to analyze continuous systems described by partial differential equations. A significant advantage is offered by the method in that the equations of motion do not have to be solved in order to examine the stability. An application of nonlocal continuum model to representative problems of nanotechnology was demonstrated in [1]. A model based on nonlocal continuum mechanics was applied to solve the buckling of multiwalled nested carbon nanotubes [2]. The detailed study on the flexural wave dispersion in single-walled nanotubes on the basis of beam models in a wide range of wave numbers was presented [3]. It was shown that the vibration analysis results based on nonlocal mechanics are in agreement withthe experimental reports in the field [4]. Based on the Donnel-Vlasov shell theory a double- elastic shell model was presented for the parametric vibrations of double-walled carbon nanotubes under time-dependent membrane forces of thermal origin [5]. The paper is concerned with the stochastic parametric vibrations of micro- and nano-rods based on Eringen's theory and Euler-Bernoulli beam theory

2. Problem formulation

The theory of nonlocal continuum mechanics assumes that the stresses at a given reference point are functions of the strain state of all points in the body. In this way the internal length scale enters into constitutive equations as a material parameter. Adopting Eringen's nonlocal elasticity [6] the nondimensionalized dynamic equation of a short nanotube has the form

(1) $$w_{,tt} + 2\beta w_{,t} + \left(f_o + f(t)\right)w_{,xx} + w_{,xxxx} + \varepsilon\left[w_{,ttxx} + 2\beta w_{,txx} + \left(f_o + f(t)\right)w_{,xxxx}\right] = 0$$

where ε - the nondimensional small scale parameter, w - the transverse beam displacement, β - viscous damping coefficient, f_o - constant axial force, $f(t)$ - time-dependent component of axial force. The instability problem is solved for simply supported edges. The trivial solution of Eq. (1) is almost sure asymptotically unstable if the measure of disturbed solution tends to infinity with probability 1.

3. Stability analysis and results

In order to examine instability we construct the energy-like Liapunov functional of the form

(2) $$V = \frac{1}{2}\int_0^1 \left[v^2 + 2\beta v w + 2\beta^2 w^2 + \varepsilon\left(v_{,x}^2 + 2\beta v_{,x} w + 2\beta^2 w^2\right) + w_{,xx}^2 - f_o\left(w_{,x}^2 + \varepsilon w_{,xx}^2\right)\right]dx$$

If the classical condition for static buckling is fulfilled the functional (2) is positive-definite and a measure of distance can be chosen as the square root of functional. If trajectories of the forces are physically realizable processes the classical calculus is applied to calculation and we have

(3) $$\frac{dV}{dt} = -2\beta V + 2U$$

where an auxiliary functional is known. In order to find a function λ satisfying inequality

(4) $$U \geq \lambda V$$

In order to find λ we solve Euler auxiliary problem and obtain the first order differential inequality with respect to functional V. The sufficient condition of the almost sure instability is as follows

(5) $$\beta \leq \left\langle \min_{n=1,2,..} \frac{\left|\beta^2 + f(t) n^2\pi^2/2\right|}{\sqrt{\beta^2 + n^2\pi^2\left[n^2\pi^2/\left(1+\varepsilon n^2\pi^2\right) - f_o\right]}} \right\rangle$$

where $\langle \ \rangle$ - mathematical averaging. Based on the formulation obtained instability domains are calculated.

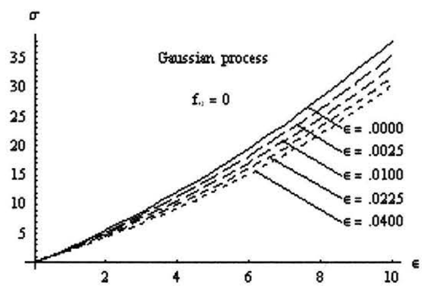

Figure 1. Changes of instability domains with dimensionless scale parameter ε.

4. References

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