Uroboros, or biology between mythology and philosophy
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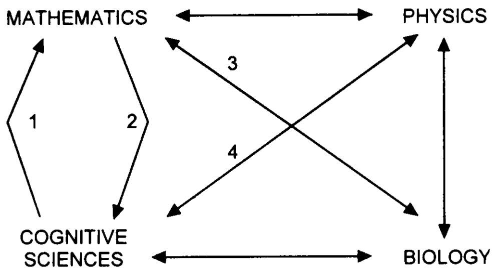
THE GEOMETRIES OF THREE ANCIENT KINGDOMS

1. OUR CONTEXT

If the present paper were to be read at a reductionist or positivist conference, the explanation of its purpose would have to go from mathematics to biology, via physics and chemistry.

But "Biology between mythology and philosophy" has a broader scope. It is neither positivist nor reductionist, but cross disciplinary, shall we say post-modern[1] – a study of the relation between life sciences and the rest of culture. Therefore, let us assume a different, non-linear classification of sciences, once suggested by Piaget[2]. As we can see in figure 1, it enhances the relations between scientific disciplines.



The classification of sciences according to Piaget:
1. The sciences of the artificial
2. Genetic epistemology

A post-modern non-reductionist extention:
3. Bio-mathematics
4. Mind-matter relationship

Fig.1

Our standpoint will be the one of biomathematics (most often bypassing the physical and chemical links); at the end of the paper, an argument in the sciences of the artificial will be presented.

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[1] J.C.B. Tiago de Oliveira, O que seria a pós-modernidade na Matemática e na Física, se existisse?, "Revista de Comunicação e Linguagens" 1988, no. 6-7.
[2] J. Piaget (ed.), Logique et connaissance scientifique, Paris 1969 Gallimard.

2. THE KINGDOMS OF NATURE

The division between the kingdoms of nature is classical and pertains to essentially all European and Asian thought. Four different types of soul, for instance, are described after Aristotle: the human (of which we will not talk), the animal, vegetal and mineral[3]. Natural sciences, the scholastic division of knowledge, and the structure of university departments are still remnant of this old differentiation. However, Leeuwenhoek's discovery of the unicellular world blurred such a clear distinction and today[4] independent new kingdoms, for example Bacteria, are recognized.

But there are instances of intermixing among the three ancient kingdoms of nature. To remind a few of them:

- viruses are alive, yet they crystallize
- Gaia's hypothesis about the Earth as a living organism
- the "chimeras" composed of cells containing both an animal and a vegetal nucleus, in the experiments of Lima de Faria
- and, above all, the work of Jagdish Chandra Bose[5] which has been perhaps the only endeavour to build a bridge joining these three kingdoms. He could do it by experimentally devising, along the animal/vegetal/mineral hierarchy, ways of finding, in the lower levels, properties of the upper ones: the fatigue doping, poisoning and dying of a galean receptor, as well as, for instance, the proof of electrical activity and autonomous movements in trees.

Bose's findings emerged in the context of his Indian philosophy, itself an integrated, unified vision of nature. However great the admiration of contemporaries (Burdon-Sanderson, Huxley, for instance) and followers, his path was not a trail for the consensus of the scientific community. But the roots of geometrization in the natural sciences simultaneously pertain to the three kingdoms; it was the crystallography school of Haüy[6] that inspired his students – Saint-Hilaire, and also De Candolle in his dream about plants as living crystals – to the realization of the idea of a unified plan for animals and of the bidimensional study of phytolaxia as the basis of plant geometry, respectively. In figure 2, for instance, we see the way Haüy discovered, by fracture, how different crystals have identical nuclei, an idea that is directly connected to the general plan for animals put forward by Saint-Hilaire. And, even if it seems

[3] T. Durali, Aristotle's thought concerning the problem of the living beings and their evolution, in: T. Zarcone (ed.), Individu et société. L'influence d'Aristote dans le monde Méditerranéen, Istanbul 1988 Isis, p. 5-30.
[4] G. Fleischaker, The myth of the putative 'first organism', this volume.
[5] P. Tompkins, C. Bind, La vida secreta de las plantas, México 1974 Diana.
[6] H. le Guayadar, A Filotaxia ou o sonho de cristal vivo, "Análise" 1986, v. 2, no. 4.

possible to configure natural beings in a unified fashion, the specificity of each of the three classes emerges in geometrical constructions and methodologies.

