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W. Nadołski, A. Pielorz

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Władysław Nadolski
Amalia Pielorz
Zakład Układów Mechanicznych

SHEAR WAVES IN BUILDINGS SUBJECT TO SEISMIC LOADING

1. Introduction

In real buildings during seismic loading occur complex physical effects and processes which have a significant influence on the displacements and strains of the building elements. An exact description of these effects and processes is virtually impossible. In view of this in technical problems we shall confine ourselves to the investigation of simplified physical effects and processes which can be described by the known mathematical apparatus. It is essential here to choose variable parameters which on the one hand describe with sufficient exactness the physical effects and processes in real buildings, and on the other hand make it possible to obtain simple mathematical relationships. For this purpose a physical model representing a simplified structure of the building has been adopted. The choice of a possibly adequate physical model is facilitated by the gathered information concerning the character and magnitude of the values of strains and displacements of the building elements. This concerns also information on the energy dissipation properties and the character of the seismic forces. It should be emphasized here that the correctness of the obtained theoretical results can be verified only experimentally by investigating a real building.

The dynamic investigations of buildings subject to seismic loading hitherto has been, as it follows from the literature [2,5,6,8] , carried out in principle by means of the methods worked out in vibration theory. In many cases, however, they are unsatisfactory, especially when we want to investigate the strains and displacements of the building columns in different cross-sections.

In this paper it has been suggested to use one-dimensional elastic waves to determine the displacements and strains of the column cross-sections in a multi-storey building with seismic forces with the simultaneous consideration of damping. Damping in real buildings is an extremely complex process. Although many theories describing damping phenomena are known, however, they are not always suitable for structural analysis. In the paper we propose to take into consideration damping through equivalent damping.

2. The physical model of a multi-storey building.

Our considerations will concern a multi-storey building with steel columns rigidly connected with the foundation and floors. We assume that the foundation and floors are undeformable and are displaced by plane motion during the time of seismic loading. The masses of the foundation and floors are denoted respectively by: m_0 - mass of the foundation, m_i - masses of the ceilings, $i=1,2,\ldots,N$.

We assume that all points of the foundation of the building have the same horizontal accelerations as the "surroundings" of the foundation. A graph of these accelerations is given as an example in Fig.1, [1] . From Fig.1 it follows that the acceleration curve $\ddot{y}/g(t)$ is irregular. Therefore in the calculations we approximate it suitably by segments of straight lines as shown in Fig.2 or 3. We carry out this approximation depending on the oscillation frequency of the horizontal acceleration given in Fig.1. When the period of oscillation is larger or slightly different from the fundamental period of free vibrations of the building, the horizontal accelerations, Fig.1, can the be approximated by the following function

$$\ddot{y}_{cal}(t) = \sum_{k=0}^{m} H(t-t_k)\left[a_k + b_k\,(t-t_k)\right] . \qquad (1)$$

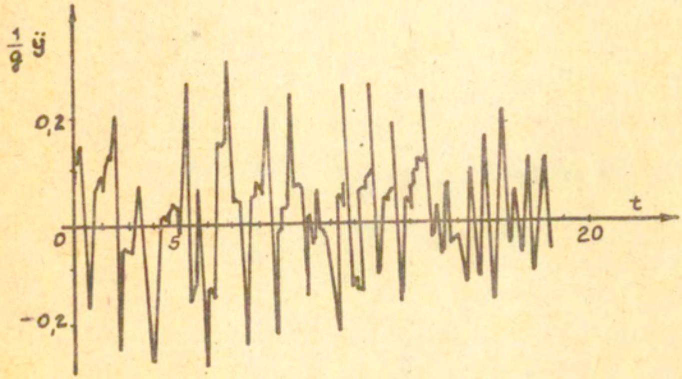


Fig.1. Diagram of relative acceleration $\ddot{y}/g$, [1]

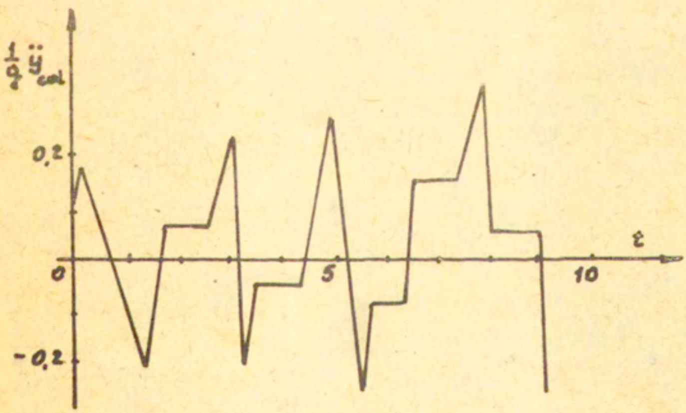


Fig.2. Approximated graph of acceleration $\ddot{y}_{cal}/g$

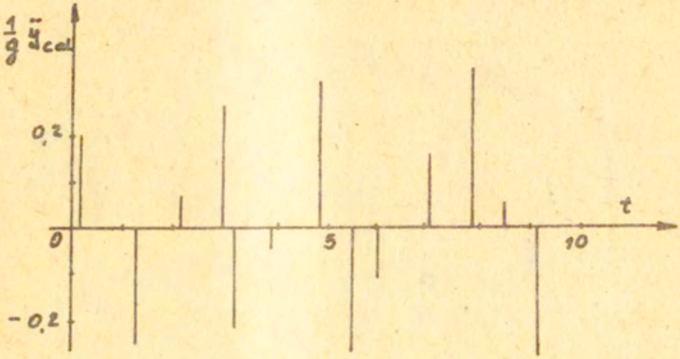


Fig.3. Approximated graph of acceleration $\ddot{y}_{cal}/g$

However, when the period is much smaller than the fundamental period of free vibrations of the building we assume the graph of horizontal acceleration to be of the Dirac type

$$\ddot{y}_{cal}(t) = \sum_{k=0}^{m} a_k \delta(t-t_k). \qquad (2)$$

In both cases of kinematic forces other kinds of waves will occur, and thereby different stresses and displacements of the column cross-sections. Ordinary waves will appear in the building columns with kinematic forces described by function (1) , and waves of strong discontinuity with forces of the Dirac type (2) .

