

STOCHASTIC SYSTEMS WITH STIFFNESS DEGRADATION DUE TO DAMAGE ACCUMULATION.

J. Trębicki

WS Atkins-Polska, Warsaw, Poland

1. Introduction

In the presentation a coupled response-degradation problem for a multidimensional vibrating system is analyzed. The analysis allows to account for the effect of stiffness degradation (during the vibration process) on the response and, in the same time, gives the actual stress values for estimation of damage accumulating in the system.

As is well known, dynamics excitation of engineering systems (including randomly varying excitation) causes variable stress generated in mechanical/structural members and, in the consequence, irreversible changes amplitudes in the material structure. These changes, known as damage accumulation, may have different physical content. But, despite the diversity of underlying physical/mechanical phenomena, it is useful to describe them jointly within a single model relating the rate of damage evolution at time with applied stress. Models of this type operate with a certain damage measure $D(t)$, which characterizes a damage state at time t . It is usually assumed that $D(t)$ is on the interval $[0, D^*]$, where D^* denotes a critical damage, and that is a non-decreasing function of time. In some situations (e.g. in the case of fatigue accumulation) external actions and generated stresses can be conveniently related to discrete values of time (e.g. by N , the number of cycles).

Since the variable stress causing damage (and, in the consequence, stiffness degradation) are generated by a vibratory system it is natural to formulate jointly the system dynamics and damage accumulation. Such an analysis allows to account the effect of stiffness degradation during the vibration process on the response and, in the same time, gives the actual stress values for estimation of damage.

2. General formulation

The coupled response-degradation problem can be formulated in the following form:

$$\dot{\mathbf{Y}}(t) + \mathbf{F}[\mathbf{Y}(t), \mathbf{D}(t), \mathbf{X}(t, \gamma)] = 0 \quad (1)$$

$$\mathbf{Q}[\mathbf{D}(t), \mathbf{Y}(t)] = 0 \quad (2)$$

$$\mathbf{Y}(t_0) = \mathbf{Y}_0, \dot{\mathbf{Y}}(t_0) = \dot{\mathbf{Y}}_0, \mathbf{D}(t_0) = \mathbf{D}_0 \quad (3)$$

where $\mathbf{Y}(t)$ is an unknown response process, $\mathbf{D}(t)$ is a degradation process, $\mathbf{F}[\cdot]$ is the given function of indicated variables satisfying the appropriate conditions for the existence and uniqueness of the solution, $\mathbf{X}(t, \gamma)$ is the given stochastic process characterizing the excitation; $\gamma \in \Gamma$, and Γ is the space of elementary events in the basic scheme (Γ, B, P) of probability theory, $\mathbf{Q}[\cdot]$ symbolizes the relationship between degradation and response process; its specific mathematical form depends on the particular situation; $\mathbf{Y}_0, \dot{\mathbf{Y}}_0, \mathbf{D}_0$ are given initial values of the response and degradation, respectively.

An important special class of the response-degradation problems is obtained if relationship (2) takes the form of differential equation, that is, equations (1), (2) are

$$\dot{\mathbf{Y}}(t) + \mathbf{F}[\mathbf{Y}(t), \mathbf{D}(t), \mathbf{X}(t, \gamma)] = 0 \quad (4)$$

$$\dot{\mathbf{D}}(t) = \mathbf{G}[\mathbf{D}(t), \mathbf{Y}(t)] \quad (5)$$

where \mathbf{G} is the appropriate function specifying the evolution of degradation; its mathematical form is inferred from the elaboration of empirical data, or - it is derived from the analysis of the physics of the process. In equation (5) dependence on $\mathbf{Y}(t)$ can be in general regarded in more relaxed sense than it is usual. Degradation rate $\dot{\mathbf{D}}(t)$ may depend on the actual values of $\mathbf{Y}(t)$, but it can also depend on some functionals of $\mathbf{Y}(t)$; for example - on the integral of $\mathbf{Y}(\tau)$, $\tau \in [t_0, t]$. In considered here damage degradation problems the damage measure $\mathbf{D}(t)$ depends on the stress range i.e. quantity related to $\mathbf{Y}_{\max} - \mathbf{Y}_{\min}$.

3. References

- [1] Sobczyk K., Trebicki J., Stochastic dynamics with fatigue-induced stiffness degradation, Probabilistic Eng. Mechanics, 15(2000) 91-99.
- [2] Barber T.T., Wen Y.K., (1981), Random vibration of hysteretic degrading systems, J. Eng. Mech. Div. ASCE, vol. 107 (EM6), 1069-1087.
- [3] Biswas M., Pandey A.K., Samman M.M., (1990), Diagnostic experimental spectral/modal analysis of highway bridge, J. Analytical and Exper. Modal Analysis, 5, 234-258.
- [4] Cawley P., Adams R.D. (1979), The location of defects in structures from measurement of natural frequencies, J. Strain Analysis, 14, 49-57.
- [5] Chondras T.G., Dimarogonas A.D., Yao J., (1998), A continuous cracked beam vibration theory, J. Sound Vibr., 215(1), 17-34.
- [6] Collins K.R., Plant R.H., Wauer J., (1992), Free and forced longitudinal vibrations of a cantilevered bar with a crack, ASME Journ. of Vibr. Acoust. Stress and Reliability in Design, 114, 171-177.
- [7] Dimarogonas A.D., (1976), Vibration Engineering, West Publishers, St. Paul.
- [8] Grigoriu M., 1990, Reliability of degrading dynamic systems, Struct.Safety, vol. 8, pp. 345-351;
- [9] Grossmayer R.L., (1979), Elastic-plastic oscillators under random excitation, J. Sound Vibr., 65(3), 353-379.
- [10] Kondo Y., (1989), Prediction of fatigue crack initiation life based on a pit growth, Corrosion, 45(1), 7-11.
- [11] Lemaitre J., Chaboche J.-L., (1990), Mechanics of solid materials, Cambridge University Press.
- [12] Lin Y.K., Cai G.Q., (1995), Advanced Probabilistic Dynamics, McGraw Hill, New York.
- [13] Sobczyk K., (1991), Stochastic Differential Equations with Applications to Physics and Engineering, Kluwer, Dordrecht-Boston.
- [14] Sobczyk K., Spencer B.F., (1992), Random Fatigue: From Data to Theory, Academic Press, Boston
- [15] Sobczyk K., Trebicki J., (1998), Stochastic response of degrading elastic systems, Proc. of IUTAM Symposium on "Rheology of Defected Bodies", (Beijing-China, Sept. 1997) Eds. Wang R., Kluwer Acad. Publ., (in print).
- [16] Soong T.T., Grigoriu M., (1993), Random Vibrations of Structural and Mechanical Systems, Prentice Hall, New Jersey.
- [17] Wen Y.K., (1986), Stochastic response and damage analysis of inelastic structures, Prob. Eng. Mech., vol. 1, No. 1, 69-57.