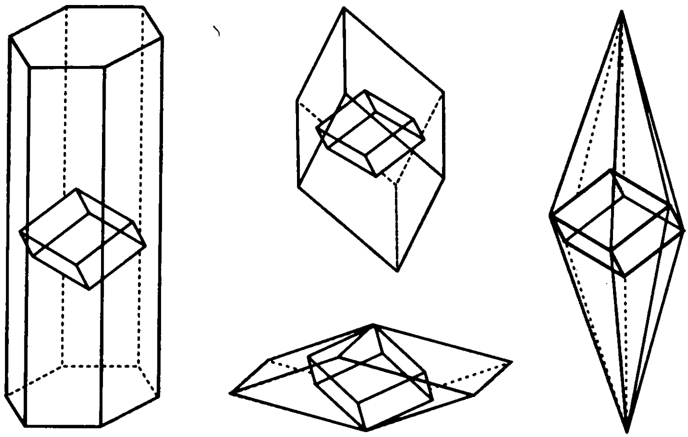

Fig. 2

3. "ON GROWTH AND FORM"

This book by D'Arcy Thompson[7] was, is, and will be the name of a great research program for finding the mathematical conditions that explain the emergence of the morphologies of the living. In fact, by its non-reductionist character, explanation is not bound to pass through physico-chemical steps; this program creates a new paradigm in transdisciplinary sciences, the connection along diagonals in Piaget's quaternary scheme.

Saint-Hilaire's general plan of the organization for animals is, for instance, the natural philosophy counterpart of an isomorphism in mathematics. On the other hand, as F. Hallé[8] once objected to René Thom, there is no common plan for plants as it exists for animals, yet they ramify recursively either by amputation or by their endogenous rhythm of growth; no two tropical trees in the same species are topologically alike, yet they are mostly self-similar. And the fact is that, since the times of De Candole and Bravais, the geometry of plants was treated as a bidimensional problem of ramification, according to an optimality principle.

[7] D'Arcy Thompson, On growth and form, Cambridge 1962 Cambridge University Press.
[8] F. Hallé, Les modèles architecturaux chez les arbres tropicaux, in: P. Delattre, G. Thellier (ed.), Elaboration et justification des modèles, Paris 1979 Maloine.

So, at the very beginning of mathematization in the science of nature, we can find:

- algebra (symmetry groups) for crystals
- smooth mappings (isomorphism) for animals
- recurrent automorphism (fractals) for plants.

The scope of application of fractal methodologies has grown during the last years. We shall rapidly survey how it became suitable for natural beings.

4. FRACTALS IN NATURE

Fractals were defined by B. Mandelbrot[9] as being self-similar, irregular, hierarchical objects, with non-integral dimensions. Classical examples include the Von Koch curve, a procedure with a zero area (after *N* steps of construction), diverging $(4/3)^N$ perimeter, and dimension log 4/log 3. Another example is the Cantor Set, with a zero length, infinite cardinality, discontinuous everywhere, and dimension log 2/log 3. Fractals are, therefore, "infinite objects" in the sense of Matsuno[10], while nature is finite. According to Dirac's Large Number Hypothesis, 78 orders of magnitude of elementary particles exist in the Universe; life has a thinner spectrum (some 20 orders between the amoeba and the whale). How can fractal formalisms be reconciled with finitude?

This question is probably solved by considering fractal geometry and a "structural science" in the sense of Küppers[11]. Provided that, in an object, three of four levels of an isomorphic structure emerge, even the unlimited complexity of fractals should prove useful. The articulation between mathematics and physical limitations is one of the major essential conflicts in science[12]. One classification of principles in physics was suggested by Drago[13]. A distinction is made between symmetry principles (essentially subsumed in a group theoretic formalism), hierarchies (ordered structures, as for instance, in irreversibility), and limitations. The latter have a vast, heterogeneous diversity: Planck's constant, the speed of light, absolute zero, the second law of thermodynamics, symmetry breakings, horizons in cosmology, etc.

In biology, at least, temporal and spatial limitations are pertinent: for all useful purposes, the unit of a limit being is the cell. Therefore, divergent exter-

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[9] B. Mandelbrot, The fractal geometry of nature, San Francisco 1982 Freeman.

[10] K. Matsuno, Beyond geometrization of biology, "Uroboros" 1991, v.1, no. 2, p. 117-139.

[11] B.-O. Küppers, Understanding complexity. Problems of reduction in biology, in A. Beckerman, H. Flohr, J. Kim. (eds.), Emergence or reduction, Berlin 1992 W. de Gruyter, p. 241-256.