We assume that the vertical columns between the foundation and first ceiling and between the floors of the building, subject to seismic loading, have the same shapes as regards elastic strains, i.e. the velocities, strains and displacements for the given height of a building are the same [1] . We assume that all the column cross-sections during seismic loading remain plane cross-sections parallel to the cross-sections at the junction points of the columns with the floors , Fig.4.

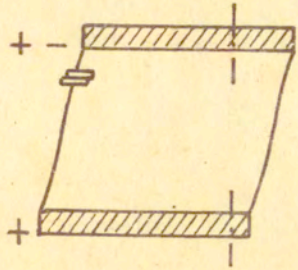


Fig.4. Scheme of elastic deformations of columns

These columns are characterized by the following parameters: G - shear modulus of elasticity, A - column cross-sectional area, k' - shear coefficient, ϱ - density, l - height of the column.

With the above assumptions we adopt for considerations the physical model of a multi-storey building with continuously distributed parameters [3] which is shown in Fig.5. In this model we consider also the equivalent damping in selected cross-sections of the columns. Damping forces loading the foundation with mass m_0 and the floors with masses m_i, $i=1,2,\ldots,N$, are assumed in the form

$$P_{Di}(t) = - D_i \frac{y_{i+1}(x,t)}{\partial t} \quad \text{for} \quad x=il, \quad i=0,1,\ldots,N-1$$

$$P_{DN}(t) = - D_N \frac{y_N(x,t)}{\partial t} \quad \text{for} \quad x=Nl \tag{3}$$

where $y_i(x,t)$ is the transverse displacement of the i-th column, D_i, $i=0,1,\ldots,N$, are the equivalent damping coefficients of the viscous type. The damping forces $P_{Di}(t)$ of selected column cross-sections take into consideration internal damping (continuously distributed) and external damping. We therefore assume that the external damping occurring between

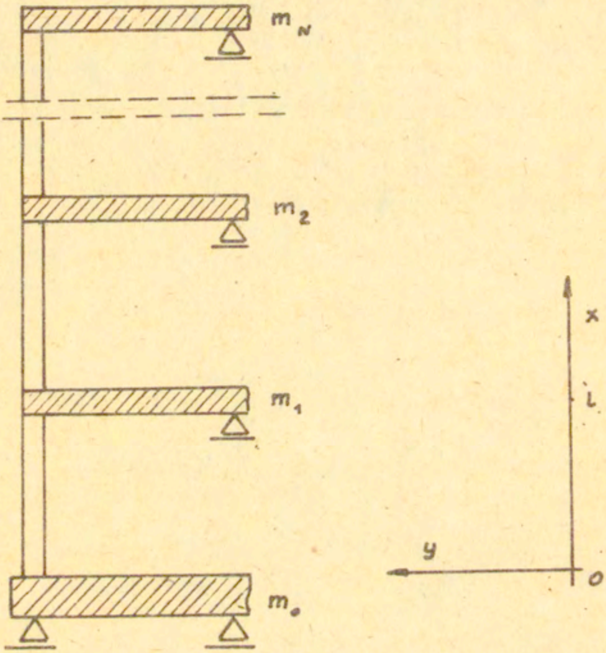


Fig.5. Model of a multi-storey building

the elements which are displaced in relative motion during seismic loading is also of the viscous type just as the internal damping.

3. Equations of motion and recurrence formulae

Investigation of the transverse displacements and strains of the building columns with the accepted assumptions can be reduced to the solution of N wave equations

$$\frac{\partial^2 y_i(x,t)}{\partial t^2} - c^2 \frac{\partial^2 y_i(x,t)}{\partial x^2} = 0\,, \qquad i=1,2,\ldots,N \tag{4}$$

with the following boundary conditions

$$k'AG\frac{\partial y_1(x,t)}{\partial x} - m_0\ddot{y}_{cal} - D_0\frac{\partial y_1(x,t)}{\partial t} = 0 \quad \text{for } x=0$$

$$y_i(x,t) = y_{i+1}(x,t) \quad \text{for } x=il, \ i=1,2,\ldots,N-1$$

$$k'AG\left(\frac{\partial y_{i+1}(x,t)}{\partial x} - \frac{\partial y_i(x,t)}{\partial x}\right) - m_i\frac{\partial^2 y_{i+1}(x,t)}{\partial t^2} - \tag{5}$$

$$- D_i\frac{\partial y_{i+1}(x,t)}{\partial t} = 0 \quad \text{for } x=il, \ i=1,2,\ldots,N-1$$

$$k'AG\frac{\partial y_N(x,t)}{\partial x} + m_N\frac{\partial^2 y_N(x,t)}{\partial t^2} - D_N\frac{\partial y_N(x,t)}{\partial t} = 0 \quad \text{for } x=Nl$$

and initial conditions

$$y_i(x,t) = \frac{\partial y_i(x,t)}{\partial t} = 0 \quad \text{for } t=0, \tag{6}$$

where $c^2 = k'G/\varrho$.

