THE INFLUENCE OF STATISTICAL FREQUENCY SCATTER ON PEDESTRIAN DESIGN LOADS FOR FOOTBRIDGES

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1. Background

Many modern footbridges are light structures in the sense that the non-permanent load is rather large compared to the permanent load, constituted by the weight of the bridge. In recent years footbridges have become longer, and more slender and flexible designs have been adopted. As a result, dynamic excitation – typically from the pedestrians using the bridge – have become a major design issue. A typical design procedure – originally introduced in [1] and later refined in [2] and [3] – consists in a Fourier expansion of typical footfall records, followed by evaluation of the response to the harmonic component closest to the base frequency of the bridge. The corresponding response acceleration of the original structure is of the form

$$\ddot{\mathbf{u}} = \omega_j^2 r_j \,\mathbf{u}_j = \frac{q_j}{2\varsigma_j m_j}\,\mathbf{u}_j$$

In this formula $\ddot{\mathbf{u}}$ is the physical acceleration, and r_j is the amplitude of the mode-shape vector $\mathbf{u}_j$ with angular frequency ω_j. The result corresponds to a simple resonance, where the load is represented by the modal load intensity q_j, while the denominator is the product of the modal damping ratio ς_j and the modal mass m_j. This formulation leads to a very simple design procedure, because it has been shown [4] that the resonant response of a mode shape with a tuned mass damper can be represented by an equivalent damping ratio, which is half the damping ratio applied in the tuned mass damper.

The advantage of the above argument is its simplicity in design, see e.g. [3]. However, it will often be a rather conservative design procedure, because it is based on the assumption that the excitation is concentrated in a very narrow frequency interval around the natural frequency of the structure. Experimental measurements of the loading from walking pedestrians indicates that the frequency has a coefficient of variation around 0.06-0.09 [5,6]. Thus, the spread of the loading frequency is much larger than the width of the resonance peak of the original structure. This means that the severity of the loading of the original bridge without additional tuned mass absorber(s) is overestimated, and that the response of the bridge after installation may be inaccurately determined.

2. Stochastic design model

The literature contains numerous formulae giving slightly different ‘optimal’ damping and frequency parameters for tuned mass absorbers. However, it has been demonstrated via analytical solutions for the extreme case of white noise excitation, that calibration of the tuned mass absorber for a resonant load gives the same response to within a fraction of a percent [7]. This observation leads to a simple procedure for evaluating the effect of frequency spreading on the bridge response – both in the original state and after installation of a tuned mass absorber.

The loading including a representative frequency spread can be represented by a simple rational function centred around the mean footfall frequency and with an appropriate coefficient of variation. When extended symmetrically to include negative frequencies this leads to a spectral density that can be generated from white noise by a second order filter. The system, consisting of

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the structure – here represented by an idealized single-degree-of-freedom system – and the tuned mass, is represented by a coupled two-degree-of-freedom system. In total the system including the tuned mass absorber can be described in terms of six state-variables driven by a white noise process. As mentioned above the tuned mass absorber parameters can be calibrated as if the load were harmonic, and thus a parametric analysis of the effect of the frequency characteristics of the loading can be carried out by computing the covariances of the system by use of Lyapunov's equation. The paper will show representative results and discuss the influence on realistic design situations for footbridges.

3. References

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