[12] J.C.B. Tiago de Oliveira, O infinito para Matemáticas e na Fisica, in: M.A. Costa Leite (ed.), Pensar a ciência, Lisboa 1986 Gradiva.

[13] A. Drago, Limitation principles in physics, in: I.T. Frolov (ed.), Logic, methodology and philosophy of science, Moscow 1987.

nal measures will not appear; even in the most blossoming cases of growth, nowhere an infinite measure appears in an organism or a fractional dimension, in the lesser scale. The concept of subfractal[14] appeared as a way of approaching textures of the living and minerals in small scales; the idea is that the slope of a log *N*/log *R* regression eventually goes to zero with decreasing *R*, without the shape losing its fractal structure. An elementary construction, in the Von Koch stage, of a subjacent line with a finite perimeter is shown in figure 3.

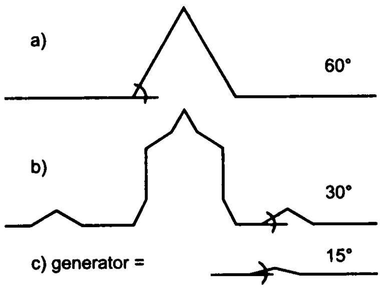

Fig. 3.

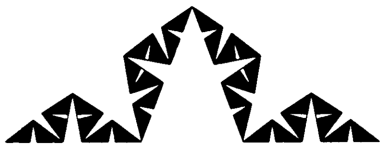

Fig. 4.

The inverse deformation on Von Koch's construction (sharpening of the apical angle) produces a fractal line with a finite area[15] as can be seen in figure 4; its dimension will be 2. These kind of objects (fat fractals, with integer dimensions) were described by Mandelbrot in a sentence: "Lebesgue-Osgood monsters are the very substance of our flesh" (as they configurate the respiratory and circulatory systems).

The next generalizations (and mathematical challenges) needed to adapt the fractals to the smoothness of the natural world include:

- fractals properties of differentiable lines, like the entropy of spinals, developped by Mendès-France
- rectifiable sets, as treated by Tricot
- non-integer differentiability, studied by Le Méhaute[16]

[14] J.P. Rigaut, in: G. Cherbit (ed.), Fractals dimensions non entéres et leur applications, Paris 1982 Masson.
[15] S. Dubuc, Courbes de Von Koch et courbes d'Osgood, "Review of Mathematics", Academy of Sciences of Canada, v. 5, August 1983.
[16] A. Le Mehauté, Les geometries fractales l'espace-temps brisé, Paris 1990 Hermes.

- sets with nontrivial contingents, paratingent in the sense of Bouligand
- the concept of local concavity (manifolds not decomposable in a finite number of locally-oriented convex segments).

All these concepts represented weakenings in the current definition of fractal objects, and their relationships is the subject of research in course. But, in each of the three realms of nature, further new questions appear.

5. GEOLOGY AND ANISOTROPY

A survey of current fractal methods in the geosciences[17] will display ideas which are common to physics, but have a further care with texture analysis over a small number of orders of magnitude. One of the procedures used, for instance, to qualify fractures, is the Cantor dust analysis[18]; its empirical application reveal anisotropic dimension, a feature that, with the geometric notions used, is an exception (the only polarized measure studied so far, Lovejoy's ellipsoidal dimension, has an axiality that does not occur in fractured soils).

Two opinions, so far, have been presented: the fractal model is insufficient (A. Ribeiro) and should de adapted; or the empirical anisotropy is due to poor resolutions (C. Tsallis). So far, the geometrical counterexamples based on Von Koch or fractal Brownian motion support this second view; but the question is open.

6. ZOOLOGY OR THE MONSTERS INSIDE

Bittner and Sernetz[19] devised a log. logistic law, modelling the eventual defractalization drift with cut-offs in a threshold (as in subfractals) as well as in an upper, through a family of log. logistic functions. There is good agreement with reality[20] in the description of the 30-times bifurcating blood system inside the human kidney; its parameters agree with a hydraulic optimality principle, and conciliate the local laminarity of blood flow with a global turbulence, the attraction of which is the interior of the vessel system.

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[17] C.V. Middleton (ed.), Nonlinear dynamics, chaos and fractals, Toronto 1991 Geological Association of Canada.
[18] B. Veide et al., Fractal analysis in rocks. The Cantor dust method, "Technophysics" 1990.
[19] K.R. Bittner, M. Sernetz, Self-similarity within limits, in: M.O. Peitgen, J.M. Henriques, L.P. Penedo (ed.), Fractal 90, Amsterdam 1991 Elsevier.
[20] H.K. Bittner, Modelling of fractal vessel systems, ibidem.