Upon the introduction of dimensionless quantities

$$\bar{x} = \frac{x}{l}\,, \quad \tau = \frac{ct}{l}\,, \quad \bar{y}_i = \frac{y_i}{y_0}\,, \quad \ddot{\bar{y}}_{cal} = \ddot{y}_{cal}\,l^2\frac{1}{c^2 y_0}\,,$$
$$k_i = \frac{A\varrho l}{m_i}\,, \quad \bar{D}_i = \frac{D_i c}{k'AG}\,, \quad i=0,1,\ldots,N, \tag{7}$$

the relations (4) - (6) for the acceleration $\ddot{y}_{cal}$ described by function (1) assume the form

$$\frac{\partial^2 \bar{y}_i(\bar{x},\tau)}{\partial \tau^2} - \frac{\partial^2 \bar{y}_i(\bar{x},\tau)}{\partial \bar{x}^2} = 0\,, \tag{8}$$

$$k_0 \frac{\partial \bar{y}_1(\bar{x},\tau)}{\partial \bar{x}} - \sum_{k=0}^{m} H(\tau-\tau_k)\left[\bar{a}_k + \bar{b}_k(\tau-\tau_k)\right] -$$

$$- k_0 \bar{D}_0 \frac{\partial \bar{y}_1(\bar{x},\tau)}{\partial \tau} = 0 \qquad \text{for } \bar{x}=0$$

$$\bar{y}_i(\bar{x},\tau) = \bar{y}_{i+1}(\bar{x},\tau) \qquad \text{for } \bar{x}=i, \quad i=1,2,\ldots,N-1$$

$$k_i\left(\frac{\partial \bar{y}_{i+1}(\bar{x},\tau)}{\partial \bar{x}} - \frac{\partial \bar{y}_i(\bar{x},\tau)}{\partial \bar{x}}\right) - \frac{\partial^2 \bar{y}_{i+1}(\bar{x},\tau)}{\partial \tau^2} - \tag{9}$$

$$- k_i \bar{D}_i \frac{\partial \bar{y}_{i+1}(\bar{x},\tau)}{\partial \tau} = 0 \qquad \text{for } \bar{x}=i, \quad i=1,2,\ldots,N-1$$

$$k_N \frac{\partial \bar{y}_N(\bar{x},\tau)}{\partial \bar{x}} + \frac{\partial^2 \bar{y}_N(\bar{x},\tau)}{\partial \tau^2} + k_N \bar{D}_N \frac{\partial \bar{y}_N(\bar{x},\tau)}{\partial \tau} = 0 \qquad \text{for } \bar{x}=N$$

$$\bar{y}_i(\bar{x},\tau) = \frac{\partial \bar{y}_i(\bar{x},\tau)}{\partial \tau} = 0 \qquad \tau = 0 \tag{10}$$

We seek the solutions of Eqs. (8) in the form

$$\bar{y}_i(\bar{x},\tau) = f_i(\tau-\bar{x}) + g_i(\tau+\bar{x}-2(i-1)) \tag{11}$$

where the function f_i represents a wave formed in the i-th column as a result of kinematic force (1) going to the right, whereas the function g_i represents a wave going to the left. In formula (11) it was taken into consideration that the first perturbation in the i-th column occurs in the cross-section $\bar{x}=i-1$ at the time $\tau=i-1$. We assume moreover that for negative arguments the functions f_i, g_i are equal zero.

Substituting (11) to the boundary conditions (9) we obtain the following set of 2N ordinary differential equations for the functions $f_i(z)$, $g_i(z)$

$$g_i'(z) = -f_i'(z-2) + f_{i+1}'(z-2) + g_{i+1}'(z-2), \quad i=1,2,\dots,N-1$$

$$g_N''(z) + r_N g_N'(z) = -f_N''(z-2) + r_{N+1} f_N'(z-2) \tag{12}$$

$$f_1'(z) = \frac{1-\bar{D}_0}{1+\bar{D}_0}\, g_1'(z) - \frac{1}{k_0(1+\bar{D}_0)} \sum_{k=0}^{m} H(z-\tau_k)[\bar{a}_k + b_k(z-\tau_k)]$$

$$f_i''(z) + r_{i-1} f_i'(z) = -g_i''(z) - k_{i-1}\bar{D}_{i-1} g_i'(z) + 2k_{i-1} f_{i-1}'(z),$$

$$i=2,3,\dots,N,$$

where

$$r_i = k_i(2 + \bar{D}_i), \quad i=1,2,\dots,N-1$$

$$r_N = k_N(1 + \bar{D}_N), \quad r_{N+1} = k_N(1 - \bar{D}_N). \tag{13}$$

The functions of the right hand sides of the first N equations of the set (12) have the arguments shifted by 2. Thus it is necessary to solve Eqs. (12) in successive intervals of the argument z beginning from even numbers. As assumed the functions f_i, g_i are equal zero for negative arguments therefore when solving Eqs. (12) in the given sequence the right sides of these equations are always known.

The set of Eqs. (12) with respect to the derivatives of the functions f_i, g_i is composed of N algebraic equations and N ordinary differential equations of the first order with constant coefficients. The latter have the form of the equation

$$y' + By = C(x) \tag{14}$$

whose solution for $x \geq x_0$ is as follows

$$y(x) = e^{-B(x-x_0)}\left[\int_{x_0}^{x} C(z)\, e^{B(z-x_0)} dz + y(x_0)\right]. \tag{15}$$

The method of solving Eqs. (12) with respect to $f_i'(z)$, $g_i'(z)$ is therefore simple: we solve them in the given sequence in the successive intervals of argument z beginning with even numbers using relationship (15) in equations of the type (14). In such a way an example of the impact of a rigid body upon a fixed rod was solved in [4] for $z \leq 8$ and in [7] the solution for a single-storey building without consideration of damping was given.

