FIRST PASSAGE FAILURE OF A LINEAR OSCILLATOR UNDER ADDITIVE AND MULTIPLICATIVE RANDOM EXCITATIONS

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1. INTRODUCTION

The focus of our paper is the following linear oscillator under both additive and multiplicative random excitations:

$$\ddot{X} + \omega_0[2\zeta + W_2(t)]\dot{X} + \omega_0^2[1 + W_1(t)]X = W_3(t), \tag{1}$$

where $W_j(t))$, $j = 1, 2, 3$, are wideband stationary processes with zero mean values. This model was studied by Ariaratnam and Tam [1] under the assumption that ζ is of order ϵ and the $W_j(t))$ are of order $\sqrt{\epsilon}$, where ϵ is a small parameter. By applying the stochastic averaging procedure, it was argued that the amplitude process $A(t) = \left(X^2 + \dot{X}^2/\omega_0^2\right)^{1/2}$ is approximately a Markov diffusion process governed by the (Itô) stochastic differential equation (SDE)

$$dA = m(A)dt + \sigma(A)dB(t)\,. \tag{2}$$

The drift coefficient $m(A)$ and the diffusion coefficient $\sigma(A)$ are given by the equations,

$$m(A) = -\alpha A + \frac{\delta}{2A}\,, \tag{3}$$

$$\sigma(A) = (\gamma A^2 + \delta)^{1/2}\,, \tag{4}$$

in which

$$\alpha = \zeta\omega_0 - \frac{\pi\omega_0^2}{8}\left[2\Phi_{22}(0) + 3\Phi_{22}(2\omega_0) + 3\Phi_{11}(2\omega_0) - 6\Psi_{12}(2\omega_0)\right]\,, \tag{5}$$

$$\delta = \frac{\pi}{\omega_0^2}\Phi_{33}(\omega_0)\,, \tag{6}$$

$$\gamma = \frac{\pi\omega_0^2}{4}\left[2\Phi_{22}(0) + \Phi_{22}(2\omega_0) + \Phi_{11}(2\omega_0) + 2\Psi_{12}(2\omega_0)\right]\,, \tag{7}$$

and

$$\Phi_{ij}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{E}[W_i(t)W_j(t+\tau)]\,\cos(\omega\tau)\,d\tau\,,\ \ i,j = 1,2,3\,, \tag{8}$$

$$\Psi_{ij}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{E}[W_i(t)W_j(t+\tau)]\,\sin(\omega\tau)\,d\tau\,,\ \ i,j = 1,2,3\,. \tag{9}$$

Ariaratnam and Tam [1] showed that the expected time $\langle T_f \rangle$ to first failure of the amplitude process $A(t)$ is given by the formulas

$$\langle T_f \rangle = \frac{1}{\eta\gamma} \int_{a_0}^{a_c} \frac{1}{u} \big[\big(1 + \frac{\gamma}{\delta} u^2\big)^\eta - 1 \big] \, du \,, \;\; \eta = \frac{\alpha}{\gamma} + \frac{1}{2} \neq 0 \tag{10}$$

$$\langle T_f \rangle = \frac{1}{\gamma} \int_{a_0}^{a_c} \frac{1}{u} \ln \big(1 + \frac{\gamma}{\delta} u^2\big) \, du \,, \;\; \eta = 0 \tag{11}$$

Here a_0 denotes the initial condition and a_c the critical level ($a_0 < a_c$). This approach would usually represent an approximation in the sense that failure for the original problem would typically be when $X(t)$ exceeds a critical threshold x_c. An approximate solution for this is obtained by studying the exceedance of $a_c = x_c$ by the amplitude process $A(t)$.

Using numerical path integration we have calculated the reliability function associated with the linear oscillator model in Eq. (1) for a range of parameter values. Since this can be done for any choice of parameter values, it provides a means of studying the limitations of the amplitude diffusion model adopted in [1], and thereby also the limitations of stochastic averaging.

The reliability is defined in terms of the displacement response process $X(t)$ in the following manner, assuming that all events are well defined,

$$R(\,T \,|\, x_0, 0, t_0) = \text{Prob}\{x_l < X(t) < x_c;\, t_0 < t \leq T | X(t_0) = x_0,\, Y(t_0) = 0\}, \tag{12}$$

where x_l, x_c are the lower and upper threshold levels defining the safe domain of operation. It has been shown [2] that

$$R(\,T \,|\, x_0, 0, t_0) \approx \int_{-\infty}^{\infty} \int_{x_l}^{x_c} \cdots \int_{-\infty}^{\infty} \int_{x_l}^{x_c} \prod_{j=1}^{n} p(z_j, t_j | z_{j-1}, t_{j-1}) \, dz_1 \cdots dz_n, \tag{13}$$

which is the path integration formulation of the reliability problem. Here, $p(z, t|z', t')$ denotes the transition probability density function of the state space vector process $Z(t) = (X(t), Y(t))^T = (X(t), \dot{X}(t))^T$, and $t_j = t_0 + j\Delta t$, $j = 1, \ldots, n$, and $\Delta t = (T - t_0)/n$.

The complementary probability distribution of the time to failure T_f, i.e. the first passage time, is given by the reliability function. The mean time to failure $\langle\, T_e \,\rangle$ can thus be calculated by the equation

$$\langle\, T_f \,\rangle = \int_0^{\infty} R(\,\tau \,|\, x_0, 0, t_0) \, d\tau \tag{14}$$

The results obtained by Eqs. (10), (11) and from Eq. (14) by path integration, can then be compared. This will shed some light on the performance of stochastic averaging methods.

2. REFERENCES

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