7. ARTIFICIAL BOTANY

Lindermayer systems are grammars of parallel rewriting, and have proved to be[21] an easy way to represent fractals for a turtle-graphical language like LOGO. In figure 5, it is seen how, with essentially three symbols (*F* = forward, +/- = left/right), an axiom (initiator) and a rewriting rule (generator), a fractal is devised. Two further symbols (brackets) enable to signal, respectively, the beginning and endings of ramifications (figure 6). And it is amazing to see[22] how the successive states of growth in bracketed systems describe the actual development of plants; to know that, for instance, Hallé's classification of architecture of tropical trees is completely described by axiomatic change. Therefore, a further step into an artificial approach to life might be taken.

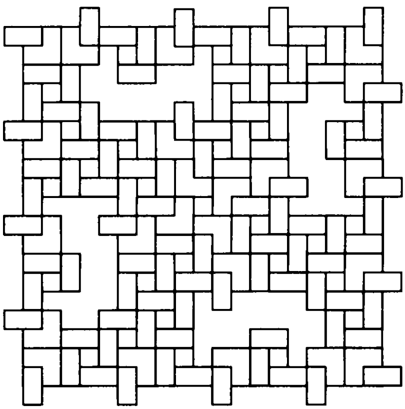

Fig. 5

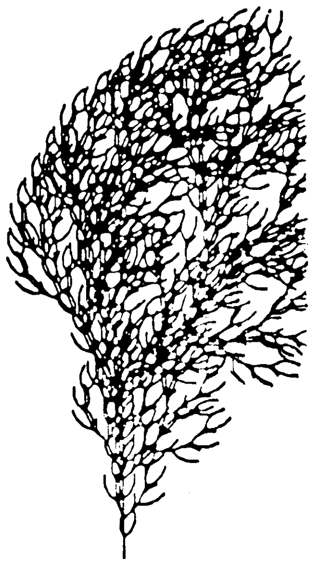

Fig. 6

[21] B. Prusinkiewicz, A. Lindenmayen, The algorithmic beauty of plants, Heidelberg 1991 Springer.
[22] K. Kelley, Designing perpetual novelty, in: J. Brockman (ed.), Doing science, New York 1991 Prentice-Hall.

8. *GEDANKENEXPERIMENTS* WITH IDEAS AND COMPUTERS

In their modelling of the embryogenesis of a worm (*Aenorhabditis elegans*) Bailly, Gaill and Mosseri[23] devised the following characteristics for a mathematical formalism (axioms) to be suitable for living dynamics, as the embryo level:

- an endomorphism (for autonomy)
- discrete (temporal alternation of activity/latency)
- recursive (internal economy)
- nonlinear (complexity growing with scale).

An additional postulate will demand this formalism to be quadratic, so that it may represent antagonistic structures, as devised by Bernard-Weil[24] and otherwise encode the binary issues. patent in symbolic dynamics[25]. Such a model exists, since the discovery by Feigenbaum of the doubling-period route to chaos, in the sequence $X_{N+1} = (RX_N)(1 - X_N)$. In their model, the authors identify each doubling with mitosis, and the chaotic zone with the differentiation of cellular lineages.

The adjunction of two extra hypothesis and a different space will be our final experiment (and adventure) with mathematical ideas:

- R changing with N, alternating between two values, A, B, with a periodicity T
- Lyapunov exponents[26].

For each (X, A, B) a measure of stability (convergence of iterates) and chaos is displayed in the A, B plane, with the sign of the exponent represented graphically. On the computer screen, the output of these experiments shows the emergence of connected, unexpected life-like and lively patterns. They seem too simple, for period 2-5; too complex to be real, for periods greater than 20; yet intuitively more similar to nature, in the 5-10 rhythmicity area.

Perhaps this type of experiment can correlate pattern with a measure of complexity (periodicity and structure in the modulation). Possibly also, the fractal way of seeing will therefore be an efficient way of simultaneously appreciating the unity and beauty of each natural object in the light of science.

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[23] F. Bailly, F. Gaill, R. Mosseri, Hierarchie de niveaux d'organisation fractales, Paris, CNRS (preprint).
[24] E. Bernard Weil, L'Arc et la corde, Paris 1977 Maloine.
[25] J. Sousa Ramos, Hiperbolicidade e bifurcação de sistemas simbólicos, Ph.D., Lisboa 1990.
[26] R. Markus, Chaos in maps with continuous and discontinuous maxima, "Computers in Physics" 1990, September-October.