From a comparison of (12) and (15) it results that the solution of Eqs. (12) depends exponentially on the constants r_i. This solution will be different for the same constants r_i and different for various r_i. We give below the solutions of Eqs. (12) for both of the mentioned cases of r_i:

I: $r_i = r, \quad i=1,2,\ldots,N$

$\underline{2n \leq z < 2(n+1)}, \quad n=0,1,\ldots$

$$g'_{in}(z) = g'_{i,n-1}(z) + H(n-1)\sum_{k=0}^{m} H(z_{nk})\Big[g^{kin} + g_L^{kin} z_{nk} + \exp(-rz_{nk})\sum_{j=0}^{2n+i-3} g_j^{kin} z_{nk}^j\Big], \quad i=1,2,\ldots,N \tag{16}$$

$$f'_{1n}(z) = f'_{1,n-1}(z) + \sum_{k=0}^{m} H(z_{nk})\Big[f^{k1n} + f_L^{k1n} z_{nk} + \exp(-rz_{nk})\sum_{j=0}^{2n-2} f_j^{k1n} z_{nk}^j\Big] \tag{17}$$

$$f'_{in}(z) = f'_{i,n-1}(z) + \sum_{k=0}^{m} H(z_{nk})\Big[f^{kin} + f_L^{kin} z_{nk} + \exp(-rz_{nk})\sum_{j=0}^{2n+i-2} f_j^{kin} z_{nk}^j\Big], \quad i=2,3,\ldots,N \tag{18}$$

II: $r_i \neq r_j$ for $i \neq j, \quad i,j=1,2,\ldots,N$

$\underline{2n \leq z < 2\,n+1}, \quad n=0,1,\ldots$

$$g'_{in}(z) = g'_{i,n-1}(z) + \sum_{k=0}^{m} H(z_{nk})[g^{kin} + g_L^{kin} z_{nk} + \tag{19}$$

$$+ \sum_{s=1}^{i} \exp(-r_s z_{nk}) \sum_{j=0}^{n-1} g_{sj}^{kin} z_{nk}^j + \sum_{s=i+1}^{N} \exp(-r_s z_{nk}) \cdot$$

$$\sum_{j=0}^{n-s+i-1} g_{sj}^{kin} z_{nk}^j], \quad i=1,2,\ldots,N$$

$$f'_{in}(z) = f'_{i,n-1}(z) + \sum_{k=0}^{m} H(z_{nk})[f^{kin} + f_L^{kin} z_{nk} +$$

$$+ \sum_{s=1}^{i-1} \exp(-r_s z_{nk}) \sum_{j=0}^{n} f_{sj}^{kin} z_{nk}^j + \sum_{s=i+1}^{N} \exp(-r_s z_{nk}) \cdot \tag{20}$$

$$\cdot \sum_{j=0}^{n-s+i-1} g_{sj}^{kin} z_{nk}^j], \quad i=1,2,\ldots,N$$

where $z_{nk} = z - 2n - \tau_k$ and $f'_{i,n-1}(z) = g'_{i,n-1}(z) = 0$ for $n=0$, $i=1,2,\ldots,N$.

Derivatives of the functions $f_i(z)$, $g_i(z)$ are denoted in the formulae (16) - (20) respectively by $f'_{in}(z)$, $g'_{in}(z)$ depending on the largest even number smaller than the value of the actual argument z. As follows from the formulae (16) - (20) the functions $f'_i(z)$, $g'_i(z)$ depend exponentially on the constants r_i, that is, on k_i and $\bar{D}_i$. The variable coefficients appearing at the exponential functions are power series with respect to the expression z_{nk}. All the constants in the formulae (16) - (20) have three upper indices. The first one is related to the k-th component of kinematic force (1), the second one to the i-th building column, and the third one to the interval to which the actual argument z belongs. The constants appearing in the power series of the formulae (16) - (18) have one lower index related to the j-th power of the expression z_{nk}. Analogously the constants in the formulae (19), (20) have two lower indices. The first one informs about the dependence upon the constant r_s, and the second one is a power index of the expression z_{nk}. Among the remaining constants in the formulae (16) - (20) the constants without a lower index are free terms, whereas the constants with the index L are beside the linear term.

From the formulae (16) - (20) it results that in order to determine the derivative of the corresponding function for an argument z from the established interval it is necessary to know the form of the derivative of this function in all of the previous intervals of argument z . The first component of these formulae informs us about it.

The constants appearing in the formulae (16) - (20) can be obtained by solving Eqs. (12) respectively with the same and with different r_i for n=0,1,... . The recurrence formulae, with respect to n , according to which these constants are determined, can be found in the Appendix.

The solution of the problem (4) - (6) has been obtained in the form (16) - (20) for kinematic force described by function (1) . Similarly a solution of this problem with force of the Dirac type (2) can be obtained.

The relationships (16) - (20) and (11) can be used to determine the velocities and strains of any cross-sections of columns in an N-storey building caused by seismic loading at any moment of time. In order to determine the displacements the solutions for the derivatives of the functions f_i, g_i should be integrated.

4. Numerical results

Numerical calculations were carried out for a single-storey building with $y_0=1$ [m] , l=3 [m] , c=3000 [m/s] , $\bar{a}_k=0$, $\bar{b}_0=8\cdot 10^{-6}$, $\bar{b}_k=(-1)^k\cdot 16\cdot 10^{-6}$, k=1,2,...,m, for a time interval $\tau \in \langle 0, 100\rangle$, where y_0 is a constant value of displacement, (7).

From the Appendix it follows that the coefficients f_j^{kin}, g_j^{kin}, f_{sj}^{kin}, g_{sj}^{kin} appearing in the formulae (16) - (20) depend on the following constants

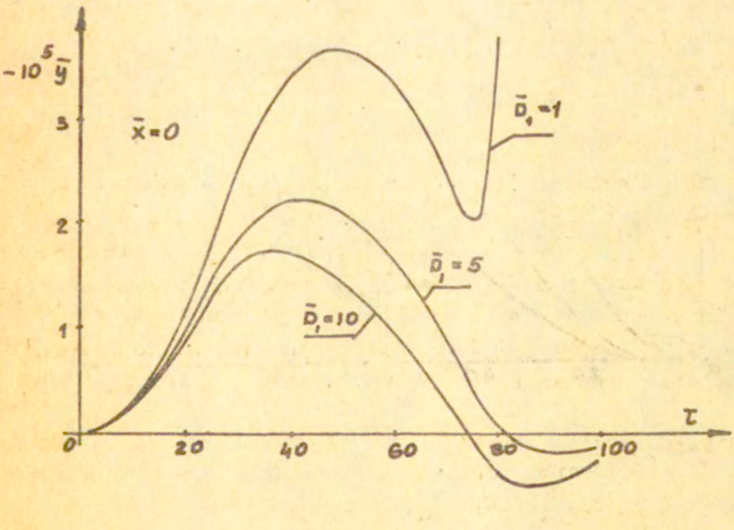
$$r_{N+1}/r,\quad r+r_{N+1},\quad 2k_i/r,\quad r_{N+1}/r_N,\quad (r_s+r_{N+1})/(r_s-r_N)\ , \tag{21}$$

$2k_i/r_i$, $k_i\bar{D}_i/r_i$, $(k_{i-1}\bar{D}_{i-1}-r_s)/(r_s-r_i)$, $2k_{i-1}/(r_s-r_i)$.

During numerical calculations for the single-storey building, i.e. for N=2, the constants (21) were found to have a significant influence on the displacements of the investigated cross-sections of the columns in the building. At absolute values of the constants (21) much higher than unity high values of the displacements are obtained. However, at values close to unity the displacements are relatively low. This is determined by the constant parameters of the building k_i and by the coefficients of equivalent damping $\bar{D}_i$, $i=1,2,\ldots,N$.

In particular for the same k_i and the same $\bar{D}_i$ (e.g.for the same masses of the floors and the same damping elements) an unstable solution for displacements of the column cross-sections was obtained. In this case some of the constants (21) were equal ± 2, ± 3.

In Fig. 6 are plotted the graphs of displacements of the cross-sections $\bar{x}=0,1,2$ of the columns of a single-storey building for $r_1=r_2=r$ (i.e. $k_1(2+\bar{D}_1) = k_2(1+\bar{D}_2)$, (13)) , with equal masses of the floors $k_1=k_2=0.4$ and for $k_0=0.15$,



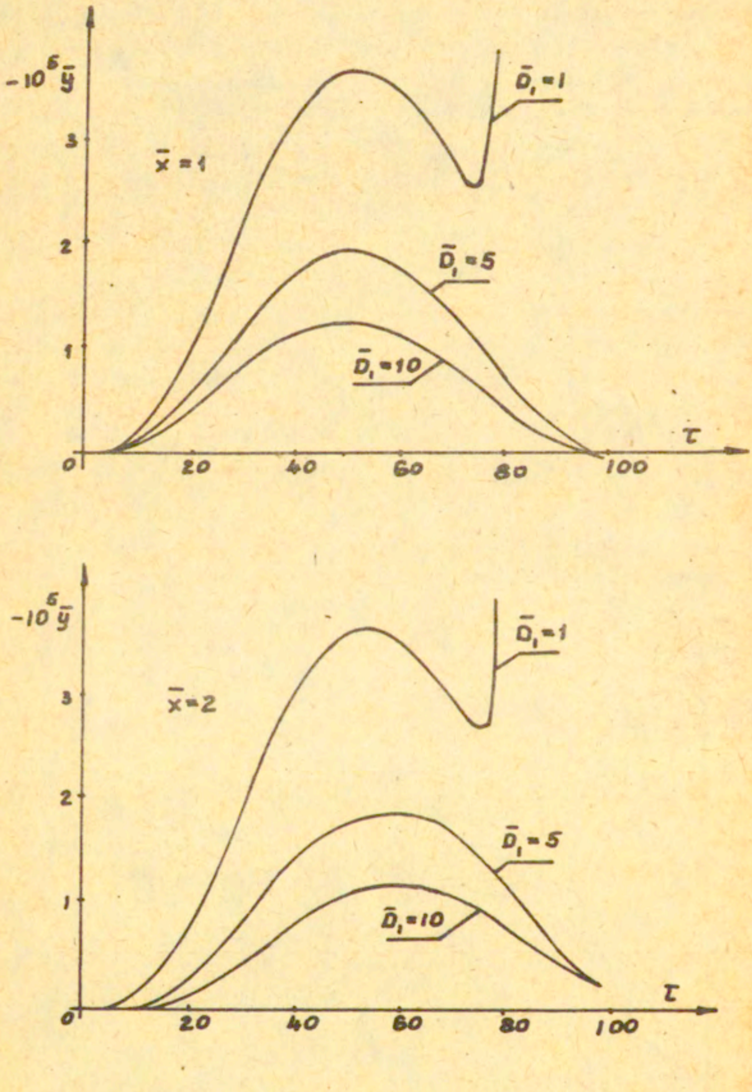
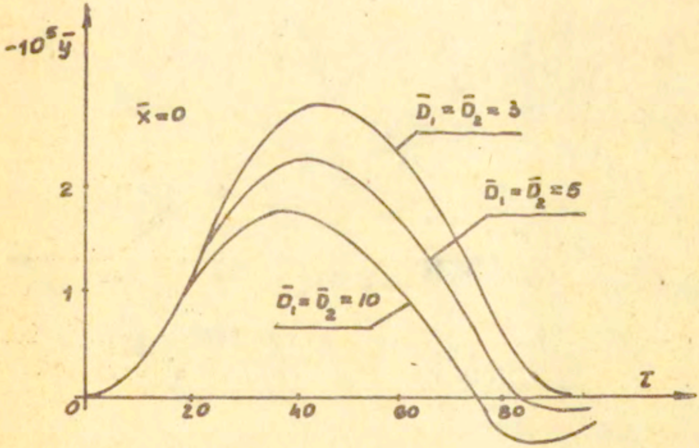


Fig.6. Graphs of displacements for $r_1=r_2=r$, $k_0=0.15$, $k_1=k_2=0.4$, $\bar{D}_0=6$, $\bar{D}_1=1,5,10$

$\bar{D}_0=6$, $\bar{D}_1=1,5,10$ and upon integration of the functions (16) - (18) . From Fig.6 it results that in the investigated time interval the displacements of the column cross-sections for $\bar{D}_1=5,10$ are of an oscillational character and decrease with increase of damping. However, for $\bar{D}_1=1$ and $\tau > 75$ the displacements very rapidly increase(the solution is unstable).

In Fig.7 are plotted correspondingly the graphs of the displacements for $r_1 \neq r_2$ with the same coefficients of equivalent damping $\bar{D}_1=\bar{D}_2=3,5,10$ with different masses of the ceilings $k_1=0.6$, $k_2=0.15$ and for $k_0=0.15$. The functions (19) and (20) were used here. The character of the obtained displacements is also oscillational and with increase of damping these displacements decrease.

Similarly graphs of the displacements can be obtained for many other values of the parameters k_1, $\bar{D}_1$ and for other cross-sections of the columns in a single-storey building such as in [7] without damping. One must bear in mind only that the values of the constants (21) should be close to unity. This remark about the constants (21) refers also to more than single-storey buildings.



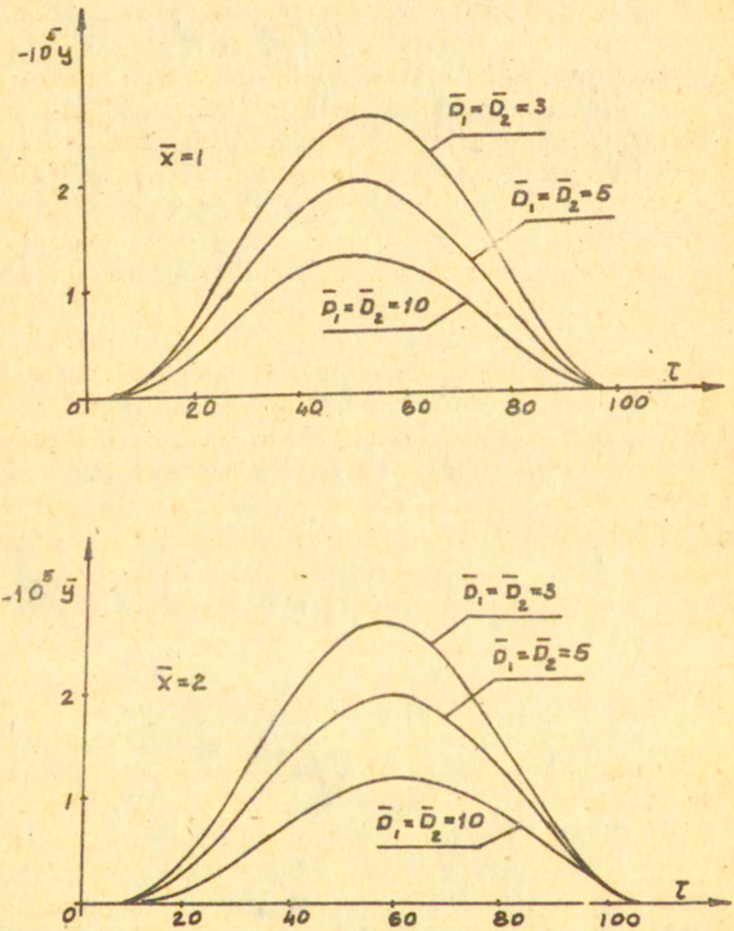


Fig.7. Graphs of displacements for $r_1 \neq r_2$, $k_0=0.15$, $k_1=0.6$, $k_2=0.15$, $\bar{D}_0=6$, $\bar{D}_1=\bar{D}_2=3,5,10$

5. Conclusions

The above considerations referred to the determination of the displacements of cross-sections of steel columns in a multi-storey building with the use of one-dimensional elastic waves taking into account reflections. An analysis was carried out for a model of the building presented in Fig.5. With accelerations of the building foundation described by formula (1) and equivalent damping of the viscous type described by formula (3) analytical solutions were obtained in the form of recurrence formulae for the functions f_i, g_i by means of which the displacements of the cross-sections of the building columns can be determined at any moment of time.

It has been noted that for absolute values of the constants (21) close to unity the displacements of the column cross-sections are relatively small. This is determined by the constant parameters k_i, $\bar{D}_i$, $i=1,2,...,N$, of the investigated system. In particular the solutions described by the formulae (16) - (18) or (19) , (20) are unstable when simultaneously the masses of the floors are equal and the values of the coefficients of equivalent damping are equal. Hence it follows that the condition for the constants (21) may be found to be very useful for the designing of multi-storey buildings subject to seismic loadings.

In dynamic investigations of a building the essential problem is to describe the internal damping (continuously distributed) and external damping that occurs between the moving elements of a building subject to seismic loading. Although numerous theories describing damping phenomena are known, however, they are not always suitable for concrete calculations. In view of this the paper takes into consideration damping through equivalent damping at the junction points of the columns with the foundation and the ceilings.

As an example calculations have been carried out for a single-storey building. Whereas all the analytical considerations have been carried out for acceleration of the foundation described by Heaviside's function (1) , Fig.2. Analogical considerations may be carried out with accelerations of the foundation of the Dirac type (2) , Fig.3.

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Appendix

1. Constants appearing in formulae (16) - (18) are following

$$g^{kin} = H(n-1)\,(f^{k,i+1,n-1} - f^{ki,n-1}) + H(n-2)\,g^{k,i+1,n-1}, \quad i=1,2,\ldots,N-1$$

$$g_L^{kin} = H(n-1)\,(f_L^{k,i+1,n-1} - f_L^{ki,n-1}) + H(n-2)\,g_L^{k,i+1,n-1}, \quad i=1,2,\ldots,N-1$$

$$g_j^{k1n} = f_j^{k2,n-1} + H(2n-3-j)\,g_j^{k2,n-1} - H(2n-4-j)\,f_j^{k1,n-1}, \quad j=0,1,\ldots,2n-2$$

$$g_j^{kin} = H(n-1)\left[f_j^{k,i+1,n-1} + H(2n+i-4-j)\,H(n-2)\,g_j^{k,i+1,n-1} - f_j^{ki,n-1}\right], \quad i=2,3,\ldots,N-1; \quad j=0,1,\ldots,2n+i-3$$

$$g_L^{kNn} = H(n-1)\,r_{N+1}\,f_L^{kN,n-1}/r$$

$$g^{kNn} = H(n-1)\,(r_{N+1}\,f^{kN,n-1} - f_L^{kN,n-1} - g_L^{kNn})/r$$

$$g_j^{kNn} = H(n-1)\left[-H(2n+N-4-j)\,f_j^{kN,n-1} + \frac{r+r_{N+1}}{j}\,f_{j-1}^{kN,n-1}\right], \quad j=1,2,\ldots,2n+N-3$$

$$g_0^{kNn} = -H(n-1)\,g^{kNn}$$

$$f^{k10} = -\frac{1}{k_0(1+\bar{D}_0)}\,\bar{a}_k$$

$$f_L^{k10} = -\frac{1}{k_0(1+\bar{D}_0)}\,\bar{b}_k$$

$$f^{k1n} = \frac{1-\bar{D}_0}{1+\bar{D}_0}\,g^{k1n} \quad \text{for } n \geq 1$$

$$f_L^{k1n} = \frac{1-\bar{D}_0}{1+\bar{D}_0}\,g_L^{k1n} \quad \text{for } n \geq 1$$

$$f_j^{k1n} = \frac{1-\bar{D}_0}{1+\bar{D}_0}\,g_j^{k1n}, \quad j=0,1,\ldots,2n-2$$

$$f_L^{kin} = \left[- k_{i-1} \bar{D}_{i-1} H(n-1) g_L^{kin} + 2 k_{i-1} f_L^{k,i-1,n} \right] / r , \quad i=2,3,\dots,N$$

$$f^{kin} = \left[2 k_{i-1} f^{k,i-1,n} - f_L^{kin} - H(n-1) (k_{i-1} \bar{D}_{i-1} g^{kin} + g_L^{kin}) \right] \quad i=2,3,\dots,N$$

$$f_j^{k2n} = - H(2n-1-j) g_j^{k2n} + \frac{1}{j} \left[(r - k_{i-1} \bar{D}_{i-1}) g_{j-1}^{k2n} + 2 k_{i-1} \cdot H(2n-1-j) f_{j-1}^{k1n} \right], \quad j=1,2,\dots,2n$$

$$f_j^{kin} = - H(2n+i-3-j) g_j^{kin} + \frac{1}{j} \left[(r - k_{i-1} \bar{D}_{i-1}) g_{j-1}^{kin} + 2 k_{i-1} \cdot f_{j-1}^{k,i-1,n} \right] \quad i=3,4,\dots,N; \quad j=1,2,\dots,2n+i-2$$

$$f_0^{kin} = - f^{kin} , \quad i=2,3,\dots,N$$

2. Constants appearing in formulae (19) and (20) are following

$$g^{kin} = H(n-1) \left(f^{k,i+1,n-1} - f^{ki,n-1} \right) + H(n-2) g^{k,i+1,n-1} , \quad i=1,2,\dots,N-1$$

$$g_L^{kin} = H(n-1) \left(f_L^{k,i+1,n-1} - f_L^{ki,n-1} \right) + H(n-2) g_L^{k,i+1,n-1} , \quad i=1,2,\dots,N-1$$

$$g_{sj}^{kin} = f_{sj}^{k,i+1,n-1} - f_{sj}^{ki,n-1} + H(n-2-j) \cdot g_{sj}^{k,i+1,n-1} ,$$

$$i=1,2,\dots,N-1; \quad s=1,2,\dots,i-1; \quad j=0,1,\dots,n-1$$

$$g_{ij}^{kin} = f_{ij}^{k,i+1,n-1} + H(n-2-j) (g_{ij}^{k,i+1,n-1} - f_{ij}^{ki,n-1}) ,$$

$$i=1,2,\dots,N-1; \quad j=0,1,\dots,n-1$$

$$g_{sj}^{kin} = f_{sj}^{k,i+1,n-1} + g_{sj}^{k,i+1,n-1} - H(n-s+i-2-j) f_{sj}^{ki,n-1} ,$$

$$i=1,2,\dots,N-1; \quad s=i+1,i+2,\dots,N; \quad j=0,1,\dots,n-s+i-1$$

$$g_L^{kNn} = H(n-1) \; r_{N+1} f_L^{kN,n-1} / r_N$$

$$g^{kNn} = H(n-1)\,(r_{N+1}\, f^{kN,n-1} - f_L^{kN,n-1} - g_L^{kNn})\,/r_N$$

$$g_{sj}^{kNn} = \frac{1}{r_s-r_N}\Big[(j+1)\,H(n-2-j)(f_{s,j+1}^{kN,n-1} + g_{s,j+1}^{kNn}) - (r_s + r_{N+1})\cdot f_{sj}^{kN,n-1}\Big], \quad s=1,2,\ldots,N-1; \quad j=n-1,n-2,\ldots,0$$

$$g_{Nj}^{kNn} = -H(n-2-j)\, f_{Nj}^{kN,n-1} + \frac{r_N+r_{N+1}}{j}\, f_{N,j-1}^{kN,n-1}, \quad j=1,2,\ldots,n-1$$

$$g_{N0}^{kNn} = -g^{kNn} - H(n-1)\sum_{s=1}^{N-1} g_{s0}^{kNn}$$

$$f^{k10} = -\frac{1}{k_0\,(1+\bar D_0)}\,\bar a_k$$

$$f_L^{k10} = -\frac{1}{k_0(1+\bar D_0)}\,\bar b_k$$

$$f^{k1n} = \frac{1-\bar D_0}{1+\bar D_0}\, g^{k1n} \quad \text{for} \quad n \geq 1$$

$$f_L^{k1n} = \frac{1-\bar D_0}{1+\bar D_0}\, g_L^{k1n} \quad \text{for} \quad n \geq 1$$

$$f_{sj}^{k1n} = \frac{1-\bar D_0}{1+\bar D_0}\, g_{sj}^{k1n}, \quad s=1,2,\ldots,N; \quad j=0,1,\ldots,n-s$$

$$f_L^{kin} = \frac{1}{r_{i-1}}\Big[2\,k_{i-1}\, f_L^{k,i-1,n} - k_{i-1}\,\bar D_{i-1}\, H(n-1)\, g_L^{kin}\Big] \quad i=2,3,\ldots,N$$

$$f^{kin} = \frac{1}{r_{i-1}}\Big[2\,k_{i-1}\, f^{k,i-1,n} - f_L^{kin} - H(n-1)\,k_{i-1}\,\bar D_{i-1}\, g^{kin} + g_L^{kin}\Big], \quad i=2,3,\ldots,N$$

$$f_{sj}^{kin} = \frac{1}{r_s-r_i}\Big[(j+1)(H(n-1-j)\, f_{s,j+1}^{kin} + H(n-2-j)\, g_{s,j+1}^{kin}) + k_{i-1}\,\bar D_{i-1} - r_s)\,H(n-1-j)\, g_{sj}^{kin} - 2\,k_{i-1}\, f_{sj}^{k,i-1,n}\Big],$$

$$i=2,3,\ldots,N; \quad s=1,2,\ldots,i-2; \quad j=n,n-1,\ldots,0$$

$$f_{i-1,j}^{kin} = -H(n-1-j)\, g_{i-1,j}^{kin} + \frac{1}{j}\Big[(r_{i-1} - k_{i-1}\,\bar D_{i-1})\, g_{i-1,j-1}^{kin} + 2\,k_{i-1}\, f_{i-1,j-1}^{k,i-1,n}\Big], \quad i = 2,3,\ldots,N; \quad j=1,2,\ldots,n$$

$$f_{sj}^{kin} = \frac{1}{r_i - r_s} \Big[- (j+1)\, H(n-s+i-2-j) \left(g_{s,j+1}^{kin} + f_{s,j+1}^{kin} \right) +$$

$$+ (r_s - k_{i-1} \bar{D}_{i-1})\, g_{sj}^{kin} + 2 k_{i-1} H(n-s+i-2-j)\, f_{sj}^{k,i-1,n} \Big],$$

$$i=2,3,\dots,N; \quad s=i,i+1,\dots,N; \quad j=n-s+i-1, n-s+i-2, \dots, 0$$

$$f_{i-1,0}^{kin} = - f^{kin} - \sum_{s=1}^{i-2} f_{s0}^{kin} - \sum_{s=i}^{N} H(n-s+i-1) f_{s0}^{kin} .$$

CONTENTS

1. Introduction 3
2. The physical model of a multi-storey building 4
3. Equations of motion and recurrence formulae 8
4. Numerical results 14
5. Conclusions 19
6. References 20
7. Appendix 21